QUANTITATIVE INTERPRETATION
OF REGIONAL GROUNDWATER FLOW PATTERNS
AS AN AID TO WATER BALANCE STUDIES

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RéSUMÉ

Interprétation quantitative des réseaux régionaux d'écoulement des eaux souterraines, dans le cadre de l'étude du bilan hydrique du bassin de drainage du Lac Good Spirit, en Saskatchewan.

Le débit naturel du bassin est une mesure de l'écoulement dynamique d'un bassin d'eaux souterraines. On peut l'évaluer à partir d'une interprétation quantitative des réseaux régionaux d'écoulement des eaux souterraines obtenue à l'aide d'un modèle mathématique approprié. Le débit naturel du bassin est un élément utile qui contribue à déterminer le bilan hydrique de l'ensemble du bassin, car il peut constituer un lien valable entre les mesures hydrologiques normales effectuées aux extrémités d'alimentation et d'évacuation du système d'écoulement.

Dans la présente étude, les réseaux d'écoulement des eaux souterraines, déterminés par une analyse à l'aide d'un modèle mathématique du bassin de drainage du lac Good Spirit dans le centre est de la Saskatchewan (Canada), sont confirmés par la cartographie sur le terrain des phénomènes d'évacuation et les mesures au piézomètre. Les réseaux d'écoulement d'une partie du bassin sont analysés quantitativement pour en arriver à son débit naturel. Cette valeur est alors comparée aux mesures et estimations disponibles des autres composants du bilan hydrique.

Abstract

The natural basin yield is a measure of the dynamic flow through a groundwater basin. It can be estimated from a quantitative interpretation of the regional groundwater flow patterns obtained as solutions to an appropriate mathematical model. The natural basin yield represents a meaningful component of basin-wide water balance determinations in that it can provide a valuable link between the standard hydrologic measurements carried out at the recharge and discharge ends of the flow system.

In this paper, the groundwater flow patterns determined by a mathematical model analysis of the Good Spirit Lake drainage basin in east-central Saskatchewan, Canada are confirmed by field mapping of discharge phenomena and by piezometer measurements. The flow patterns in a portion of the basin are analyzed quantitatively to arrive at the natural basin yield. The value is in good correlation with the available measurements and estimates of the other components of the water balance.

1. Introduction

The "average annual" water balance equation can be stated as:

\[ P = R + U + E \]  

(1)

where:

- \( P \) = average annual precipitation;
- \( R \) = average annual runoff;
- \( R_s \) = average annual surface water component of surface runoff;
- \( R_g \) = average annual groundwater component of surface runoff.
\( R_g = \) average annual groundwater component of surface runoff;
\( U = \) average annual subsurface outflow;
\( E = \) average annual evapotranspiration.

Equation (1) is valid in undeveloped basins where there is no surface or subsurface inflow. It must be applied to a period of study spanning several climatologically representative years. Over such a period, the average annual values of the change in surface water, groundwater, and soil moisture storage can be assumed negligible.

It is probably safe to say that the terms in equation (1) that have proved most resistant to accurate measurement or reliable estimation are the two groundwater terms \( R_g \) and \( U \). In this paper, a method is presented whereby quantitative interpretation of steady-state regional groundwater flow patterns (representative of the average conditions over the period of study) can be used to determine these groundwater terms. An added feature of the method is that an examination of the existing groundwater flow patterns can provide an insight into the areal variations of surface runoff and evapotranspiration.

In the first section of the paper, the nature of regional groundwater flow is discussed, and the simulation of flow patterns using a mathematical model is briefly described. In the second section of the paper, the method is applied to the Good Spirit Lake drainage basin in east-central Saskatchewan, Canada.

2. Regional groundwater flow patterns

2.1 Definitions

Tóth (1963) defined a (groundwater) flow system as “a set of flow lines in which any two flow lines adjacent at one point of the flow region remain adjacent through the whole region, and that can be intersected anywhere by an uninterrupted surface across which flow takes place in one direction only”.

The three-dimensional closed system that contains the entire flow paths followed by all water recharging the system has been termed by Freeze and Witherspoon (1967) a groundwater basin.

The surface of the basin can be divided into recharge areas where the direction of groundwater flow is away from the water table, and discharge areas where the direction of groundwater flow is toward the water table.

Natural groundwater recharge is that water which percolates down through the unsaturated zone to the water table and actually enters the dynamic groundwater flow system. This definition excludes that portion of the moisture surplus that increases the soil moisture content but does not enter the flow pattern itself. The term is not to be confused with the actual areal precipitation which in some cases may lead to groundwater recharge and in other cases may not.

Natural groundwater discharge is that water which is discharged from the flow system by means of baseflow, “artesian feeding” of lakes and rivers, springs, seepage and evapotranspiration.

