METHODS OF DETERMINING THE SURFACE DISTRIBUTION OF EXCESSIVE RAINFALLS

Jaroslav PETRLÍK
Hydraulic Research Institute, Prague

RÉSUMÉ

On a employé pour cette étude comme matière de base les enregistrements ombrographiques des averses, obtenus durant les années 1925-1949 dans les régions de Prague, de Brno, d’Ostrava et de Bratislava, dont l’intensité maximale était plus grande que 0,10 mm/min ou égalait cette valeur.

Les averses soumises à l’examen furent caractérisées d’abord par la définition de la relation entre leur intensité maximale et leur durée. Ensuite, on a fixé comme leur second caractère les relations entre les intensités des averses entières et les intensités de leurs étapes les plus fortes. Pour compléter la caractéristique des averses examinées on a élaboré en forme statistique leur fréquence pendant différents mois de l’année et pendant différentes heures du jour.

Au cours de cette étude on a choisi d’abord un groupe d’averses (à différente durée et à différente intensité) et on examinait la diminution de leur intensité maximale du lieu qu’elles avaient atteint, c’est-à-dire, on déterminait les courbes de la diminution horizontale de l’intensité. On a fixé 168 courbes. À la base de leur analyse mathématique on a fixé empiriquement une équation générale définissant la relation examinée. L’avantage de cette équation repose en ce qu’elle rend possible la fixation d’une courbe complète de la diminution horizontale de l’intensité, même dans le cas où le réseau des stations ombrographiques est moins dense, éventuellement dans le cas où les enregistrements sont incomplets (du point de vue de la répartition superficielle). L’équation empirique ci-dessus citée fut employée pour la détermination de la répartition superficielle des averses durant la période de 25 ans (depuis 1925 jusqu’à 1949) dans les quatre régions des plus grandes villes de la Tchécoslovaquie.

1. Evaluation of basic data

In the Hydraulic Research Institute in Prague the investigation of the surface distribution of heavy rainfalls in the area of the four largest towns in Czechoslovakia (Prague, Brno, Ostrava and Bratislava) was carried out. Raingauging data of the 25-year period 1925-1949 were elaborated.

In the region of Prague (about 200 km²) there were during the mentioned period 23 rain-gauging stations, in the Brno-region (about 100 km²) 9 stations, in the Ostrava-region (about 50 km²) 8 stations and in the Bratislava-region (about 60 km²) 9 stations.

For the investigation of the surface distribution all rainfalls reaching or exceeding a maximum intensity of 0,10 mm/min were used.

Approximately 350 rainfalls were treated in each region. The rainfalls were divided into four categories according to their duration in the place of maximum precipitation. The means of these categories were determined so as to form a geometrical series using as first term 10 min and a quotient of 2.5. Their precise values were: 10, 25, 62 and 155 min; for simplicity sake they were marked as 0¹⁰, 0²⁵, 1⁰⁰, 2³⁰ hours.

For each region two basic characteristics were determined for the rainfalls. In the first place this was the relation between the maximum intensity of the total rainfall \((c_i m)\) in mm/min and its duration in the place of maximum intensity \((T)\) in min; this relation was defined by the exponential function

\[ c_i m = d \cdot T^m \]  

where \( d \) and \( m \) are constants. The validity of this relation was verified in a bilogarithmic system by means of the coefficient on linear correlation and its standard deviation.
As second characteristic feature the relation between the duration of the maximum intensity of the rain \( (j'_{max}) \) and the maximum intensity of the total rain \( (c_{max}) \) were determined within the time categories 0^25, 1^00 and 2^30 hours; both intensities in mm/min. This relation was defined by the linear function

\[
{j'_{max}} = A \cdot c_{max} + B
\]

(2)

where \( A \) and \( B \) are constants. The validity of the relation was also confirmed by means of the coefficient of linear correlation and its standard deviation. The mentioned relation was not defined by the time category 0^10 hours, since with regard to the scale of the rain-gauge records it was not possible to determine the duration of the maximum rain-intensity reliably.

To illustrate the close relationship of equations (1) and (2) some results of the survey of the Prague region which is the largest of the investigated regions are shown. Equation (1): the coefficient of linear correlation \( r = -0.48 \) and its standard deviation \( \sigma_r = 0.041 \). Equation (2): for the categories 0^25, 1^00 and 2^30 \( r = +0.79; +0.69; +0.88; \) and \( \sigma_r = 0.043; 0.053; 0.053 \) respectively.

To complete the characteristics of the used material relative frequencies of the occurrence of heavy rainfalls in the single months and relative frequencies of the occurrence of the beginnings of heavy rainfalls in the single hours of the day were determined for each studied region.

