HEAT TRANSFER COEFFICIENTS FOR NATURAL WATER SURFACES

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SUMMARY

It is shown in this paper that heat losses from open water surfaces will be approximately proportional to the difference in temperature between the water surface and the air, provided the energy gained by short-wave radiation equals that lost by evaporation. The heat transfer coefficient for a small, sheltered lake at Ottawa was measured during 4 autumn cooling periods and found to be 23.4 cal/sq cm/24 hr/°C. Calculated values from other studies for latitudes 40 to 60°N indicate that the coefficient varies from 40 to 60 cal/sq cm/24 hr/°C for average exposure conditions. Lower values occur when short-wave radiation exceeds evaporation loss; the higher values during mid-winter conditions result when evaporation loss exceeds short-waves radiation.

RÉSUMÉ

On indique dans cette étude que les pertes de chaleur, qui se produisent à la surface des étendues d'eau, sont approximativement proportionnelles à la différence de température entre la surface de l'eau et de l'air, à condition que l'énergie obtenue par radiations (ondes courtes) soit équivalente à celle perdue par évaporation. Le coefficient de transfert de chaleur pour un petit lac abrité à Ottawa a été mesuré durant quatre périodes de refroidissement en automne et il s'est trouvé être de 23.4 cal/cm^2/24 hr/°C. Les valeurs calculées provenant d'autres études, pour des latitudes allant de 40 à 60°N, indiquent que le coefficient varie de 40 à 60 cal/cm^2/24 hr/°C dans des conditions moyennes d'exposition. Des valeurs plus faibles sont obtenues lorsque les radiations (ondes courtes) sont supérieures aux pertes par évaporation; au cœur de l'hiver des valeurs plus élevées sont obtenues lorsque les pertes par évaporation sont supérieures aux radiations (ondes courtes).

A method of estimating the rate of heat loss from water surfaces is needed for such problems as predicting frazil and surface ice formation or determining the heat required to keep ice from forming under given weather conditions. Newton's Law of cooling, which states that the rate of cooling of a body is proportional to the difference in temperature between the body and its surroundings, has been the basis of many empirical formulae used in practice. Because of the difficulties associated with measuring the rate of heat loss from natural water surfaces, it is usually not feasible to check the accuracy and limitations of these formulae. It is the purpose of this paper to present some measurements of heat losses from a lake and to use these measurements, along with other observations reported in the literature, to evaluate the validity of Newton's Law of cooling for predicting the rate of heat loss under given weather conditions.

BASIC PRINCIPLES

The reason for the relationship between rate of heat loss and the difference in temperature of a body and its surroundings has been explained for a dry surface under laboratory conditions. Under these conditions heat loss is by convection and long-

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wave radiation. The heat loss by convection is proportional to $h\ (T_o - T_a)$, where T_o and T_a are the temperatures of the body and the air surrounding the body, and h is a coefficient which will vary with the shape and dimensions of the surface and the characteristics of the air stream. The heat loss by long-wave radiation is proportional to $\alpha\ (T_1^4 - T_2^4)$, where T_2 and T_1 are the absolute temperatures of the body and its surroundings, and α is a constant that depends on the emissivity of the surface. If the temperature difference $(T_1 - T_2)$ is small in comparison to T_1 or T_2, the expression for long-wave cooling will be approximately equal to $\alpha\, 4T_2^3\ (T_1 - T_2)$ (1). Thus both convection and long-wave radiation components of heat transfer are approximately proportional to temperature difference.

Although there is justification for the use of this relation for a dry surface under laboratory conditions it is not so apparent why there should be a correlation between rate of heat loss and temperature difference for a wet surface under atmospheric conditions. Under these conditions the surface can receive heat by short-wave solar radiation and condensation and lose it by evaporation. Solar radiation varies with cloud amounts, latitude, time of year, condition of surface — factors not dependent on the difference between surface and air temperature. The rate of evaporation depends on vapour pressure gradient, wind velocity, surface roughness and surface temperature, the latter through the dependence of saturated vapour pressure of air on temperature. As solar radiation and evaporation are essentially independent of air-surface temperature difference, it is necessary to consider the heat balance at a water surface in detail in order to determine under what conditions Newton's Law of cooling is valid.