The water table is considered to be an imaginary surface beneath ground level at which the absolute pressure is atmospheric. It is assumed to be the upper boundary of the saturated flow system.

The existence of a three-dimensional groundwater flow system implies the existence of a corresponding three-dimensional potential field. A suitable potential function is the hydraulic head which is the sum of the pressure head and elevation head. At any point in the potential field, the hydraulic head equals the elevation, above a standard datum, of the liquid level in a piezometer inserted at that point.
2.2 Mathematical Model Approach

It is possible to simulate steady-state regional groundwater flow patterns in a three-dimensional, non-homogeneous, anisotropic groundwater basin by obtaining solutions to an appropriate mathematical model.

The partial differential equation that describes the flow is the steady state form of Richards' equation:

$$\frac{\partial}{\partial x} \left[ K(x, y, z) \frac{\partial \phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K(x, y, z) \frac{\partial \phi}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K(x, y, z) \frac{\partial \phi}{\partial z} \right] = 0 \quad (2)$$

where:

\begin{itemize}
  \item $x, y, z$ = coordinate directions;
  \item $K(x, y, z)$ = non-homogeneous, anisotropic permeability function, in which the principal directions of anisotropy coincide with the coordinate directions;
  \item $\phi$ = hydraulic head.
\end{itemize}

This equation is developed from Darcy's law and the steady-state equation of continuity.

The boundary conditions are determined on the assumption that the groundwater basin is bounded on the bottom by a horizontal impermeable basement, on the top by the water table, and on all sides by imaginary vertical impermeable boundaries which simulate the groundwater divides.

The boundary value problem described by equation (2) and the appropriate boundary conditions can be solved using a finite-difference technique and a digital computer. With this method of solution the continuum of points making up the field and its boundaries is replaced by a finite set of points (nodes) arranged in a three-dimensional grid over the basin. The partial differential equation is then replaced by a system of finite-difference equations which can be solved by the extrapolated Liebmann method of successive overrelaxation. A complete description of the method and its mathematical development is included in Freeze and Witherspoon (1966).

In order to determine the regional groundwater flow pattern in a given basin using this mathematical model approach, it is necessary to know:

1. The dimensions of the basin;
2. The water-table configuration;
3. The permeability configuration resulting from the subsurface stratigraphy.

The computer output is in the form of printed values of the hydraulic head at each node in the grid and machine plotted equipotential nets in vertical cross-sections through the basin. Flowlines can be constructed to complete the flow-net.

2.3 Field Mapping and Measurement

It is possible to "map" regional groundwater flow patterns using field techniques. Such an approach can be used to confirm flow patterns determined from mathematical models.

Field methods can be divided into two classes: (a) those which provide direct quantitative measurements of the groundwater flow system, and (b) those which are qualitative and interpretive in nature. In the first class would fall direct measurements of hydraulic head by means of piezometer nests. In the second class are hydrogeochemical correlation, and the mapping of surficial evidence of groundwater discharge. In the Canadian prairies, this latter method was pioneered by Meyboom.
(1963). He referred to the following surface manifestations of groundwater discharge as "groundwater outcrops".

(1) Saline sloughs, lakes and rivers;
(2) Saline soils and their associated vegetation;
(3) Springs, seepage areas and swamps;
(4) High water-table levels, and flowing wells.

2.4 Natural Basin Yield

The potential pattern determined by a mathematical model and confirmed by field measurement, can be used to construct a basin-wide quantitative groundwater flow net. Such a flow net can be interpreted in terms of the quantity of flow through the basin. For an undeveloped basin under natural conditions this quantity of flow has been defined by Freeze and Witherspoon (in press) as the natural basin yield. In that the steady-state theory on which the concept is based, gives the long-term average of the potential distribution, it is reasonable to consider the natural basin yield as an average annual value of the dynamic groundwater flow. Because this flow enters the system in recharge areas and leaves from discharge areas, natural basin yield can be considered as a measure of the average annual groundwater recharge (which equals the average annual groundwater discharge).

In basins where large permeability contrasts or high factors of anisotropy exist, construction of a quantitative flow net becomes exceedingly complex, and in some cases virtually impossible. Under such circumstances, the natural basin yield can be calculated by determining the quantity of recharge and discharge over the basin. This requires systematic application of Darcy's law at the top of each nodal column, using the known permeability values and the near-surface hydraulic gradients calculated from the computer output. This approach is used in the following treatment of the Good Spirit Lake drainage basin.

The practical application of the concept of natural basin yield is limited to certain hydrologic environments. In particular, the method is not applicable in regions with strongly fluctuating water tables or where the direction of groundwater flow reverses with the season. It should also be noted that the concept is defined in terms of natural flow in virgin basins and does not consider the effects of groundwater development. The natural basin yield is presented here as an average annual value. It can be shown, however, that it does not fluctuate significantly with time and is relatively independent of rainfall conditions. It represents a unique property of a basin and one which can be of use in water-balance studies.

