ESTIMATION OF MAXIMUM DISCHARGES
AND HYDROGRAPHS OF FLOOD

Professor D.L. SOKOLOVSKY and Engineer I.A. SHIKLOMANOV
Hydrometeorological Institute Leningrad, USSR

ABSTRACT

Some methods of estimation of the maximum discharges and hydrographs of flood
with application of the electronic modelling equipment are described in this paper.
On the basis of this estimation the theory of "time of travel" is laid to take into
account the transformation of the course of water yield at hydrograph of runoff in
accordance with the so-called genetic formula known in mathematics as Duhamel
integral and used in the unitgraph theory. As the principal parameter of flood the
value of time of lag between maximum rainfall and runoff is considered, as well as the
coefficient of the shape of flood defined as relation between the time of fall and the
time of rise of the hydrograph.

In the process of estimation the characteristic of time of travel may be considered
either constant during flood (linear scheme) or changing which depends on the
variation of discharges (unlinear scheme).

In the graph of the course of the water yield (the course of rain or melting taking
into account the runoff coefficient) we may with the help of this methods define the
maximum discharges of rainfloods on different streams either investigated or not
investigated. In the latter case the parameters of flood are defined in accordance with
the area, slope and shape of this basin, the extent of the wooded land and other
physiographic and morphometric characteristics of a catchment area.

The methods have been tested on many streams under different conditions and
proved very fruitful as to the economy in time and the accuracy of results especially
in comparison with other methods.

RÉSUMÉ

On présente dans cette communication quelques méthodes d’estimation des débits
maxima et de détermination des hydrogrammes de crue en utilisant des modèles
électroniques dont l’équipement est décrit.

Sur la base de cette estimation, la théorie du temps de propagation intervient pour
tenir compte de la transformation du débit et de l’hydrogramme d’écoulement en accord
avec la formule dite génétique connue en mathématiques comme l’intégrale de Duhamel
et utilisée dans la théorie de l’hydrogramme unitaire. La valeur de l’écart entre les
maxima de la pluie et de la crue (lag) est pris comme paramètre principal, ainsi que
le coefficient de forme de la crue, défini comme le rapport entre le temps de descente
et le temps de montée de l’hydrogramme. On peut considérer le temps de propagation
comme étant constant durant la crue (système linéaire) ou aussi comme se modifiant
(système non linéaire).

It is a matter of general experience that the most rational scheme of flood formation
can be based upon the theory of "time of travel" describing the process of hydrograph
origination in a gauging section (outlet) as a result of snow-melt water and rain water
running off the slopes, their travel through the channels and the summation of elemen-
tary discharges formed in different parts of the basin. This scheme is analytically
expressed by the so-called genetic formula which may be written as follows:

\[ Q_t = \int_0^{t-\tau} h_{t-\tau} f_{t-\tau} \, d\tau, \]  \hspace{1cm} (1)

where

\( Q_t \) discharge in the gauging section at the moment \( t \);
\( h_{t-\tau} \) intensity of precipitation (minus infiltration losses) falling in the basin at the
moment \( t-\tau \);
distribution curve of the unitary runoff-plots or the so-called "time-area-concentration curve" (travel time curve).

The above mentioned scheme of the flood formation described by the genetic formula of runoff can be modelled by means of an electric circuit comprising a linear system of \( n \) elements connected in series with resistance \( R \) and capacitance \( C \), or \( RC \)-elements. The scheme forms the basic principle of the computing block in a special electronic analogue computer (modelling equipment) \( 3\text{MY} \) extensively used in hydrological organizations of this country and referred to as \( 3\text{MY} \) \text{PP-27}.