Calculation of Over-all heat transfer coefficient

The net loss of heat from a water body for a given period must equal the heat lost by long-wave radiation, convection and evaporation less the heat gained by short-wave radiation. This requirement can be expressed by the following energy balance equation :

$$+Q_t = Q_{lw} + Q_e + Q_c - Q_{sw} \tag{1}$$

where

Q_t = total heat loss
Q_{sw} = net short-wave radiation
Q_{lw} = net long-wave radiation
Q_e = heat associated with evaporation
Q_c = heat associated with convection

Using available formulae, simplified expressions for Q_{sw}, Q_{lw}, Q_e and Q_c are as follows :

(a)

Q_{sw} = (1 − a) R_{sw} cal/sq cm/24 hr

where

a = albedo of surface
R_{sw} = incoming short-wave radiation (cal/sq cm/24 hr)

(b)

Q_{lw} = 12.0 ΔT cal/sq cm/24 hr
ΔT = $T_w - T_a$

where

T_w = temperature of water surface (°C)
T_a = temperature of air (°C)

(This formula was obtained from Johnsson's approximation for long-wave radiation; it assumes the surrounding objects and terrain to be at the same temperature as the air, and water to have an emissivity of 1.0 (2).)

(c)

$$Q_e = 23\ \Delta e \text{ cal/sq cm/24 hr (wind speed of 160 km/day) at 2-metre level}$$
$$= 38\ \Delta e \text{ (wind speed of 320 km/day)}$$

where

$\Delta e = (e_w - e_s)$

e_w = saturation vapour pressure of air at temperature of surface (mb)

e_s = vapour pressure of air (mb)

(This simplified formula for evaporation from Penman [3])

(d)

$$Q_c = 14\ \Delta T \text{ cal/sq cm/24 hr (wind speed 160 km/day)}$$
$$= 24\ \Delta T \text{ cal/sq cm/24 hr (wind speed 320 km/day)}$$

where

ΔT = difference in temperature between air and water surface (°C) (obtained from Q_e by assuming that Bowen's ratio [4] is valid)

Substituting these expressions for the various components of the heat balance into equation (1) gives :

$$Q_t = (1 - a)\ R_{sw} - 23\ \Delta e - 26\ \Delta T \tag{2}$$

(cal/sq cm/24 hr) (wind speed 160 km/day)

and $$Q_t = (1 - a)\ R_{sw} - 38\ \Delta e - 36\ \Delta T \text{ (wind speed 320 km/day)}$$

From these formulae it is apparent that if the energy gained by short-wave radiation balanced that lost by evaporation equation (2) would simplify to :

$$Q_t = Q_{lw} + Q_c = 26\ \Delta T \text{ (average wind speed 160 km/day)} \tag{3}$$
$$= 36\ \Delta T \text{ (average wind speed 320 km/day)} \tag{4}$$

Under this condition, the heat loss from water surfaces would be approximately proportional to the difference in temperature between the air and the surface, in agreement with Newton's Law of cooling.

From the preceding discussion it may be seen that an over-all heat transfer coefficient h_0, relating net heat loss to air-water temperature difference ($Q_t = h_0\,\Delta T$), can be expected to vary not only because of wind velocity but also because of the size of the evaporation loss in relation to incoming short-wave radiation. If incoming short-wave radiation exceeds evaporation, h_o should be smaller for a given temperature difference, (i.e. Q_t reduced); if evaporation exceeds incoming short-wave radiation, then h_o should be larger for the same temperature difference (i.e. Q_t increased).

THE MEASUREMENT OF HEAT TRANSFER COEFFICIENTS FOR A SMALL LAKE

During the past four years observations have been made on water temperature at McKay Lake, a small lake located within the city of Ottawa (latitude 45° N longitude 75° E). These observations were used to check the validity of Newton's cooling equation and to obtain information on the possible variations in the coefficient h_0.

This small lake was chosen for heat loss observations because of several advantages. It is within 4 kilometres of a standard meteorological station and the weather records from this station could be used in the analysis. It is about 450 metres long and 200 metres wide, with a maximum depth of 10 metres, and did not require an excessive number of man hours to obtain the necessary water temperature records. The lake is fed by ground water with no marked inflow or outflow of surface water except during periods of heavy rain or melting snow. It is surrounded by trees and houses and thus is an example of a water body "sheltered" from the wind.