2.5 Recharge-Discharge Profile

An interesting property of regional groundwater flow is the variation in the rate of groundwater recharge and discharge from place to place over the basin (Freeze and Witherspoon, in press). The positions of concentration of recharge and discharge depend on the water table configuration and the geometry of the subsurface permeability variations (Freeze and Witherspoon, 1967). In general, groundwater discharge tends to be concentrated in major valleys and above the permeability pinchout of subsurface aquifers. High rates of recharge usually occur near the groundwater divides beneath topographic highs, in the vicinity of steep valley flanks, and above high permeability lenses. Sloping aquifers and aquicludes which outcrop at the surface can create patterns of recharge and discharge that are difficult to anticipate and that result in concentrated rates at various points on the surface depending on the configuration of the sloping layers.
When the results of regional flow pattern analysis are presented as two-dimensional vertical sections across the basin, the values of the rate of recharge or discharge at various points along the water table can be plotted to form a recharge-discharge profile. To present the results of a three-dimensional analysis, a map of the basin can be prepared that shows contoured values of the recharge and discharge rates.

2.6 Application to Water Balance Studies

The use of mathematical models to determine natural basin yield can be an important tool in the determination of basin-wide water balances. In the simplest case, where discharge areas are limited to river valleys or lake bottoms, the natural basin yield is a measure of \( R_s \), the average annual groundwater component of surface runoff. In more complex cases such as where surface runoff is intermittent or drainage poorly integrated, discharge may be divided between baseflow (during streamflow periods), evapotranspiration from the streambed (during dry periods), and evapotranspiration from other discharge areas such as sloughs, swamps, and saline flats. Quantitative groundwater flow nets can also be used to compute, directly, the average annual subsurface outflow, \( U \). The analysis of the Good Spirit Lake drainage basin provides an example of the use of the method in an environment where \( R_s = U = 0 \) and the interpretation becomes relatively simple.

Because the groundwater flow pattern must be in dynamic equilibrium with the other components of the hydrologic cycle, its configuration has an important effect on the quantity and areal concentration of such parameters as infiltration, evapotranspiration and surface runoff. The recharge-discharge profile provides an insight into the areal variations in the values of these hydrologic parameters. For example, if one analyzes the streamflow records at several stream-gauging stations in a basin, one will find that the baseflow varies along the length of the stream, depending on the position of the gauging site in the recharge-discharge profile. In fact, the stream will be influent as it crosses a recharge area and effluent as it crosses a discharge area.

Areal variations in the rates of groundwater recharge and discharge can also help to explain variations in the rates of evapotranspiration and infiltration. In discharge areas where there is a constant supply of water from below, the actual evapotranspiration will presumably equal the potential evapotranspiration (as calculated by some method such as Thornthwaite, 1948, or Penman, 1948), whereas in recharge areas the actual evapotranspiration will be less than the potential and must be calculated by a soil moisture budget technique such as that of Holmes and Robertson (1960). Similarly, soil moisture flow in the unsaturated zone during infiltration will be influenced by the ability of the saturated zone to accept the percolating water into the regional groundwater flow system. The quantity of water which can be absorbed into the flow system at any given place is dependent on the location of the site in the recharge-discharge profile.

3. Good Spirit Lake Drainage Basin

3.1 Location, Topography and Drainage

The Good Spirit Lake drainage basin, occupies an area of 435 square miles in the aspen parkland region of east-central Saskatchewan, Canada (inset, fig. 1) at latitude 51°30′, longitude 103°00′. The area is being studied as a “representative basin” in Canada’s International Hydrologic Decade program.

The topography and drainage of the study area are shown in figure 1. The total relief is of the order of 400 feet. Intermittent drainage is provided during the spring
months from the northwestern upland into Good Spirit Lake via Spirit Creek and its tributaries. Good Spirit Lake is approximately 3 miles wide by 7 miles long but is extremely shallow; it has a maximum depth of less than 15 feet. In dry years, the lake does not overflow and the basin becomes one of internal drainage. After several wet years in succession, however, overflow to the south into Horseshoe Lake has been recorded. Horseshoe Lake in turn drains into Whitesand River, a tributary of the Assiniboine.

The boundary of the basin shown by the dashed line in figure 1 is somewhat arbitrary. The lack of integrated drainage and the presence of considerable depression storage, which are common features of the semi-arid glaciated Canadian prairies, lead to fluctuations in the size of the drainage area under the influence of variations.
in annual rainfall and runoff. The uncertainty of the position of the topographic divide and hence the groundwater divide, led the author to choose a study area which extends to the bounding streams on either side of Good Spirit basin. Reference to figure 1 shows the large horseshoe encompassing the basin that is created by Whitesand River and Crooked Hill Creek.

