NONSTEADY FLOW TO A WELL IN AN INFINITE ANISOTROPIC AQUIFER

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ABSTRACT

Well flow equations presently used in the analysis of pumping tests and the prediction of water levels have been derived under the assumption that aquifers are isotropic. These existing equations are not applicable to anisotropic aquifers such as fractured rocks.

In this paper, an equation is derived for the drawdown distribution around a well discharging at a constant rate from an infinite anisotropic aquifer. Drawdowns computed by this equation are compared and found to be in good agreement with those observed in an electric-analog model constructed for this purpose. It is shown that pumping test data from a minimum of three observation wells can be analyzed to obtain the components of the transmissibility tensor along an arbitrarily chosen set of axes, and that these components, in turn, can be used to determine the principal transmissibilities and the orientation of the principal axes. The method is illustrated with an example.

RESUME

Écoulement irrégulier en direction d'un puits dans une nappe anisotrope illimitée

Les équations d'écoulement des puits actuellement utilisées pour l'analyse des tests de pompage et les prévisions des niveaux d'eau ont été établies sur le postulat que les aquifères sont isotropes. Ces équations ne peuvent être appliquées aux aquifères anisotropes que sont les roches fissurées.

L'auteur de cet article établit une équation donnant la répartition du rabattement de la nappe autour d'un puits au débit constant, foré dans une nappe anisotrope illimitée. Si l'on compare les rabattements calculés d'après cette équation à ceux que l'on observe sur un modèle électrique analogique construit à cette intention, on constate que les uns et les autres concordent. L'auteur montre aussi qu'en analysant les données des tests de pompage effectués sur un minimum de trois puits d'observation, on peut obtenir les composants du tenseur de transmissivité pour un ensemble d'axes arbitrairement choisis. Ces composants peuvent eux-mêmes être utilisés pour déterminer les transmissivités principales et l'orientation des axes principaux. La méthode est illustrée par un exemple.

INTRODUCTION

Equations presently used in analyses of pumping tests and predictions of water levels have been derived under the assumption that the aquifers are isotropic. These existing equations are not applicable to anisotropic aquifers such as some fractured rocks in which joint patterns cause variations of the permeability in different directions.

This paper derives an equation for the nonsteady drawdown distribution around a well discharging at a constant rate from an homogeneous anisotropic aquifer of infinite areal extent.

The flow of ground water in aquifers follows Darcy's law which states that the velocity is proportional to the negative gradient of the hydraulic head. In vectorial form this can be written as

$$\vec{V} = -K \text{grad } h$$

where $\vec{V}$ is the velocity vector, the constant of proportionality $K$ the permeability (hydraulic conductivity) of the aquifer, and $h$ the hydraulic head.

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In anisotropic aquifers the velocity vector and the hydraulic gradient vector are generally not parallel. The constant of proportionality $K$ is then a symmetric tensor of the second rank (Ferrandon, 1948; Scheidegger, 1954; Liakopoulos, 1962), usually referred to as the "permeability tensor", which transforms the components of the hydraulic gradient into those of the velocity. The velocity and the hydraulic gradient have the same direction only along one of three orthogonal axes called the "principal axes" of the permeability tensor. The anisotropy of an aquifer can be defined by the orientation of the principal axes and the magnitudes of the components of permeability along them.

For the two-dimensional flow problem treated in this paper use of the "transmissibility tensor" $T$, which is the product of the two-dimensional permeability tensor and the thickness of the aquifer, is convenient. In matrix notation the two-dimensional symmetric transmissibility tensor can be written as

$$T = \begin{bmatrix} T_{xx} & T_{xy} \\ T_{xy} & T_{yy} \end{bmatrix}$$

where $x$ and $y$ are an arbitrary set of orthogonal axes. For the principal axes $\xi$ and $\eta$ the above equation reduces to

$$T = \begin{bmatrix} T_{\xi\xi} & 0 \\ 0 & T_{\eta\eta} \end{bmatrix}$$

$T_{\xi\xi}$ and $T_{\eta\eta}$ being the maximum and minimum transmissibilities, respectively.

Analysis

The distribution of drawdown around a well of constant discharge which fully penetrates an infinite anisotropic artesian aquifer is described by the following boundary value problem

$$s(x, y, t) = 0$$

where $s$ is the drawdown, $T_{xx}$, $T_{yy}$, and $T_{xy}$ the components of the transmissibility tensor, $S$ the storage coefficient, $Q$ the discharge of the well, $\delta$ the Dirac delta function, $x$ and $y$ the coordinates of an arbitrary set of orthogonal axes with origin at the well, and $t$ the time since the flow started.