To determine the reaction of the linear electric system \( i(t) \) caused by time-variable voltage \( U_t \), while investigating the transients in electric circuits, the so-called Duhamel integral is often used, which under the zero initial conditions may be written as:

\[
    i(t) = \int_0^{t=\tau} u_{t-\tau} P_\tau \, d\tau, \tag{2}
\]

where

\( P_\tau \) is integral characteristic of the system, called weight function and representing the reaction of the system to a unit impulse applied at its input. For the values of \( R \) and \( C \) similar for each element, the weight function will be:

\[
    P_{n(t)} = \frac{n^{n-1}}{T^n(n-1)!} \cdot e^{-t/T}, \tag{3}
\]

where

\( T = R \cdot C \) is time constant of a single element;
\( n = \) number of \( RC \) elements.

It is evident from the analogy of expressions (1) and (2) that, if we apply at the input of an electric circuit comprising \( RC \) elements voltage \( U_t-\tau \), which is as variable in time as the precipitation \( h_t-\tau \) in the catchment area, and if we select the weight function of the system \( P_\tau \) in accordance with the time — area — concentration curve (time of travel curve) \( f_\tau \), then the reaction at the input \( (t) \) will correspond (in a definite scale) to the discharges \( Q_\tau \) in the gauging section. This curve is expressed therewith as function (3) depending on two parameters \( T \) and \( n \), possessing a great flexibility and adequately describing the natural shape of the observed unigraphs.

The parameters of function (3) \( T \) and \( n \) can be determined in a rapid and reliable way by selecting actual precipitation, applying the electronic analogue computer \( 3\text{MY} \), proceeding from the best conformity of the observed and the machine—calculated hydrographs in the gauging section of the catchment area. To match the parameters \( T \) and \( n \) the observation data from 120 highest floods on 55 streams of the Far East and the south of the USSR European territory have been used, with the catchment areas of 0.70 to 20000 sq. km and the time unit of 10 minutes to 24 hours. The analysis of the values \( T \) and \( n \) obtained for all the floods and their comparison to different hydrograph characteristics have proved that there exists an approximate relationship between the value \( \tau_{\text{total}} = T \cdot n \cdot M_e \) (where \( M_e = \) machine scale multiplier) and the value of lagging time \( t_{\text{lag}} \) of the maximum discharge relative to the gravity centre of effective precipitation. The linear relationship obtained will be:

\[
    \tau_{\text{total}} = 1.30 \cdot t_{\text{lag}}, \tag{4}
\]

The value of lag time \( t_{\text{lag}} \) in our hydrological literature has been recommended back in 1937 as the integral characteristic of the basin and the time of travel, being at present adopted for the analysis of the floods by the hydrologists of many countries. The advantages of the lag time characteristic \( t_{\text{lag}} \) consists in its concreteness and in the
possibility of objective definition on the basis of observation data, while the generally used for the flood estimates conception of the summary time of travel or time of concentration cannot be determined unambiguously on the basis of observation data and, while possessing a very modest precision, it is interpreted by many authors in a different way.

As regards the \( n \) parameter, the latter, as shown by the analysis of estimated floods, is in an approximate inverse relationship with the hydrograph form factor
\[
\gamma = \frac{t_f}{t_r} \quad (t_f = \text{time of the hydrograph fall with cutting the ground train}; \ t_r = \text{time of the hydrograph rise})
\]

\[
n = \psi(\gamma)
\]

The obtained relationships (4) and (5) enable us to establish the numerical value of the parameters of the time—area—concentration curve (3) \( T \) and \( n \) according to the known values \( t_{lag} \) and \( \gamma \), which are easy to determine for the streams, where the rainfall and runoff are observed, with the aid of the information on the preceding floods. For the streams which have not been investigated those values may be obtained from the characteristics of catchment areas, defined with the use of survey maps.