3.2 Water Table Configuration

Water table records from 9 shallow observation wells in the basin show that the average annual depth to the water table varies from 5 to 20 feet. In that the total relief over the basin is 400 feet, it is clear that on a regional scale the water table reflects the overlying topography. On a local scale it appears that only the more...
pronounced topographic highs and lows are reflected by the water table configuration.

For the purposes of specifying the upper boundary of the mathematical model, it was assumed that the water table configuration is identical to the regional topography. Topographic maps with a scale of 1:50,000 and a contour interval of 25 feet were used. With this contour interval and a wide spacing of nodes in the mathematical model, the minor relief is not reflected in the water table configuration. The seasonal water table fluctuations in the Good Spirit basin, as recorded in the 9 observation wells, are in the range of 2 to 5 feet. This is less than 1% of the total saturated depth of the near-surface regional groundwater flow system.

No investigations have yet been carried out to determine if there are seasonal reversals in water table slope (and thus in flow direction) around the sloughs in the basin. The chosen water table is assumed to represent the average long-term position of the water table and the results, the average annual conditions.

3.3 Permeability Configuration

Figure 2 is a map of the surficial geology of the area (after Cherry, 1966). There are three types of deposits: lake sediments of sand and silt, outwash gravel and glacial till. The lake deposits overlie the glacial till to a depth of 0-40 feet in the vicinity of Good Spirit and Horseshoe Lakes. Between the two lakes and to the west of Good Spirit Lake, the sand has been reworked into dune deposits. Farther west, the lake sediments interfinger with shallow outwash gravels which overlie the glacial till. Throughout the remainder of the area, glacial till is exposed at the surface, although in some places, it is veneered with thin discontinuous outliers of lake silt or outwash gravel. The thickness of the glacial till varies from 30 feet in the Whitesand River valley in the southeast to over 300 feet in the northern upland. It is underlain throughout the area by a very thick section (> 1000 feet) of marine shale of Cretaceous age. The surface of the bedrock formation shows less relief than the present topography. For the purposes of the mathematical model, the surface has been taken as a horizontal plane at elevation 1500.

The most important hydrologic property of the geological deposits is their permeability. In order to determine permeability values for each of the formations, tests were carried out, using the method of Hvorslev (1951), in each of 25 piezometers which bottomed in the various formations. The piezometers were arranged in nests of 1, 2 or 4 piezometers at 8 different sites. The locations of the sites are shown on figures 3 and 4; the depths of the piezometers and the formations in which they bottom are listed in table 1.

Permeability values obtained from Hvorslev tests are a combination of the vertical and horizontal components. However, the method does not indicate a value for the degree of anisotropy so it is possible to interpret the results in an infinite number of ways depending on the assumed anisotropy of the formation. In table 1 three interpretations of the results of the permeability tests in the 25 piezometers are presented: (1) the isotropic case where the horizontal permeability $K_h$ is assumed equal to the vertical permeability $K_v$, (2) $K_h = 10 K_v$ and (3) $K_h = 100 K_v$. For each case, both the $K_h$ and $K_v$ values are presented in inches per hour. An examination of the results shows that the horizontal permeability $K_h$ is only slightly changed by an increase in the order of magnitude of the vertical permeability. In other words, the Hvorslev test is strongly biased in favour of $K_h$ and can therefore be used as a measure of the horizontal permeability. In the absence of suitable field methods, the degree of anisotropy must be estimated. Table 2 lists the median values of the field results quoted in Table 1 and the corresponding values of horizontal permeability used in the mathematical model. The assumed anisotropy factors and vertical perme-
<table>
<thead>
<tr>
<th>Piezometer Nest</th>
<th>Number</th>
<th>Depth-Feet</th>
<th>Formation</th>
<th>Permeability — inches/hour</th>
<th>Permeability — inches/hour</th>
<th>Permeability — inches/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$K_h:K_v = 1:1$</td>
<td>$K_h:K_v = 10:1$</td>
<td>$K_h:K_v = 100:1$</td>
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<td></td>
<td></td>
<td></td>
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<td>$K_h$</td>
<td>$K_v$</td>
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<td>$6.1 \times 10^{-3}$</td>
<td>$8.0 \times 10^{-3}$</td>
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<td></td>
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<td>Till</td>
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<tr>
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</tr>
<tr>
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<td>Till</td>
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<td>$1.1 \times 10^{-3}$</td>
<td>$1.5 \times 10^{-3}$</td>
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<tr>
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<td>$2.2 \times 10^{-2}$</td>
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<td>4</td>
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<td>Lake sediment, sand</td>
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<td>$0.23$</td>
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<td>$8.9 \times 10^{-5}$</td>
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<tr>
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<td>Shale</td>
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<td>$1.7 \times 10^{-4}$</td>
<td>$2.2 \times 10^{-4}$</td>
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<tr>
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<td>Till</td>
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<td>$6.3 \times 10^{-3}$</td>
<td>$8.1 \times 10^{-3}$</td>
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<tr>
<td></td>
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<td>Shale</td>
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<td>$2.4 \times 10^{-2}$</td>
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<tr>
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<td>$9.5 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-3}$</td>
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<tr>
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<td>Shale</td>
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<td>$9.8 \times 10^{-3}$</td>
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</table>

