QUANTITATIVE INTERRELATION OF EROSION AND RIVER REGIME BY REGIME THEORY METHODS

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SUMMARY

The three basic regime theory equations for self-adjustment of controlled sediment-bearing channels of small bed-load charge are stated. The dynamical significance of these and derived relations is discussed. Practical extension to large bed-load charge and to the unsteady meandering conditions of rivers is outlined. Case histories illustrate that most problems of erosion and deposition are soluble to a useful degree of accuracy by calculation or by model.

RÉSUMÉ

Les trois équations fondamentales de régime pour les chenaux contrôlés transportant des sédiments avec faible charge sont présentées. La signification dynamique et des relations dérivées en sont discutées. Une extension pratique aux fortes charges de sédiments et aux conditions non permanentes des rivières avec méandres est mise en relief. Il est montré que la plupart des problèmes d'érosion et de dépôts peuvent être résolus, avec un degré utile suffisant, par calculs ou par modèles.

INTRODUCTION

For scientific study most practical erosion problems reduce to those of transport of incohesive bed-material and of overcoming the cohesion of erodible side-material. The attack on the combined problems has to follow the inductive methods of classic physics since the mechanism of turbulent flow is unknown even for the relatively simple problem of flow of a pure Newtonian fluid in a circular pipe. Laboratory investigations to date have the major limitations that (i) they do not deal with either erosion from or deposition on sides, (ii) they may show a phase of transport that does not occur in the field, (iii) there are gaps and indefinitenesses. Regarding item (ii) Bhattacharya (1) quotes Indian references and has demonstrated that material even slightly coarser than coarse sand can be transported in bed-movement without dunes in a laboratory flume; he found also that, as discharge intensity increases, the bed-load charge below which duneless behaviour occurs decreases progressively till behaviour comes into agreement with prototype expectation. So transport formulas based on laboratory data may not apply in the field; examples are given by Bondurant (2) for suspended load and by Blench (3) for bed-load. Field studies have the outstanding handicap that bed-load cannot be measured; their advantages include accurate measurement of some quantities, very large range for others (table 1), and a multitude of unassociated observers over many years.

The present paper concerns, on the one hand, the scientific quantitative field approach in term of channel self-adjustment, as developed by Gerald Lacey initially (4,5) and, on the other, its application in the field where, although the basic principles still apply, there are various disturbing factors calling for empirical coefficients (some based on carefully chosen laboratory observations) and caution in posing the problems so as to avoid pitfalls. The basic approach is known as "regime theory", in which "theory" has its dictionary implication of "a system of" and not the dictionary alternative of "speculation". The basic and applied materials are presented separately because the author believes that the basic development by Lacey has avoided the

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pseudo-scientific speculations of engineering fluid mechanics and produced functional forms that are dynamically correct for certain simple conditions. Only an outline of essentials is attempted since a two semester graduate course is needed for detailed treatment; fuller information is in references 4,5. Nothing in the paper concerns a permanently immobile bed; the difference between a bed that has not started to move and one that is moving or has ceased to move is as radical as that between a flow that is laminar and one that is turbulent or is on the verge of ceasing to be turbulent.

**Regime theory basis**

When canals carrying a bed-load of sand and a suspended load of finer materials are built in alluvium to no other rule than a conventional flow formula of rigid boundary hydraulics they refuse, in general, to remain as designed. If built too tight or too broad they widen or contract (by depositing suspended material on the banks). If built too deep or shallow they adjust depth. If built too steep or too flat they degrade or aggrade. In fact, they gradually adjust breadth, $b$, depth $d$, and slope $S$ to obey laws concerning erosive action on sides, transport of bed-material in the presence of suspension, and the equality of resistance to driving force; this last corresponds to the flow formula of rigid boundary hydraulics which has so many competing forms because it cannot be deduced from first principles.