Thus, the main hydrological characteristic—the value of lag time \( t_{lag} \), which is necessary for the definition of the value \( \tau_{total} = T \cdot n \cdot M_t \) according to the relationship (4), is determined for the investigated streams on the basis of direct observation data and for the non-investigated streams—on the basis of approximate empirical formulae, obtained for various physiographic conditions as follows:

\[
t_{lag} = K_p \frac{F^m \cdot B^z}{I^y} \cdot \left( \frac{Q_{1\%}}{Q_{max}} \right)^x \cdot f_{sw} \cdot f_w
\]

(6)

where
- \( K_p \): coefficient of proportionality;
- \( F \): catchment area;
- \( I \): stream gradient;
- \( B \): form factor of the catchment area;
- \( Q_{1\%} \): maximum discharge of one per cent frequency;
- \( Q_{max} \): maximum discharge during flood, for which the value \( t_{lag} \) is determined;
- \( f_{sw}, f_w \): factors taking into account the grade of land swamping and woodiness;
- \( m, z, y, x \): exponents.

For streams of the steppe and wood-steppe zones in the European part of the USSR:
- \( m = x = 0.33; \ y = z = 0.25 \); for the rivers of the Far East: \( m = 0.20; \ y = x = 0.33; \ z = 0. \)

Formula (6) enables us to determine the value of the lag time for non-investigated catchment areas which corresponds to the maximum discharges of different frequency. The form factor of the floods \( \gamma \) is completely determined by the physiographic and hydrographic conditions of the region and, while varying within rather narrow limits, it may be determined for different catchment areas according to the data of analogue basins.

The above described scheme for the estimation of maximum discharges and hydrographs of floods with the aid of electronic analogue computer \( 3M_y \) has been tested on many investigated and non-investigated streams in different regions of the USSR, and ensured quite satisfactory results in comparison with observed maxima and hydrographs of floods.

In particular, to check the above described scheme of estimation, values have been determined with the aid of the electronic analogue computer \( 3M_y \) ПР-27 for maximum
discharges on 26 small non-investigated streams with catchment areas of 3.0 to 1900 sq.km according to the initial data on rainfall in July, 1958, in the basin of the Upper Amour which resulted in a very high flood with the probable frequency of 1:100 years. When the values of $Q_{\text{max}}$ obtained on the electronic analogue computer 3MY were compared to the maximums determined in those catchment areas according to the traces of high water and to hydraulic formulae directly after the flood has passed the conformity was rather good, with a mean deviation of $+19\%$. This mean error of the maximum discharge values estimated on the electronic analogue machine 3MY is smaller than for various formulae, including the regional formula of the maximum runoff with accurately defined parameters. Moreover, the estimation on 3MY enabled to define at the same time with the maximum discharges the whole run-off the flood hydrograph, having a complicated two- and three peak shape, which is of great importance for the storage reservoir projects.

On the basis of the above described scheme for the estimation of floods with applying the electronic analogue computer 3MY np-27 in the steppe and wood-steppe zones of the European part of the USSR, a practical method has been developed for estimating maximum rainstorm discharges and hydrographs with 1% frequency for small non-investigated streams. The method includes the definition of the estimated rainstorm, the values $t_{\text{ag}}$ and parameters $T$ and $n$ according to the relationships (6), (5) and (4), recommendations on the definition of the runoff coefficient $\alpha$ and the form factor $\gamma$, as well as reduction of the rainfall layer in the area.

The developed method has been tested on 57 catchment areas of 0.12 to 495 sq.km situated in the Ukrainian steppe and wood-steppe zones. The values obtained with the use of 3MY np-27 for maximum discharges at 1% frequency on those streams have been compared to the discharges defined according to the observation data. The mean estimated error proved to be $\pm20\%$. The advantages of this method as compared to the existing empirical formulae consist in the genetic basis, physical evidence, economy in time of calculations and mainly in the possibility of obtaining not only the maximum ordinate, but also the whole estimated hydrograph.

The comparison of hydrographs has proved that the shape of the estimated floods determined by the 3MY shows, in general, a good conformity with the typical hydrographs obtained for small non-investigated catching areas of the region under consideration.