* Piezometer U-1 gives anomalous permeability measurements, so has been ignored in the calculation of median permeabilities listed on Table 2.
abilities built into the model are also listed. The anisotropy values have been chosen on the basis of past experience, a few reported results, and geological considerations (for example: the homogeneous, isotropic nature of dune sand, the presence of horizontally oriented sand lenses of higher permeability within the glacial till, and the sedimentary nature of the shale).

The decrease in permeability with depth within the glacial till at any given site, which is indicated by the field results (table 1), is not included in the mathematical model.

3.4 Flow System as Determined by Mathematical Model

The mathematical model approach described in section 2.2 has been applied to the Good Spirit Lake drainage basin. A $30 \times 46 \times 20$ array of nodal points has been used.
used to obtain the finite-difference solution. Figure 3 shows the nodal grid in the two horizontal coordinate directions. The nodes are numbered from \( I = 1 \) to \( I = 30 \) in the \( x \)-direction (north-east) and from \( K = 1 \) to \( K = 46 \) in the \( y \)-direction (north-west). The nodal spacing in each of the horizontal directions is 4610 feet. In the vertical direction the nodes are numbered from \( J = 1 \) at the basal elevation of 1050 to \( J = 20 \) at elevation 2000. The vertical nodal spacing is 50 feet.

The upper boundary of the mathematical model is the water table. The value of the hydraulic head at the water table (ie — the elevation of the water table) is inserted at the nodal position closest to the true position of the water table at each areal nodal coordinate. As noted in section 3.2 the water table was assumed to reflect the regional topography.

The external boundaries of the mathematical model are shown on figure 3. They are vertical impermeable boundaries which extend to the full depth of the model.
TABLE 2
Field and Model Permeabilities

<table>
<thead>
<tr>
<th>Formation</th>
<th>Field No. of Samples</th>
<th>Field Median* permeability value</th>
<th>Mathematical Model K_h</th>
<th>K_v</th>
<th>K_h : K_v</th>
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<tr>
<td>Shale</td>
<td>5</td>
<td>$2.2 \times 10^{-4}$</td>
<td>$10^{-4}$</td>
<td>$10^{-6}$</td>
<td>100:1</td>
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<td>Glacial till</td>
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<td>$1.7 \times 10^{-3}$</td>
<td>$10^{-3}$</td>
<td>$10^{-4}$</td>
<td>10:1</td>
</tr>
<tr>
<td>Outwash gravel</td>
<td>1</td>
<td>14.1</td>
<td>1</td>
<td>1</td>
<td>1:1</td>
</tr>
<tr>
<td>Lake sand</td>
<td>1</td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* From Table 1.

On the basis of earlier hypothetical results (Freeze and Witherspoon, 1967), the assumption that there is no interbasinal groundwater flow as inferred by this boundary condition, appears to be well-founded for the particular combination of stratigraphy and water table configuration present in the Good Spirit basin. The basal impermeable boundary has been chosen at elevation 1050, 450 feet below the upper boundary of the low-permeability shale formation.

The geological stratigraphy has been determined on the basis of a study by Cherry (1966). Geological logs prepared during piezometer installation provided a second source of information. The three vertical cross-sections shown in figure 5 provide an illustration of the stratigraphy as simulated in the model.

In order to obtain a converged solution (tolerance 0.01 feet of hydraulic head) to the mathematical model, it was necessary to carry out 93 iterations using the extrapolated Liebmann method. The computer time needed for the computations on the IBM 7094 at the University of California at Berkeley was 1 hour and 40 minutes.

The results of the mathematical model are shown in two ways: as a map of groundwater discharge (fig. 3) and as flow patterns in representative vertical cross-sections (figure 5). In figure 3, the nodal columns where the hydraulic head increases with depth are labelled with a small circle. The areas enclosed by the dashed line encircling these nodes are the discharge areas. The major discharge areas are seen to be Spirit Creek, Good Spirit Lake, Whitesand River, Horseshoe Lake, Stonewall Lake and Crooked Hill Creek (see figure 1 for the locations of these names).