Water temperature profiles were obtained at several representative locations at weekly intervals during the autumn cooling periods for the years 1959, 1960 and 1961, and at bi-weekly intervals during all of 1962.

The surface temperature at a given time was assumed equal to the average of several water temperature readings measured in the upper 10 to 20 cm at several locations in the lake. The average water surface temperature for a given one-week period was assumed equal to the mean of the average water surface temperature at the beginning and end of the period. Average air temperature was obtained by averaging the daily maximum and minimum air temperatures recorded at a standard weather station located on National Research Council grounds, Ottawa.

The depth of the lake was measured at a number of sites and a bottom contour map drawn. The volume of water contained in consecutive 5-ft layers was then calculated and the heat loss in each layer was estimated from the change in water temperature that took place over a given period. The rate of heat loss from the lake surface for a given period was obtained by adding the contributions from all the layers and dividing by the surface area. In these calculations it was assumed that the amount of heat brought into or out of the lake by ground water could be neglected. The heat loss from snow that fell into the open water was allowed for in the calculations.

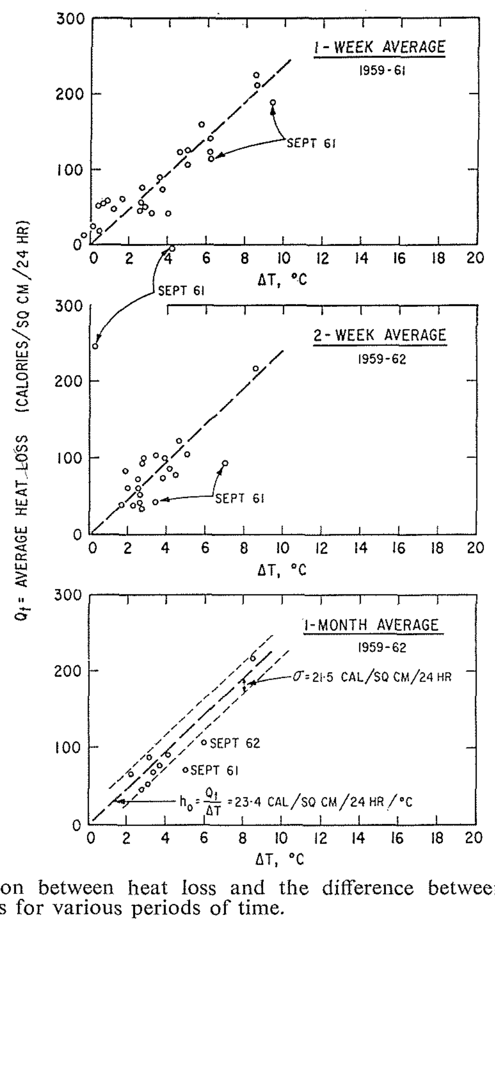

Fig. 1 — Relation between heat loss and the difference between air and water temperatures for various periods of time.

The values obtained for the rate of heat loss from McKay Lake were compared to the differences between the water surface and the air temperature for weekly, bi-weekly and monthly periods (Figure 1). It is apparent that the correlation between measured heat loss and temperature difference improves with length of observation period. On a monthly basis the variability of factors that affect heat loss, such as wind velocity, incoming short-wave radiation, and air vapour pressure, tend to average out and a reasonably good relationship between heat loss and temperature difference can be established.

The slope of the line in each of the graphs is a measure of h_o. On a monthly basis the average heat transfer coefficient h_o equals 23.4 cal/sq cm/24 hr/°C. The standard error was calculated to be 21.5 cal/sq cm/24 hr. Individual values of h_o tend to be unreliable when the temperature differences are small.