The theory of integral transforms is used in solving the problem. By using the Laplace transformation with respect to $t$ and initial condition 2 the problem can be expressed as

$$T_{xx} \frac{\partial^2 \delta}{\partial x^2} + 2 T_{xy} \frac{\partial^2 \delta}{\partial x \partial y} + T_{yy} \frac{\partial^2 \delta}{\partial y^2} + \frac{Q}{p} \delta(x) \delta(y) = S \frac{\partial \delta}{\partial t}$$

$$s(x, y, 0) = 0$$

$$s(\pm \infty, y, t) = 0$$

$$s(x, \pm \infty, t) = 0$$

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$$s(\pm \infty, y, p) = 0$$

$$s(x, \pm \infty, p) = 0$$
where $s$ is the Laplace transform of $s$, and $p$ the parameter of the transformation.

Application of the complex Fourier transform with respect to $x$ to the above equations results in

$$- T_{xx} \alpha^2 w - 2i\alpha T_{xy} \frac{\partial w}{\partial y} + T_{yy} \frac{\partial^2 w}{\partial y^2} + \frac{Q}{\sqrt{2\pi p}} \delta(y) = Spw \quad (8)$$

$$w(x, \pm \infty, p) = 0 \quad (9)$$

where $w$ is the transform of $s$, $\alpha$ the parameter of the transformation and $i = \sqrt{-1}$.

Transforming once more through use of the complex Fourier transform with respect to $y$ yields an explicit expression for the transform of the solution

$$z = \frac{Q}{2\pi \rho} \frac{1}{T_{xx} \alpha^2 + 2T_{xy} \alpha + T_{yy} \beta^2 + Sp} \quad (10)$$

where $z$ is the transform of $w$ and $\beta$ the parameter of the transformation.

Equation 10 is obtained irrespective of the order that the three transformations are made. Consequently, the order of inversion is irrelevant and can be chosen for convenience. Taking first the inverse Fourier transform with respect to $y$, one obtains

$$w = \frac{Q}{2\sqrt{2\pi}} \exp \left\{ (iy\alpha T_{xy} - y) [(T_{xx} T_{yy} - T_{xy}^2) \alpha^2 + ST_{yy}p]^{-1} \right\} \frac{1}{p [T_{xx} T_{yy} - T_{xy}^2 \alpha^2 + ST_{yy}p]^{-1/2}} \quad (11)$$

Then, taking the inverse Laplace transform through use of the convolution integral results in

$$L^{-1}\{w\} = \frac{Q e^{\eta x T_{xy}/T_{yy}}} {2\pi \sqrt{2ST_{yy}}} \int_0^t \exp \left\{ -\frac{\nu^2 S}{4T_{yy} \tau} - \frac{(T_{xx} T_{yy} - T_{xy}^2) \alpha^2 \tau} {ST_{yy}} \right\} d\tau \quad (12)$$

where $L^{-1}$ denotes the inverse Laplace transform of $w$. Finally, taking the inverse Fourier transform with respect to $x$, and after some mathematical manipulation we obtain the formal solution

$$s = \frac{Q}{4\pi \sqrt{T_{xx} T_{yy} - T_{xy}^2}} W(u_{xy}) \quad (13)$$

where $W(u)$ is the negative exponential integral, known in the field of hydrology as the "well function", defined as

$$W(u) = \int_u^\infty e^{-\mu} \frac{d\mu}{\mu}$$

and in which

$$u_{xy} = S \left( \frac{T_{xx} y^2 + T_{yy} \alpha^2 - 2 T_{xy} xy} {T_{xx} T_{yy} - T_{xy}^2} \right) \quad (14)$$

If the coordinate axes $x$ and $y$ coincide with the principal axes $\xi$ and $\eta$ of the transmissibility tensor, equation 13 reduces to

$$s = \frac{Q}{4\pi \sqrt{T_{\xi\xi} T_{\eta\eta}}} W(u_{\xi\eta}) \quad (15)$$
where $T_{\xi\xi}$ and $T_{\eta\eta}$ are the "principal transmissibilities" and

$$u_{\xi\eta} = \frac{S}{4t} \left( \frac{T_{\xi\xi}^2 + T_{\eta\eta}^2}{T_{\xi\xi}T_{\eta\eta}} \right)$$ (16)

Equation 15 is similar to one given by Collins [1961].