Basic regime theory states, as three formulas, certain discoveries about the self-adjustment of the three dimensions $b$, $d$, $S$ of such channels to final equilibrium values. Its data were subject to major conditions: (i) bed-load charge $C$, was very small. ($C$ is measured in parts per hundred thousand of the ratio of weight discharge of bed-load to weight discharge of water). (ii) Suspended load varied seasonally up to about 1% by weight of the water and its effect, though noticeable, could be averaged out in determining functional forms. (iii) Discharge was effectively constant. (iv) Meandering was prevented. (v) Sides were cohesive and erodiible. (vi) Bed-load was of sand and moved in dunes.

**Basic regime forms and their significance**

Algebraic manipulation allows any three basic independent regime formulæ to be converted to three independent alternatives. The following three are selected and worded to illustrate that there seems to be remarkable dynamical significance and simplicity and a generalisation from rigid boundary hydraulics.

**First.** Channels having the same intensity of load adjust themselves till the Froude Number in terms of a representative depth is the same for all. This observation is used to define a practical bed-factor:

$$ F_b = V^2/d $$

where $V$ is mean velocity of flow and the depth $d$ is an average that smooths out the dunes.

**Second.** Channel banks will erode when $q \mu V^3/b$ exceeds a limit depending on their nature, and will grow when the factor falls short of a lower limit depending on the suspended load: $q$ is mass density and $\mu$ is viscosity. This fact is used to define a practical side factor:

$$ F_s = V^3/b $$

**Third.** Channel slope will adjust till the rate of working by gravity per unit mass per unit time:

$$ gVS = fn(F_b, VF_s) $$

where $V$ is kinematic viscosity.

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Immediately obvious features of these relations include the following. The Froude Number in terms of depth, which dimensional analysis indicates must have a meaning, has a particularly simple application. The factor that concerns bank erodibility is a standard expression of rigid boundary hydraulics when dealing with the square of the shear stress in a laminar film; so it seems that the canals examined had "hydraulically smooth" sides as appearances suggest. The rate of working criterion, independent of discharge, has the simplicity and appearance of the kind of minimal principle that occurs in classic physics and dynamics to express laws of nature. Incidentally it contradicts the two common mutually contradictory engineering beliefs that self-adjustment occurs, for a given bed-load intensity of given material till (a) the velocity (b) the shear stress at the boundary is fixed.

Not so obvious, but very significant, results arise from manipulating the three equations to give:

$$V^2/gdS \alpha (Vb/v)^{3/4}$$  \hspace{1cm} (4)

$$V \alpha (d/v)^{1/4} \sqrt{g/dS}$$  \hspace{1cm} (5)

where $x = (Vb/v)^{1/2}/F_b$.

Equation (4) is a generalised Blasius smooth pipe one. Figure 1 shows it plotted on a generalised standard Stanton friction factor diagram for a range covering most regime data, labelled "King Data Range". On the same diagram are samples showing extreme and mean data from the well-known Nikuradse (labelled N) and Colebrook-White (labelled CW) pipe observations and several plots from large hydroelectric
conduits. It is to be noted that canals in the field, and large conduits, cover a range right outside that of the laboratory and that the dotted extrapolation of the Nikuradse smooth-pipe data (which yields the speculative and dynamically impossible logarithmic flow formula of most fluid mechanics texts) does not receive convincing support from large conduits. The remarkable feature of finding that a generalised “smooth boundary” flow formula is deriveable from the experimental data is that it was not foreseen. Yet, in retrospect, whatever formula was found should have been expected to be of smooth boundary type since “smooth” in rigid boundary hydraulics is a relative term meaning that the flow adjacent to the boundary is in a special phase of flow (laminar) which permits the shear stress there to be expressed in terms of the fluid property viscosity and not at all in terms of the material of the boundary which acts only as an anchor. In bed-load transport the fluid is a water-sediment complex in which the coarser part of the sediment differentiates out to the lower zone of flow, the boundary is effectively a special phase of the complex, and the resistance has to be expressible in terms of the complex — explicitly or implicitly. Unless the nature of the flow were radically different from that associated with a rigid boundary a dynamically significant flow formula has to be a generalisation of that for rigid boundary.