The deviations shown for weekly periods, September 1961, can be explained in part by above-average incoming short-wave radiation and below-average evaporation during this month. Part of the explanation may also be that during this month there were periods of unusually high day-time air temperatures and above-average wind speeds, so that convective heat gains during daylight hours possibly equalled or even exceeded the heat losses at night. Under these circumstances it is possible to have a net gain of heat by a water body even though average weekly air temperature is less than the water surface temperature. As appropriate weather records were not available over the lake surface, these September anomalies could not be analysed in greater detail. The deviations during September 1961, and to a lesser degree those in September 1962, suggest that the relation between heat loss and temperature difference will not be the same during early autumn (and presumably summer months) as during the late cooling periods.

Analyses of McKay lake values in relation to those reported in the literature

The previous section showed that $Q_t = h_o \ \Delta T$ is reasonably valid on a monthly basis for McKay Lake. This suggests that during the autumn cooling period evaporation losses are approximately equal to net short-wave radiation and that total heat loss equals the sum of convection plus long-wave radiation. In this section the validity of these relationships is further investigated by analysing published observations and comparing them with the McKay Lake observations.

In the McKay Lake calculations, values for incoming short-wave radiation were obtained from the standard meteorological records for Ottawa ([5]). The short-wave radiation absorbed was estimated assuming values of albedo given by Budyko ([6]) for water surfaces at latitude 45° N, months September to July. Evaporation was calculated with Penman's evaporation formula and appropriate weather parameters. Combined long-wave radiation and convection losses were then obtained for each observation period by subtracting the sum of net short-wave radiation and evaporation from the total heat loss.

The first comparison of the size of the heat exchange components is shown on Figure 2, where monthly short-wave radiation is plotted against monthly evaporation. Values for short-wave radiation evaporation, long-wave radiation, and convection ([2 7 8 9]) for latitudes 40° to 60° N have been used in this analysis directly as reported

Figure 2 shows that, on the average, monthly values of incoming short-wave radiation do balance calculated evaporation during the autumn cooling period, but not during the mid-winter months. During winter months incoming short-wave radiation is relatively small and evaporation from open water is relatively large because of large air-water vapour-pressure differences.

If evaporation loss is approximately equal to short-wave radiation then $Q_t = Q_{lw} + Q_c$. Figure 3 compares the sum of $Q_{lw} + Q_c$, combined long-wave radiation

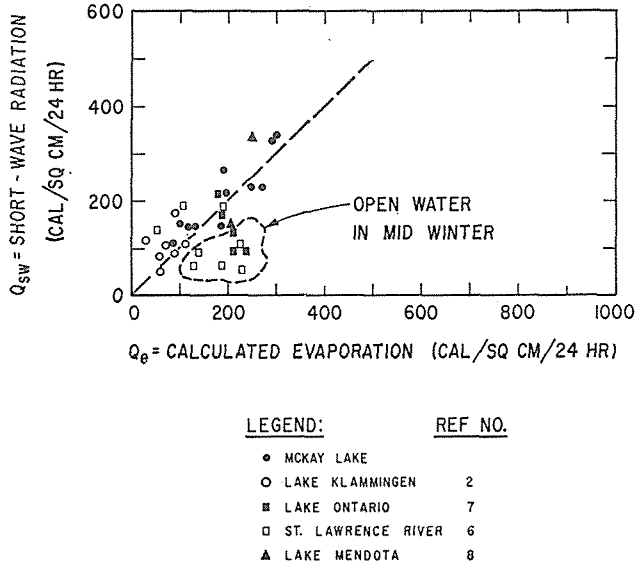

Fig. 2 — Comparison of monthly evaporation and short-wave radiation

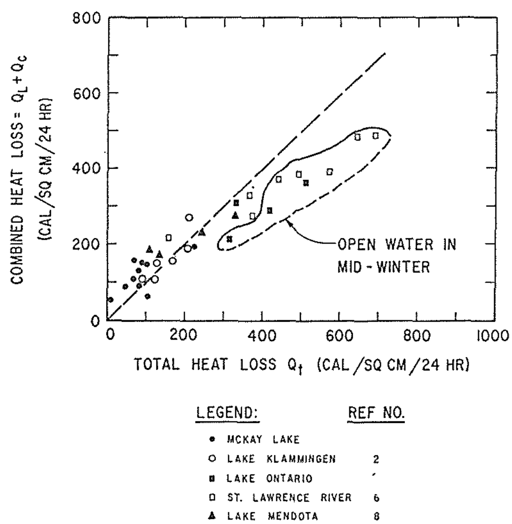

Fig. 3 — Total heat loss versus sum of net long-wave radiation and convective heat loss.