1.1. Methods of determining the surface distribution of excessive rainfalls.

An ideal, although laborious method of determining the surface distribution of heavy rainfalls is the plotting of isohyetal lines for each heavy rain and the planimetering of areas which they demarcate. From data obtained in this way it is possible to determine safely the areas hit by the different types of rain and further curves of horizontally diminishing intensities (from maximum intensity to zero intensity). This method, however, requires a large area, equipped with a dense and uniform system of rain-gauging stations in order to find a sufficient number of rainfalls hitting the territory lying within the entire internally investigated area. Since in the majority of cases of the investigation of precipitation regime it is not possible to count with these conditions, it was necessary to find another method for the determination of the distribution of heavy rainfalls, which is less pretentious as to the amount of basic data. Besides that in the rain gauging material of the Prague, Brno, Ostrava and Bratislava regions there were few rainfalls recorded which covered the area lying completely inside these regions.

For this reason those rainfalls (without regard to the region, intensity and duration) were selected for which it was possible to find at least in some directions the total curve of horizontally diminishing intensity (from maximum to zero intensity); the great majority of these rainfalls was recorded in the Prague-region. Figure 1 illustrates the Prague-region with a network of rain-gauging stations. In figure 2 the intensities (in mm/min) of the heavy rain of May 7, 1948 from the same area are given as example. For each rainfall one or two curves were determined. The total number of these curves were 168. Analysing them it was found that the majority of them can be considered as a parabola of second order in the semilogarithmic system, where the axis of distances of intensities \( d \) from the corresponding maximum intensity \( (i_{max}) \) is logarithmic and the axis of intensities linear. Hence the most frequent form of the curve of horizontally diminishing intensity was defined analytically by means of the empirical equation

\[
i = i_{max} + a \log (d + 1)^2
\]

(3)

where \( a \) is constant and \( d \) the distance (in km) of the place of intensity \( i \) from the place of maximum intensity \( i_{max} \) (both intensities in mm/min).

The abscissa of the maximum of this parabola is equal to one. This finding was
evaluated statistically by treating the fundamental values of the single curve. The procedure of treatment is shown in the following paragraphs.

Since the heavy rainfalls in question had different maximum intensities and different durations, the mentioned values corresponding to the different rainfalls were transferred to the same basis according to the transformation equations:

\[ I = \frac{i}{i_m} \times 100\%; \quad D = \frac{\log (d + 1)}{\log (d_m + 1)} \times 100\% \]

where \( d_m \) (in km) is the distance of the place with zero intensity from the place with maximum intensity. The relative data obtained in this way were listed in Table I based on the class interval equal to 11 per cent for both variables.

Analogous to the empirical equation (3) is in the transformed system \((D ; I)\) the equation

\[ I = 100 - a \cdot D^2 \]

From the transformation equations (4) it follows logically that this equation must comply with the condition

\[ 0 = 100 + a \cdot 100^2 \]

Complying with this condition equation (5) changes after re-arrangement to

\[ I = 100 - \left( \frac{D^2}{10} \right) \]

Hence equation (7) is a complete analogy to the empirical equation of horizontally diminishing intensity (3) in the system \((D ; I)\). Since it is a precisely defined function, i.e. it does not contain any general coefficients which would have to be found by the
### TABLE 1

*Relative parameters (%) of horizontally diminishing intensity*

<table>
<thead>
<tr>
<th>D</th>
<th>0</th>
<th>1-11</th>
<th>12-22</th>
<th>23-33</th>
<th>34-44</th>
<th>45-55</th>
<th>56-66</th>
<th>67-77</th>
<th>78-88</th>
<th>89-99</th>
<th>100</th>
<th>Σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>168</td>
</tr>
<tr>
<td>1-11</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td>19</td>
<td>0</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>12-22</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>12</td>
<td>14</td>
<td>7</td>
<td>0</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>23-33</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>10</td>
<td>11</td>
<td>4</td>
<td>0</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>34-44</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>45-55</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>56-66</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>13</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>67-77</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>78-88</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>89-99</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>168</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>171</td>
<td></td>
</tr>
</tbody>
</table>

| Σ | 168| 0    | 18    | 12    | 29    | 39    | 65    | 45    | 33    | 168   | 577 |   |

---

**PRAGUE AREA - HEAVY RAINFALL OF MAY 7, 1948.**

---

**Fig. 2**
method of the least squares, it would be possible to determine its residua directly from the data given in Table 1. These residua were determined for the arguments \( D \) in the interval (1;99) and their absolute values are shown in the following Table 2; the extent of their class intervals followed naturally from the values of the intervals in Table 1.