Equation (5), unlike its alternative expression equation (4), is of “rough boundary” type with an equivalent roughness height”, which is what one look at the dunes would cause nobody to expect. Presumably x measures an effective roughness height and we note that it shows that effective dune height decreases as \( F_b \) increases. This is consistent with the fact that, as the Froude Number increases to unity, at which critical flow occurs, dunes flatten and finally vanish. At some Froude Number greater than 1.0 antidunes start. “Rough”, in rigid boundary parlance, means that the shear stress at the boundary is expressible in terms of “roughness height”, and equation (5) is merely equation (4) with the boundary properties, as defined by \( F_b \) and \( v_f \), made explicit. Apparently the regime boundary is “smooth” and “rough” at the same time. The coexistence of these states is concordant with belief in a universal flow formula which is, in turn, concordant with belief in a universal velocity distribution formula since a flow formula can be derived by integrating a velocity distribution one. The universal flow formula postulate is discussed (4,10).

The criterion \( (\sqrt{V^2/gdS}) \div (Vb/y)^{1/4} \) from equation (4) may be called the King Number after its founder (4) and is 3.63 if \( d \) is depth from surface to bed and \( b \) is mean breadth in terms of \( d \); more generally it will be a function of suspended load charge.

Apart from their obvious dynamical significance it seems that the basic regime equations permit a generalisation of rigid boundary ideas. It is particularly interesting that they support the functional form of the Blasius smooth boundary formula against the more modern speculative logarithmic one which is based on what Sutton (9) calls the “crude empiricism of the mixing length formulas”.

**Derived regime forms for canal practice**

For practical applications the basic equations (1, 2, 3) are inconvenient. The first two convert to:

\[
b = \sqrt{F_b Q/F_s}
\]

(6)

\[
d = \sqrt{F_s Q/F_b^2}
\]

(7)
The third yields, without the functions of \( C \):

\[
S = \frac{5/6}{F_{bo}} \left( \frac{F_a}{KQ^{1/6}} \right)^{1/12} f'(C)
\]

\[
= \frac{7/8}{Kb^{1/4}d^{1/8}} f''(C)
\]

\[
= \frac{11/12}{Kb^{1/6}Q^{1/12}} f'''(C)
\]

The three functions of \( C \) have been introduced to allow for bed-load charge \( C \), were found by analysing what seemed the legitimate portions of standard laboratory data from all available sources, are rough and empirical and are devoid of any known dynamical meaning. As \( C \) tends to zero the functions all tend to unity and the equations then become those derived from equation (4) and have the full dynamical significance of the basic regime formulas from which they were derived. Values of the \( C \) functions can be found from formulas and graphs in references (4,5). In fact many natural channels, even with suspended loads up to 1% seasonally, display functional values exceedingly close to 1.0; however, values up to about 3.0 are possible. \( K \) is 3.63g/m\(^{1/4}\) if \( d \) is depth from surface to average level bed and \( b \) is mean breadth in terms of \( d \); it is not much different if \( b \) is surface breadth and \( d \) is mean depth in terms of \( b \). For streams in hot countries \( K \) may be taken as 2,000, and in cold countries 1800 for calculations about average annual behaviour. The term \( F_{bo} \) means the value of \( F_b \) for \( C \) tending to zero. In the practical range the rough empirical formula:

\[
F_b = F_{bo}(1 + kC)
\]

can be used, where \( k \) is about 0.12 for natural bed materials and is considerably different for the uniformised materials used in some laboratory experiments.

In practical application the equations (8) are transport ones with about the same background as the various non-regime formulas — du Boys, Meyer-Peter, Straub, etc. That background is small straight flumes, artificial bed-load constitution and no suspended load.