For small values of its argument, that is for $u < 0.02$, the well function appearing in equations 13 and 15 can be closely approximated [Cooper and Jacob, 1946] by

$$W(u) = -0.5772 - \log_2 u = 2.303 \log_{10} \frac{2.25}{4u}$$

Substituting this approximation in equations 13 and 15 one obtains solutions for relatively large values of time as

$$s = \frac{2.303 Q}{4\pi \sqrt{T_{\xi\xi} T_{\eta\eta}}} \log_{10} \left[ \frac{2.25t}{S} \left( \frac{T_{xx} T_{yy} - T_{xy}^2}{T_{xx}y^2 + T_{yy}x^2 - 2T_{xy}xy} \right) \right]$$ (17)

for an arbitrary set of axes, and

$$s = \frac{2.303 Q}{4\pi \sqrt{T_{\xi\xi} T_{\eta\eta}}} \log_{10} \left[ \frac{2.25t}{S} \left( \frac{T_{\xi\xi} T_{\eta\eta}}{T_{\xi\xi}^2 + T_{\eta\eta}^2} \right) \right]$$ (18)

for the principal axes.

As an examination of the drawdown equations indicates, for a given time, lines of equal drawdown around a well pumping from an anisotropic aquifer have the form of concentric ellipses (fig. 1) with transverse axes along the maximum transmissibility axis $\xi$ and conjugate axes along the minimum transmissibility axis $\eta$.

DATA: $T_{\xi\xi}=12cm^2/sec$ $T_{\eta\eta}=3cm^2/sec$
$S=0.0001$ $Q=5.46 liters/sec$
$t=400hrs$

Fig. 1
The analytical solution obtained in the previous pages was verified by an electric analog model consisting of a rectangular resistance-capacitance network. The model was designed for an aquifer having principal transmissibilities $T_{xx}$ and $T_{yy}$ of 37 cm$^2$/sec. and 11 cm$^2$/sec and a storage coefficient of 0.048. The well discharged at a rate of 6.67 liters/sec. A node spacing corresponding to a 50 meter orthogonal grid was chosen. One quadrant of the infinite space bounded by the $\xi$ and $\eta$ axes was modeled. To simulate an infinite aquifer the model was extended in both directions beyond the limits that would be reached by the effect of pumping within the period of measurement.

![Fig. 2](image_url)

Measured and computed drawdowns at each node point, for $10^5$ seconds of pumping, are shown on figure 2. Variation of the computed and observed drawdown with time at $\xi = 100$ meters and $\eta = 100$ meters is shown on figure 3. The curve representing the observed drawdown was traced from a photograph of the oscilloscope screen. The measured and computed values in the two figures agree closely, the slight differences being due to instrumental error.

**APPLICATION TO PUMPING TESTS**

The use of analytical solutions in quantitative hydrologic studies requires that the transmissibility and storage coefficient of the aquifers be determined or estimated by some means. These formation constants are usually determined from analyses of pumping tests, which consist of relating observed drawdowns to theoretical drawdown equations for the flow system under consideration. For anisotropic aquifers the
appropriate theoretical equations are equations 13 or 17 if the principal axes are not known, and equations 15 or 18 if the principal axes are known. The method of analysis is essentially the "type-curve" or, if applicable, the "straight-line" method, both of which are well known to hydrologists from their use in the analyses of tests of isotropic aquifers. The observed drawdown $s$ is plotted against time $t$ or reciprocal of time $1/t$ for each observation well. Because of the absence of radial symmetry, the composite drawdown graph ($s$ against $r^2/t$, where $r$ is the radial distance) and the distance-drawdown ($s$ against $r$) plots which are used in tests of isotropic aquifers,

![Figure 3](image)

cannot be used in tests of anisotropic aquifers. Also, since there are four constants to be determined (the three transmissibility components $T_{xx}$, $T_{yy}$ and $T_{xy}$ and the storage coefficient $S$) a minimum of three observation wells at different distances and different directions from the pumping well are necessary.

Both the type-curve and the straight-line methods of analysis for anisotropic aquifers are outlined below for the case where only three observation wells exist and the directions of the principal axes are not known. When data from more than three observation wells are available the same approach can be used by grouping them into sets of three.

**Type-curve method**

1. Choose a convenient rectangular coordinate system with the origin at the pumping well and record the $x$ and $y$ coordinates of each observation well.
2. From tables of the well function \( W(u) \) [Wenzel, 1942] prepare a type curve of \( W(u_z) \) against \( u_z \) on logarithmic paper. The curve so obtained is known as the type curve.