**TABLE 2**

<table>
<thead>
<tr>
<th>residuum</th>
<th>0</th>
<th>11</th>
<th>22</th>
<th>33</th>
<th>44</th>
<th>55</th>
<th>66</th>
<th>77</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>62</td>
<td>81</td>
<td>39</td>
<td>30</td>
<td>19</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>cummulative frequency</td>
<td>62</td>
<td>143</td>
<td>182</td>
<td>212</td>
<td>231</td>
<td>238</td>
<td>240</td>
<td>241</td>
</tr>
<tr>
<td>cummulative relative frequency</td>
<td>26</td>
<td>59</td>
<td>76</td>
<td>88</td>
<td>96</td>
<td>99</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

The magnitude and the frequency of these residua \((\Delta I)\) represent the degree of reliability of the empirical equation of horizontally diminishing intensity \((3)\). For this reason these residua were not characterized by the square deviation as the only data as is normally the case. The fundamental characteristics show the relative cumulative frequency (determined with an accuracy of 1 per cent). From the table of residua it can be further seen that 95 residua are smaller than \( \pm 10 \) per cent, 155 residua smaller than \( \pm 20 \) per cent and 173 residua smaller than 25 per cent.

On the basis of these data it can be stated that the empirical formula \((3)\) shows with a probability of 0.39 the curve of horizontally diminishing intensity of excessive rainfalls with a mean inaccuracy of less than 10 per cent of its maximum intensity; with a probability of 0.64 with a mean inaccuracy of less than 20 per cent; and with a probability of 0.72 with a mean inaccuracy less than 25 per cent. These data characterized the degree of reliability of the empirical equation \((3)\).

For its determination in practice one additional intensity value besides the data on maximum intensity, together with the distance of the corresponding places is sufficient. It is evident that a greater number of intensity values substantially increases its precision, especially intensity data from places with sufficiently different distances from the place of maximum intensity. In this case the coefficient \( a \) of this equation is determined by the method of the least squares.

The distance of the place with zero intensity from that of maximum intensity \( d_m \) (in km) is found by solving the mentioned equation together with the equation of the abscissa

\[
i = i_m + a \log (d + 1) \]

\[
l = 0
\]

after re-arrangement

\[
d = 10^l - i_m \cdot a^{-1} - 1 = d_m.
\]

The investigation of the distribution of heavy rainfalls by means of the mentioned empirical equation \((3)\) can be done in two ways:

1. The curves of horizontally diminishing intensities are determined separately for several directions according to the quantity of data available. In this case it is possible to determine for the intensity in the interval \((0; i_m)\) or graphically the isolynthetale lines corresponding to the actual situation. Data obtained using the empirical equation \((3)\) can then be used as supplement to the incomplete rain-gauge recordings.
2. The mean curve of horizontally diminishing intensity is determined directly by means of all fundamental data available. By doing this the actual isohyetal lines are substituted by concentrical circumferences with the centrum in the place of maximum intensity. This procedure is naturally less precise, since in the majority of cases basic material for the different directions is available in different amount. The more precise calculation of coefficient \( a \) from equation (3) evaluating the obtained data according to their frequency in the single directions from the place of maximum intensity must necessarily be considered as problematic as the rain-gauge records are of unequal quality. For this reason it is advisable to use this procedure only in the case of systematic treatment of a large quantity of basic material for long time series. The lesser accuracy of this method appears in the greater dispersion of the results. For information sake the size of the area hit by the intensity \( i \geq 0.15 \) mm/min was determined for the heavy rain shown in figure 2, and that on the one hand by planimetry of the area limited by the corresponding isohyetal line and on the other hand using the mentioned empirical equation (3), determined by method 2. The difference between the two data is less than 5 per cent of the area determined by planimetry.

2. Conclusion

The aim of this study was to develop a method permitting the investigation of surface distribution of heavy rainfalls in the regions of the main cities of Czechoslovakia.

Doing these investigations in the conventional manner difficulties are encountered in the majority of cases caused by the not sufficiently dense system of rain gauging-stations. For this reason an equation of the horizontally diminishing intensity of heavy rainfalls was determined empirically which allows the extrapolation of the missing data and even with a not sufficiently dense network of rain-gauging stations to characterize the precipitation regime of the considered regions for the requirements of projection.

For the investigation of the surface distribution of heavy rainfalls in the areas of the four largest cities in Czechoslovakia in the period 1925 to 1949, mainly average curves of horizontally diminishing intensity for the single rainfalls were used. The surface data obtained were expressed as function of the corresponding maximum intensities.