**Applications to River Problems**

Rivers may be considered as canals that have meandered and are subject to wide variations of discharge and charge. The laws that apply to canals to apply them, but there is practically never time for \( b, d, S \) to adjust to any particular discharge. There are disturbing factors such as log jams, ice jams, heterogeneous banks, snags and slides. However (4) there are good reasons to think that suitable ruling dimensions can be related to a “dominant” or “formative” discharge by formulas of the same functional forms as the basic regime ones, but with correction coefficients. A special coefficient is needed in the slope formulas (8) to allow for meandering and, pending proper research on the matter, the author assumes that meandering of average intensity approximately doubles the slope indicated by the basic formula. The same correction is needed in non-regime transport formulas although it is seldom applied in textbooks. Coefficients applied to straight-channel depths can be used to estimate scour depths round standard bends and at standard obstacles such as bridge piers and spur noses, at least as limits. In many practical problems a few observations on a river yield its own coefficients, or problems can be stated to permit unknown coeffi-
The case histories below will indicate that cautious application of theory is relatively simple provided the limits of strict applicability of the basic formulas are kept in mind.

Meander dimensions require a formula not contemplated by basic regime theory. The Ingls formula:

\[ L_m = \text{const.} \, Q^{1/2} \]

is statistical. \( L_m \) is meander wave-length and \( Q \) is a representative discharge, e.g. the 100 year flood. The author prefers to amend it to:

\[ L_m = \text{const.} \, \sqrt{F_b \, Q/F_s} \]  \hspace{1cm} (9a)

when he believes that a working bed-factor and a working side-factor can be estimated. The formula may be taken to state that, all other things being the same between two rivers, meander length is as the square root of representative discharge.

**Case Histories**

(i) **Breadth and depth variation “along the river”**

Leopold and Maddock (7) analysed data of river gauging sites by systems at representative discharges for each site. Most of the systems of their figure 9 are fitted, within the limits of statistical significance, by:

\[ b = A Q^{1/2} \]  \hspace{1cm} (10)

\[ d = B Q^{1/3} \]  \hspace{1cm} (11)

where \( A, B \) have fixed values for any one system. The regime theory explanation of equation (10), for a system whose channels have different bed materials, is that a large bed-factor is likely to be associated with a large side-factor (e.g. gravel bed, gravel banks), so \( F_b/F_s \) in equation (6) will not be very variable and its square root will be even more steady. Also, in any one system, the hydrograph form is unlikely to vary much between channels, so the use of a standard discharge statistic is consistent. When both equations (10) and (11) hold very accurately then \( F_b \) and \( F_s \) should be fairly uniform through a system. Stated otherwise the equations say that “All other factors but discharge being fixed for a set of channels, breadth and depth vary as the square root and cube root respectively of representative discharge”. Some algebra converts this into the regime equations (6, 7).

(ii) **Effect of removing suspended load from a river**

The closure of Hoover Dam, U.S.A., in 1938 removed suspended load completely from the Colorado river immediately downstream. The effect of this on the gauging site at Yuma has been plotted strikingly by Leopold and Maddock (7). Accepting their maximum discharge for this plot as bankfull, and computing bed and side factors for this discharge by equations (6, 7), it seems that the side factor changed from about 0.4 to 0.15, the bed-factor changed from about 6.1 to 1.6. As the intervening large reservoir must have been trapping bed-load before Hoover Dam was built it seems that changes of side factor must be attributed to suspended load removal alone. The magnitude of the change is consistent with knowledge of canal side factors, and its existence agrees with fairly general opinion that sandy banks can be held together by depositions of suspended load and will fall apart when the depositions
cease. The bed-factor changes can be attributed only to something like a cutoff, or interference with river works.

A further known case is where soil-conservation practices were followed by a river of about 100,000 cusecs peak flood and sandy banks doubling its incised channel breadth within a few years over several miles. At the same time specific gauges dropped—i.e. the river degraded slightly in terms of water-levels—although bedlevels rose very conspicuously. Equations (6, 7) show that the river might broaden and become more shallow from either bed-factor increasing or side-factor decreasing; but the equations (8) show that the former cause would require the river to aggrade whereas the latter would require it to degrade. So the conservation practices, which consisted mainly of altering cultivation to stop sheet erosion, must have reduced the suspended cohesive load in the river; in fact the reduction of side factor to about one quarter is comparable with what happened on the Colorado river, as described above.