Figure 5 shows three representative vertical cross-sections through the mathematical model. Their location can be determined by reference to figure 3. The dashed lines are lines of equal hydraulic head (equipotential lines). Construction of flowlines on these sections is complicated by three factors:
1. The diagrams are drawn with an exaggerated vertical scale;
2. The formations are anisotropic and the degree of anisotropy is not the same in all formations;
3. The sections, being arbitrarily oriented parallel to the nodal mesh, are not necessarily parallel to the direction of flow in the three-dimensional model.
Under these circumstances, construction of flowlines with quantitative significance appears to be an impossible task. Instead random flow directions are indicated on the three sections. These directions have been constructed taking into account the three factors listed above, for the particular point where the arrows intersect the equipotential lines. In general, the flowlines are not perpendicular to the equipotential lines.

Above each cross-section in figure 5 a recharge-discharge profile is shown. At any given point, the rate of groundwater recharge or discharge is indicated by the \( \nu \)-coordinate of the profile. These values were determined through application of Darcy's law, at each water table node, using the assumed permeability value and the hydraulic gradient in the direction of flow, as determined from the mathematical model. At the right hand side of the profile, the scale is given in inches per hour as originally calculated. At the left hand side, the units have been converted into inches per year for use in water balance calculations. It should be noted that the scale is logarithmic. The recharge rate in the sand and gravel is several orders of magnitude higher than that in the glacial till. This fact is demonstrated more forcefully in figure 6, a map of the areal variation in the rates of recharge and discharge over a portion of the basin. This illustration will be discussed in detail in sections 3.6 and 3.7.
3.5 Field Correlation

In order to determine the validity of the results of the mathematical model, two field methods have been employed. The first is field mapping of the groundwater discharge areas by the methods outlined in section 2.3, and the second is direct measurement of the hydraulic head in piezometer nests.

Figure 4 indicates the results of the field mapping. Comparison of figure 4 with figure 3 shows that the results of the field mapping and the results of the mathematical model are in good agreement. There are, however, two discrepancies which deserve comment. One occurs in the vicinity of piezometer nest E south of Horse Lake. Here, there is no obvious field evidence of groundwater discharge but the mathematical
model predicts an upward gradient. As will be noted in the following paragraph, the piezometer results at site $E$ confirmed the model’s prediction.

The second notable difference between the two maps is on the sand-gravel plain west of Good Spirit Lake. The local discharge areas along the east-west trending depressions which traverse the plain (figure 4) are the result of topographic relief of the order of 10 feet. Such relief is not apparent on the 25 foot contoured topographic maps used to determine the elevations of the water table in the mathematical model. The discrepancy is unimportant on a regional scale, however, as the streamflow which arises from the groundwater discharge disappears into the sand farther downstream in many cases. In effect, the streams themselves are a part of the groundwater flow system and the entire plateau can be considered a recharge area.

The results of the piezometer measurements are given in table 3. The water-level elevations in the piezometers (which equal the hydraulic head at the base of the piezometers) are listed, together with the dates on which the readings were made. The measurements were taken on different dates at the various piezometer nests due to problems of winter access and due to the fact that certain piezometers stabilized later than others following a withdrawal of water for chemical sampling in the late fall of 1966.

The results are further presented in the form of vertical hydraulic gradients. For example, at nest $A$, the head decreases 2.80 feet in 152 feet of depth (from 30 to 182) leading to a downward vertical gradient of $0.0184$. In table 3 a positive gradient means the direction of groundwater movement is downward (recharge); a negative gradient means groundwater movement is upward (discharge). Sites $A$ and $R$ are in recharge areas; sites $J$, $U$, $B$, $E$ and $D$ are in discharge areas. Reference to the locations of the piezometer nests on figures 3 and 4 shows that this is in agreement both with the results of the mathematical model and, except for site $E$ mentioned above, with the field mapping.

The vertical hydraulic gradients determined from the mathematical model at the nodal columns closest to the piezometer sites (i.e. for nest $A$, $l = 14$, $K = 30$, see figure 3) are also presented in table 3. At nests $A$, $U$, $B$ and $D$ the agreement is good between the model and field results. At $J$ and $E$, the results are within an order of magnitude, and at $R$ there is a discrepancy of one order of magnitude. This correlation is quite satisfactory, especially if it is realized that one of the most important controlling parameters, the permeability, varies over 6 orders of magnitude.

The comparison of the horizontal gradients in the model with those of the piezometer measurements for the spans $A-J$, $R-J$, and $E-D$ are indicated in table 3. Here the agreement is near-perfect. Reference to the cross-sections in figure 5 shows that the predominant direction of groundwater flow is horizontal. The good agreement between the horizontal hydraulic gradients in the model and those measured in the field is therefore an indication that the mathematical model is quantitatively representative of the groundwater flow.