3. Plot observed values of the drawdown \( s \) against reciprocal time \( 1/t \) for each of the three observation wells on logarithmic paper to the same scale as the type curve.

4. Superpose the observed data plot on the type curve and, keeping the coordinate axes of the two plots parallel, find, for each well, the best fit of the data on the type curve. Choose a match point for each well and record the dual coordinates \( W(u_z), s, u_z \) and \( 1/t \) of each match point.

5. Substitute the values of \( W(u_z) \) and \( s \) from each match point into equation 13 and solve for \( (T_{xz}T_{yy} - T_{x}^2y) \). All three match points should yield the same, or approximately the same, value for \( (T_{xz}T_{yy} - T_{x}^2y) \). If they do not, judgement must be used to obtain an "average" value.

6. Substitute the values of \( u_z \) and \( 1/t \) from each match point and the value of \( (T_{xz}T_{yy} - T_{x}^2y) \) obtained in step 5 into equation 14 and, using the coordinates of the observation well corresponding to each match point, solve the resulting three equations for the products \( ST_{xz}, ST_{yy} \) and \( ST_{xy} \).

7. Solve these products for \( T_{xx}, T_{yy} \) and \( T_{xy} \) in terms of \( S \) and, substituting these into the expression \( (T_{xz}T_{yy} - T_{x}^2y) \) whose value is known from step 5, obtain \( S \).

8. Having found \( S \), calculate \( T_{xx}, T_{yy} \) and \( T_{xy} \) from the products obtained in step 6.

Straight-line method

The straight-line method of analysis can be used only if all or the latter part of the observed drawdown data for all three observation wells falls within the range of time for which equation 17 is applicable.

1. Same as step 1 of type-curve method.

2. Plot observed values of drawdown \( s \) in each observation well against time \( t \) on semilogarithmic paper with \( t \) on the logarithmic scale. If the latter data plot as a straight line equation 17 probably applies.

3. For each well draw a straight line through those points that plot as a straight line. An examination of equation 17 shows that this straight line has a slope \( (As \) per log cycle) given by

\[
\frac{As}{cycle} = \frac{2.303 Q}{4\pi \sqrt{T_{xx}T_{yy} - T_{xy}^2}}
\]  

(19)

and a \( t \)-intercept \( t_o \) given by

\[
t_o = \frac{S}{2.25} \left( \frac{T_{xx}y^2 + T_{yy}x^2 - 2T_{xy}xy}{T_{xx}T_{yy} - T_{xy}^2} \right)
\]  

(20)

4. Find the intercept \( t_o \) and the slope \( As/cycle \) of each line. All three lines should have the same, or approximately the same, slope. If differences exist obtain an average value.

5. Substitute the slope in equation 19 and calculate \( (T_{xx}T_{yy} - T_{x}^2y) \).

6. Substitute the intercept \( t_o \) of each line and the value of \( (T_{xx}T_{yy} - T_{xy}) \) obtained in step 5 into equation 20 and, using the coordinates of the observation well corresponding to each intercept, solve the resulting three equations for the products \( ST_{xx}, ST_{yy} \) and \( ST_{xy} \).

7. Follow the same procedure as in steps 7 and 8 of the type curve method to calculate \( S, T_{xx}, T_{yy} \) and \( T_{xy} \).
After the components $T_{xx}$, $T_{yy}$ and $T_{xy}$ of the transmissibility tensor are obtained from the type-curve or the straight-line method, the principal transmissibilities $T_{\xi \xi}$ and $T_{\eta \eta}$ and the orientation of the principal axes can be determined by making use of tensor properties. The following relations, obtained from the invariants and the rules of transformation of tensors, apply for all symmetric tensors of the second rank:

$$T_{\xi \xi} = \frac{1}{2} \left\{ (T_{xx} + T_{yy}) + \left[ (T_{xx} - T_{yy})^2 + 4 T_{xy}^2 \right]^{\frac{1}{2}} \right\} \tag{21}$$

$$T_{\eta \eta} = \frac{1}{2} \left\{ (T_{xx} + T_{yy}) - \left[ (T_{xx} - T_{yy})^2 + 4 T_{xy}^2 \right]^{\frac{1}{2}} \right\} \tag{22}$$

$$\theta = \arctan \left( \frac{T_{\xi \xi} - T_{xx}}{T_{xy}} \right) \tag{23}$$

where $\theta$ is the angle between the $x$ and the $\xi$ axis, positive in a counterclockwise direction from the $x$-axis, and restricted for convenience to the interval $0 \leq \theta < \pi$.