(iii) Effect of binder on banks

Schumm (8) has correlated $b/d$ of river channels with weighted mean proportion of binder (defined necessarily in a somewhat arbitrary manner) in banks; the weighting was done in terms of bed and bank lengths exposed to flow in a section. The regime equations lead to

$$F_{g}^{1/3} \alpha dS \cdot f'(C) = b^{1/3}$$  \hspace{1cm} (12)

A plot of proportion by weight of binder in the banks alone against $dS/b^{1/3}$ on single log paper was found, by the author, to show wide scatter but to indicate a rough rule of side-factor varying as the 0.7 power of binder proportion. The wide scatter had to be expected from having to omit $f'(C)$ because $C$ was not measured, making no allowance for degree of meandering, etc.

(iv) Inevitability of regime slope

Highway engineers are notorious for unlined ditches along new highways, and the consequent canyoning. In fine-grained soils regime slopes of the order of 1 foot per mile are inevitable. A comparable large-scale case occurred when very heavy coniferous forest was stripped for a Canadian townsite. The natural drainage channel for an estimated 500 cusec peak had maintained itself, with the aid of deadfalls, at some 50 ft drop in a mile through silty soil. Temporary drops in the cleared channel washed out violently in the first flood and the channel is now controlled by concrete drops and runs in regime between them at about 1 foot per mile grade. In another case a reservoir spillway designed for about 100,000 cusecs discharged down a valley of glacial till with occasion rock outcrops; the valley had never carried more than a trickle of water. After 10 years there is now a 20 ft drop beyond the small outcrop on which the spillway is built and a drop at each rock outcrop downstream; regime slope has been practically attained. This was all foreseen and allowed for.

(v) Effect of meandering on regime slope

In two cases meandering gravel torrents were canalised straight through a new townsite on the assumption that their new regime slopes would be one half the old; rock rapids were provided to allow for this. They have not changed grade in the ten years of their existence.

Assisted cutoffs have been designed for sand and for gravel rivers, in Canada, using equations (8) and allowing the standard meander correction coefficient of two. All have developed and none has “silted up” instead of developing.

Cases are recorded of streams being “improved” by straightening and of then cutting down where straightened and depositing sand on the flood plain downstream.
That this must happen is an obvious deduction from equations (8) applied in the straightening and downstream. It is obvious also that the river will eventually return to its initial regime unless the straights are prevented from meandering.

(vi) Effects of barrages

Foy (9) has analysed the effects of long-term barrage building on the Indus system and found that all rivers tend to rise to attain approximately their old slopes. Deviations of slope are explicable by eqns (8). Canals offtaking from the barrages reduce dominant discharge downstream and cause slightly enhanced eventual slope; also, canal heads are designed to exclude sediment, so the bed-factor of a river downstream of a barrage increases and a further increase of regime slope occurs. Thus it is commonly observed that, immediately after building a barrage specific gauges downstream drop rapidly during a couple of years and then gradually recover till, eventually, they are higher than initially.

Engineering literature sometimes suggests that engineers who create large reservoirs cannot believe that the consequent delta must work upstream indefinitely as well as into the lake, but they do not have the time sense of geologists. To assume that it does not is to assume that, as a river lengthens its delta into the reservoir, it must flatten its regime slope without reason, in contradiction of equations (8), so as to maintain a fixed elevation of flow at some fixed upstream point for ever. Although the ultimate fate of the river is clear, despite the very long time required for it to fill up its flood plain as well as its bed, the detailed mechanism of deposition is complex. It seems that, as coarse bed material drops out at the upstream end of the backwater curve, finer material is left to deposit further downstream, so the slope that develops into the advancing delta is less than the original regime slope of the river. Analysis suggests that aggradation by means of relatively finer bed-material will proceed up a river at a mile or so per annum, when load is considerable, till a control point is reached; thereafter the normal size material will move downstream in an grading wave. The phenomenon is somewhat comparable with a surge moving upstream till it is reflected from a control point, but the "sediment surge" requires years or decades to reach a reflecting point.