### 3.6 Natural Basin Yield of Sand-Gravel Plain

The Good Spirit basin can be divided into three sub-basins on the basis of the groundwater flow regime. The results of the mathematical model have been analyzed in detail for the central sub-basin which includes the sand-gravel plain to the west of the lake and a portion of the till plain to the east. The areal variation in the rates of recharge and discharge over the sub-basin is shown in figure 6. The contours were determined from the recharge-discharge profiles for various cross-sections, of which those shown on figure 5 are representative. It can be seen that on the major portion of the sand gravel plain west of the lake the average annual rate of recharge ranges from 1 to 5 inches whereas on the till plain east of the lake it is less than 0.01 inches.
### TABLE 3

**Hydraulic Head Measurements and Hydraulic Gradient Calculations in Field and Model**

<table>
<thead>
<tr>
<th>Piezometer Nest</th>
<th>Number</th>
<th>Depth-Feet</th>
<th>Formation</th>
<th>Date of Reading</th>
<th>Water level Elevation ft. a.s.l.</th>
<th>Hydraulic Head measured in piezometers</th>
<th>Hydraulic Gradient ft/ft.</th>
<th>Vertical***</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>182</td>
<td>Sand lens in till</td>
<td>29/1/67</td>
<td>1761.99</td>
<td></td>
<td>+ 0.0184</td>
<td>+ 0.0135</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>Till</td>
<td></td>
<td>1763.20</td>
<td></td>
<td>A to J</td>
<td>0.0071</td>
<td>A to J</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60</td>
<td>Till</td>
<td></td>
<td>1764.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>Till</td>
<td></td>
<td>1764.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>1</td>
<td>197</td>
<td>Shale</td>
<td>29/1/67</td>
<td>1604.49</td>
<td></td>
<td>-0.0040</td>
<td>-0.0193</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>149</td>
<td>Sand lens in till</td>
<td></td>
<td>1604.13</td>
<td></td>
<td>R to J</td>
<td>0.0023</td>
<td>R to J</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>80</td>
<td>Till</td>
<td></td>
<td>1604.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>60</td>
<td>Till</td>
<td></td>
<td>1604.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>1</td>
<td>200</td>
<td>Shale</td>
<td>8/1/67</td>
<td>1652.95</td>
<td>+0.0189</td>
<td>+ 0.0018</td>
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<td></td>
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<td></td>
<td>2</td>
<td>100</td>
<td>Shale</td>
<td></td>
<td>1654.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60</td>
<td>Till</td>
<td></td>
<td>1655.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>Till</td>
<td></td>
<td>1656.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>16</td>
<td>Outwash Gravel</td>
<td>29/1/67</td>
<td>1635.32</td>
<td>-0.0028</td>
<td>-0.0034</td>
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<td></td>
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<tr>
<td>U</td>
<td>1</td>
<td>149</td>
<td>Shale</td>
<td>12/2/67</td>
<td>1595.53*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>Shale</td>
<td></td>
<td>1593.97</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>64</td>
<td>Sand lens in till</td>
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<td>1593.88</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>22</td>
<td>Lake sediment, sand</td>
<td></td>
<td>1593.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>200</td>
<td>Shale**</td>
<td>12/2/67</td>
<td>1580.84**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>100</td>
<td>Shale</td>
<td></td>
<td>1583.73</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>Shale</td>
<td></td>
<td>1583.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>60</td>
<td>Shale</td>
<td>29/1/67</td>
<td>1583.39</td>
<td>-0.0163</td>
<td>-0.0023</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>2</td>
<td>30</td>
<td>Shale</td>
<td></td>
<td>1582.90</td>
<td></td>
<td>E to D</td>
<td>0.0020</td>
<td>E to D</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>90</td>
<td>Shale</td>
<td>29/1/67</td>
<td>1558.91</td>
<td>-0.0102</td>
<td>-0.0066</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>30</td>
<td>Shale</td>
<td></td>
<td>1558.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Piezometer U-1 gave anomalous permeability measurement (table 1) so hydraulic head measurement is also open to question. Vertical gradient at nest U is calculated on basis of upper 100 feet.

** Piezometer B-1 had not stabilized. Vertical gradient at nest B is calculated on basis of upper 100 feet.

*** A positive gradient means groundwater movement is downward (recharge); a negative gradient means groundwater movement is upward (discharge).
On the basis of figure 6, but in slightly more detailed form, it is possible to calculate the total annual quantity of recharge in the Good Spirit Lake sub-basin. Similarly, one can calculate the total quantity of discharge. As the sub-basin has no groundwater outflow it is clear that: Recharge = Discharge = Natural Basin Yield. The quantitative values are:

- Natural Basin Yield of Good Spirit Lake Sub-basin: 12,010 acre-feet
- Natural Basin Yield of Sand-Gravel Plain (west of lake): 11,500 acre-feet
- Natural Basin Yield of Till Plain (east of lake): 510 acre-feet

96% of the total flow in the sub-basin arises in the sand-gravel plain west of the lake.