When the runoff velocities during the formation of some individual flood phases differ sharply from one another and the above described linear scheme (which is based on the assumption of constant velocities of travel during the flood) does not offer adequate results, then the estimation is performed with the non-linear scheme taking into account the variation in the time of travel depending on the discharges in the gauging section. Estimation of floods with the variable time of travel is effected with the use of universal nonlinear 3MY (in particular HM-7) by the solution of the following system of equations:

\[
\begin{align*}
\frac{dQ_1}{dt} &= \frac{1}{\tau_1} (Q_1 - Q_1) \\
\frac{dQ_2}{dt} &= \frac{1}{\tau_2} (Q_2 - Q_2) \\
&\vdots \\
\frac{dQ_n}{dt} &= \frac{1}{\tau_n} (Q_n - Q_n) \quad (7)
\end{align*}
\]
used at $\tau_1, \tau_2, \ldots, \tau_n$ const by G. P. Kalinin and P.I. Milyukov for the river bed and by Nash for the catchment area, who divided the whole catchment area (river bed) into $n$ stretches, assuming that the controlling action of each stretch corresponds to the controlling action of the storage reservoir.

In equations (7): $g_t$ = water supply to the basin,

$Q_n$ = discharges in the gauging section, $\tau_1, \tau_2, \ldots, \tau_n$ = time of travel in the stretches, depending on water discharges.

Equations (7) at $\tau_1 = \tau_2 = \ldots = \tau_n = \text{const}$ are solved with the Duhamel integral, the time-area-concentration curve being expressed as a function similar to (3).

A scheme for the solution of equations (7) has been composed on the 3My MH-7, which permits to estimate the floods for catchment areas possessing no hydrometric observation data. The values $\tau_1 = \tau_2 = \ldots = \tau_n$ are determined therewith in accordance with the minimum lag time of the catchment area and taking into account the constant coefficient, which indicates the relation of the minimum time of travel during the flood to the mean time. The dependence of the time of travel on discharges in the gauging section are assumed in accordance with formula (6) and are given in graphical form in the nonlinearity-blocks of the machine.

The comparison of hydrographs estimated on MH-7 by means of the nonlinear scheme and the observed hydrographs for 50 floods differing in shape, assuming the actual runoff coefficient, produced quite satisfactory results with the mean estimation error of maximums for all peaks equalling $\pm 13\%$, while the error of $\leq \pm 5\%$ has been assured on 46%.

The results obtained permit us to presume that the nonlinear scheme of flood estimation reflects adequately the dynamics of the natural process of forming the runoff from the basins and may be used for practical estimates on different streams either investigated or non-investigated.

The above described schemes of flood estimates may be similarly applied in principle to the spring flood. In fact, the main condition of applying the genetic formula (1) — the uniformity of water yield on the whole catchment area, is fulfilled for the spring flood on the rivers of the plains even in a greater degree than for rainfall floods, as the intensity of the rainfall has a greater variation in time and space. To check the applicability of the estimation scheme with applying 3My np-2 to the spring floods, the machine-estimations of spring flood hydrographs have been performed on large and small streams using daily- and hour time unit. Parameters $T$ and $n$ for each flood have been defined therewith by the relationships (4) and (5) in accordance with the real values $\tau_{ag}$ and $\gamma$, while the initial data were presented by the course of the water yield of the indicator-basins or the water yield of snow calculated from the intensity of snow-melting by conventional methods. The comparison of natural hydrographs during spring flood and the hydrographs estimated on 3My has shown a quite satisfactory conformity both for the shape of hydrographs and for the maximum ordinate. Thus, a conclusion can be made about the possibility of using the above described scheme for spring flood estimation and forecasts both on small streams with sharply expressed diurnal variation of stream flow and for larger rivers, where no diurnal variation of stream flow during spring period is observed.

In the absence of hydrometric observations the estimation may be effected according to the estimated course (variation) of water yield and the runoff coefficient with the definition of values $\tau_{ag}$ and $\gamma$ in accordance with the characteristics of catchment areas for various physiographic regions.

347