(vii) Stability of multiple channels

Islands in mobile bed streams are usually impermanent, but sometimes a channel bifurcation remains remarkably permanent. Two permanent bifurcations of a major tidal river in Canada were studied quantitatively. One offtake had finer sand than its parent, the other had coarser. These differences are normal consequences of curvature of approach flow and occur in all systems. Since degrees of meandering of the channel were comparable the equations (8) could be applied with the same meander correction coefficient which then cancelled out. It was found that, replacing bed-factors by their equivalents in terms of sediment size, the equations (8) showed the dominant discharges to be closely in accord with measurement. Had the approach conditions upstream been able to change, the delicate distribution of sediment into offtakes would have been upset and the relative channel capacities would have altered progressively till the conditions which determined sediment balance struck a new regime or till secondary channels died completely.

(viii) Models

A set of European and American models, some with artificial light-weight bed material, was analysed by plotting the ultimately accepted breadth and depth scales against each other (12). By equations (4) and (5), accepted as valid for modelling subject to conditions explained in (9):

\[ d = (F_b)^{1/3} b^{1/3} \]

(13)
where $R$ means that each term is the ratio of model to prototype value. It was found that almost all the points fell in a band corresponding to this equation with the bracketed expression allowed to range from $\frac{1}{4}$ to 2; five tidal models fell on the line for the bracketed expression equal to 1.0. There is a fairly well-known "empirical rule" that the depth scale of a model should be the two-thirds power of the breadth scale. This rule is virtually an instruction to make the bed and side factors the same in model and prototype. There is some merit in this procedure but it is not strictly necessary: for tidal models general opinion is that the Froude Number in terms of depth (i.e. the bed-factor) should be the same in model and prototype otherwise there is little hope of scaling tidal currents.

**CONCLUSION**

This necessarily brief exposition aims largely at showing that there is an inductively derived and dynamically suggestive set of formulas that permits solving, to a useful degree of quantitative accuracy, most problems of river regime. A major utility of such a system is that it avoids the almost inevitable growth of wrong physical concepts occasioned by the use of speculative or groundless formulas. Of course, no set of equations true for ideal conditions can be of much practical use without knowledge and experience of the complex natural conditions to which application has to be made. Thus the author insists that graduates specializing in river engineering must study geomorphology under a geologist in addition to acquiring fundamental hydraulic knowledge concerning both rigid and mobile boundary flow; thereafter engineering experience will permit expertise to be achieved. Reference (11) indicates some of the consequences of inadequate basic knowledge.

**TABLE 1**

*Comparative ranges of some regime data*

<table>
<thead>
<tr>
<th></th>
<th>Indian Canals</th>
<th>Flumes of Gilbert</th>
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<td>D mm</td>
<td>0.10 — 0.60</td>
<td>0.30 — 7.0</td>
</tr>
<tr>
<td>Grading</td>
<td>Log. prob.</td>
<td>Uniformised</td>
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<tr>
<td>$C$ per $10^5$</td>
<td>0 — about 3</td>
<td>0 — 3,000</td>
</tr>
<tr>
<td>Suspended</td>
<td>0 — 1 %</td>
<td>Nil</td>
</tr>
<tr>
<td>Water temperature</td>
<td>50 — 86 °F</td>
<td>Prob. 55 deg. F</td>
</tr>
<tr>
<td>Sides</td>
<td>Clay, smooth</td>
<td>Wood or glass, smooth</td>
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<tr>
<td>$b/d$</td>
<td>4 — 30</td>
<td>1 — 25</td>
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<tr>
<td>$V^2/d$ ft/sec$^2$</td>
<td>0.5 — 1.5</td>
<td>1 — 150</td>
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<tr>
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<td>$10^6$ — $10^8$</td>
<td>$10^5$ — $10^6$</td>
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<tr>
<td>Bed phase</td>
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REFERENCES