and convection loss, with the total heat loss Q_t. This illustrates that during the autumn months the sum of $Q_{lw} + Q_c$ tends to exceed the net heat loss, whereas during the mid-winter months Q_t exceeds the sum of $Q_{lw} + Q_c$. Monthly average sums of $Q_{lw} + Q_c$, obtained from the same sources, are plotted against monthly average air-water temperature difference in Figure 4. Equations (3) and (4) are also plotted on Figure 4 as straight lines :

$$\frac{Q_{lw} + Q_c}{\Delta T} = 26 \text{ and } 36 \text{ cal/sq cm/24 hr/}^\circ\text{C}.$$

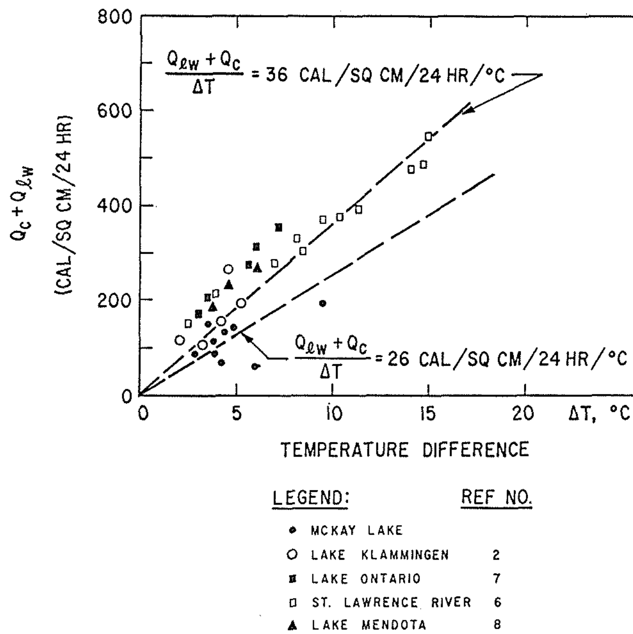

Fig. 4 — Sum of net long-wave and convective heat loss plotted against temperature difference.

The majority of the values of $\frac{Q_{lw} + Q_c}{\Delta T}$ obtained from the literature are somewhat higher than 36, the value calculated for an average wind speed of 320 km/hr; the values obtained for McKay Lake are close to 26, the value calculated for average wind speed of 160 km/day. For small differences of temperature (less than 5°C) there is considerable scatter to the values obtained for the sum of $Q_{lw} + Q_c$.

From the foregoing analysis it may be concluded that during the cooling period the sum of long-wave and convective heat is approximately proportional to the difference between the average air and water surface temperature. The net short-wave radiation, for latitudes 40° to 60°N, is approximately equal to the evaporation loss; it exceeds it during the autumn cooling period and is less than it for open water surfaces during mid-winter months. As a consequence, the sum of the convective and long-wave radiative heat loss is approximately equal to the total heat loss Q_t. This indicates that Newton's law of cooling can be a useful approximation of the rate of heat loss from an open water surface during autumn and winter if proper account is taken of site and average weather conditions.

Limits to heat transfer coefficient

It is evident that if Newton's law of cooling is used to estimate the rate of cooling of water bodies, it is necessary to know the approximate variation $f\ h_o$ under different conditions of exposure. In Table I are given values for h_0 obtained from observations reported in the literature ([2,7 to 14]). Values are listed as measured if the heat loss was obtained by taking temperature profiles at regular intervals; and listed as calculated if the heat loss was obtained by another method, usually the energy balance method. All values are for latitudes 40° to 60° N, except those for the ice surface ([10]) which were apparently calculated for more northerly locations.

Some of the information contained in Table I is presented in graphical form on Figure 5. Reported monthly values of heat loss are plotted against air-water temperature differences to show the range in over-all heat transfer coefficients for different degrees of exposure and time of year.