**ILLUSTRATIVE EXAMPLE**

A 12-hour pumping test was conducted to determine the hydraulic properties of an anisotropic aquifer. The well $PW$ was pumped at a rate of 12.57 liters/sec. and the drawdown was observed at three observation wells $OW-1$, $OW-2$ and $OW-3$ located as shown on figure 4. The drawdown data are given on table 1. The problem is to find the storage coefficient and principal transmissibilities of the aquifer and the direction of the principal axes.

![Fig. 4](image-url)
Solution

The coordinate axes are chosen with the $x$-axis passing through $OW-1$ and the coordinates of the observation wells are determined as

$$
\begin{align*}
OW/1- & \quad x = 28.3 \text{ m} ; \quad y = 0 \\
OW/2- & \quad x = 9.0 \text{ m} ; \quad y = 33.5 \text{ m} \\
OW/3- & \quad x = -19.3 \text{ m} ; \quad y = -5.2 \text{ m}
\end{align*}
$$

(24)

A semilogarithmic plot of the data (fig. 5) shows that the straightline method of analysis is applicable. The slope of the lines drawn through the latter part of the data is the same for all three lines and has the value of 1.15 meters per log cycle. The $t$-intercepts are:

$$
\begin{align*}
(t_0)_1 & = 0.37 \text{ min.} \\
(t_0)_2 & = 0.72 \text{ min.} \\
(t_0)_3 & = 0.24 \text{ min.}
\end{align*}
$$

(25)

From equation 19 we obtain

$$
(T_{xx} T_{yy} - T^2_{xy}) = \left[ \frac{2.303 \times 12.57 \text{ liters/sec.}}{4\pi \times 1.15 \text{ m} \times 1000 \text{ liter/m}^3} \right]^2 = 4 \times 10^{-6} \text{ m}^4/\text{sec}^2.
$$

(26)
Substituting from (24), (25) and (26) into equation 20 we find

\[(28.3)^2 ST_{yy} = 2.00 \times 10^{-4} \text{ m}^4/\text{sec.}\]

\[(33.5)^2 ST_{xx} + (9.0)^2 ST_{yy} - 2(33.5)(9.0) ST_{xy} = 3.89 \times 10^{-4} \text{ m}^4/\text{sec.}\]

\[(5.2)^2 ST_{xx} + (19.3)^2 ST_{yy} - 2(5.2)(19.3) ST_{xy} = 1.39 \times 10^{-4} \text{ m}^4/\text{sec.}\]

Solving these three equations simultaneously we obtain

\[
\begin{align*}
T_{xx} &= \frac{2.5 \times 10^{-7}}{S} \text{ m}^2/\text{sec.} \\
T_{yy} &= \frac{2.5 \times 10^{-7}}{S} \text{ m}^2/\text{sec.} \\
T_{xy} &= -\frac{1.5 \times 10^{-7}}{S} \text{ m}^2/\text{sec.}
\end{align*}
\]

(27)

Substituting from (27) into (26) gives

\[
\frac{6.25 \times 10^{-14} - 2.25 \times 10^{-14}}{S^2} = 4 \times 10^{-6}
\]

or

\[
S = 10^{-4}
\]

(28)

Finally, by substituting from (28) into (27) we have

\[
T_{xx} = 2.5 \times 10^{-3} \text{ m}^2/\text{sec.} = 25 \text{ cm}^2/\text{sec.}
\]

\[
T_{yy} = 2.5 \times 10^{-3} \text{ m}^2/\text{sec.} = 25 \text{ cm}^2/\text{sec.}
\]

\[
T_{xy} = -1.5 \times 10^{-3} \text{ m}^2/\text{sec.} = -15 \text{ cm}^2/\text{sec.}
\]

With the components of the transmissibility tensor now known, the principal transmissibilities are obtained from equation 21 and 22 as

\[
T_{\xi\xi} = \frac{1}{2} \{(25 + 25) + [4 \times (-15)^2]\} = 40 \text{ cm}^2/\text{sec.}
\]

\[
T_{\eta\eta} = \frac{1}{2} \{(25 + 25) - [4 \times (-15)^2]\} = 10 \text{ cm}^2/\text{sec.}
\]

The angle \(\theta\) between \(x\) and \(\xi\) axis is found from equation 23 to be

\[
\theta = \arctan \left[ \frac{40 - 25}{-15} \right]
\]

\[
= \arctan (-1)
\]

\[
= 135^0
\]
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REFERENCES