TABLE I

Summary of calculated and measured values of h_0 over-all heat transfer coefficient for various water bodies

Water Body	Maximum Depth (metres)	Period	No. of Periods	Average h_0 (cal/sq cm/ 24 hr/°C)	Measured or Calculated	Reference No.
McKay Lake	10	monthly	10	23.4	measured	
L. Klam-mingen	40	monthly	7	43.5	calculated	([2])
L. Mendota	25	monthly	3	47	measured	([9])
L. Ontario	225	monthly	2	49.5	calculated	([8])
			3	78		
St. Lawrence River	10	monthly	8	57	calculated	([7])
St. Lawrence River	10	approx. weekly	7	47	measured	([11])
10-ft tank	.3	5 to 10 hr evening	20	36.5	measured	([12])
River Lea	1.0	daily	3	52	measured	([13])
River Meuse	4.0		average conditions	40	measured	([14])
Ice Surface			average conditions	24 (still air) 60 (15 mph wind)	calculated	([10])

The average value for the over-all heat transfer coefficient h_0 varies from 40 to 60 cal/sq cm/24 hr/°C for most situations. For a sheltered lake or ice surface under relatively still air conditions the average value is from 20 to 30 cal/sq cm/24 hr/°C.

For an extremely large lake under mid-winter conditions the calculated average value of h_0 is about 75 cal/sq cm/24 hr/°C. As it is not known how representative the reported values for this lake will be for other water bodies, the upper limit for h_0 of 75 must be used with caution.

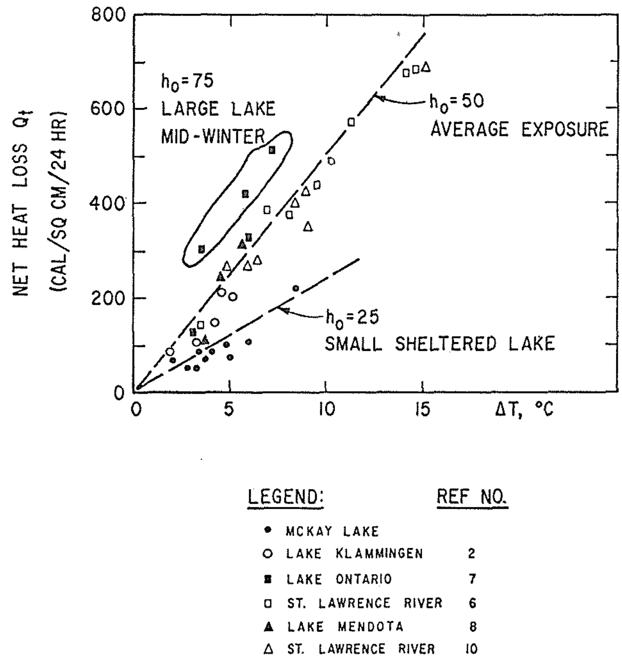

Fig. 5 — Relation between total heat loss and air-water temperature difference.

Conclusions

Heat losses from open water surfaces are approximately proportional to the difference in temperature between the water surface and the air, provided the energy gained by short-wave radiation equals that lost by evaporation. In this case heat losses would be equal to the sum of convection and long-wave radiation and be approximately proportional to 26 ΔT cal/sq cm/24 hr for average wind speed of 160 kilometres per day, and 36 ΔT for average wind speed of 320 kilometres per day.

Average values of h_0, the over-all heat transfer coefficient relating total heat losses to temperature difference ($Q_t = h_0 \Delta T$) were measured for four autumn cooling periods for a small sheltered lake. It was found that relatively large errors can be expected if calculations were made on a weekly basis. The average monthly value of h_0 obtained was 23.4 cal/sq cm/24 hr/°C.

Values of h_0 obtained from other studies, latitude 40° to 60°N, indicate that for average conditions of exposure h_0 is probably between 40 to 60 cal/sq cm/24 hr/°C, with lower values when short-wave radiation exceeds evaporation loss and higher values during mid-winter conditions when evaporation loss exceeds short-wave radiation.

In conclusion it can be stated that if site conditions and air-water temperature differences are known, Newton's law of cooling can be used as an approximate method of estimating rate of heat loss from natural water surfaces on a monthly basis during the autumn cooling period.

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