3.7 Preliminary Water Balance Calculations

Preliminary water balance calculations have been carried out for the sand-gravel plain to the west of the lake in the Good Spirit Lake sub-basin. In the recharge portion of the plain we can simplify our original equation:

\[ P = R_s + R_g + U + E \] (3)

by noting that \( R_s = 0 \), \( U = 0 \), and \( R_g \) = natural basin yield (NBY). Therefore:

\[ P = NBY + E \] (4)

The nearest meteorological station with a suitable period of record is 20 miles south of Good Spirit Lake at Yorkton, Saskatchewan. For the 10-year period, 1955-1964, the average annual precipitation at Yorkton was 16.68 inches. The average annual potential evapotranspiration has been calculated using the methods of Penman (1948) and Thornthwaite (1948). The actual evapotranspiration has been estimated by applying the moisture budget technique of Holmes and Robertson (1960). The results are listed in table 4. The average annual actual evapotranspiration, based on Penman data, for the sand-gravel plain is 13.26 inches.

**TABLE 4**

*Evapotranspiration Calculations*

<table>
<thead>
<tr>
<th>Yorkton, Sask. 1955-1964</th>
<th>Thornthwaite Method</th>
<th>Penman Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average annual precipitation</td>
<td>16.68 inches</td>
<td>16.68 inches</td>
</tr>
<tr>
<td>Average annual potential evapotranspiration</td>
<td>21.43 inches</td>
<td>21.49 inches</td>
</tr>
<tr>
<td>Holmes and Robertson Moisture Budget</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average annual actual evapotranspiration (for gravelly sandy loam with soil moisture capacity of 4.0 inches).</td>
<td>13.15 inches</td>
<td>13.26 inches</td>
</tr>
<tr>
<td>Average annual moisture surplus</td>
<td>3.53 inches</td>
<td>3.42 inches</td>
</tr>
</tbody>
</table>

*Note:* The Penman calculations were carried out with the original Penman "f" factors, with albedo values of 0.20 (April-October) and 0.75 (November-March), and with sink strength coefficients of \( a = 0.18 \), \( b = 0.55 \).
We have now obtained, independently, values for the three parameters in equation (4). The results are listed in table 5. The error in the totals for the two sides of the equation is 2.8%.

**TABLE 5**

*Preliminary Water-balance Calculations for Recharge Area on Sand-Gravel Plateau in Good Spirit Lake Sub-basin*

<table>
<thead>
<tr>
<th>Symbol in Equation (4)</th>
<th>Inches over the recharge area</th>
<th>Acre-feet</th>
<th>Totals Acre-feet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precipitation</td>
<td>$P$</td>
<td>16.68</td>
<td>64800</td>
</tr>
<tr>
<td>Natural Basin Yield</td>
<td>$NBY$</td>
<td>2.96</td>
<td>11500*</td>
</tr>
<tr>
<td>Evapotranspiration</td>
<td>$E$</td>
<td>13.26</td>
<td>51500</td>
</tr>
</tbody>
</table>

* From section 3.6

The difference between the precipitation and the actual evapotranspiration in the Holmes and Robertson moisture budget technique is termed the moisture surplus. This is the quantity of water available for surface runoff and groundwater recharge. As no surface runoff takes place from the sand-gravel plain, the average annual moisture surplus should equal the average annual groundwater recharge. Referring to table 4, the Penman moisture surplus is 3.42 inches; the average annual groundwater recharge (which equals the $NBY$ in table 5) is 2.96 inches. This "inches over the recharge, area" value of $NBY$ is, of course, the areal average. The actual distribution of recharge as determined from the mathematical model, is shown on figure 6. The variations in the rate of recharge infer that the soil moisture capacity is not 4.0 inches everywhere on the recharge area. The capacities must vary with the soil moisture profile, the depth to the water table etc., as well as the soil type. It is clear that one cannot fully comprehend soil moisture phenomena without reference to the groundwater flow conditions, nor vice versa.

On the till plain east of the lake the soil moisture capacity is of the order of 8.0 inches, the actual evapotranspiration is 15.44 inches and the moisture surplus 1.24 inches. As the groundwater flow pattern analysis showed the groundwater recharge to be less than 0.01 inches, it is clear that virtually all the moisture surplus must become surface runoff in this area.

In a discharge area, equation (4) is not valid. Here, there is a continuous source of upward rising groundwater to help feed the evapotranspiration demands. In general:

$$P + \text{Groundwater Discharge} = E$$

In the small area bordering the west side of Good Spirit Lake which is a part of the major discharge area (fig. 6), the groundwater discharge is in the order of 1.0 inch per year. The annual actual evapotranspiration is therefore $16.68 + 1.0 = 17.68$ inches. The actual evapotranspiration would equal the potential of 21.49 inches (table 4) if the groundwater discharge were $21.49 - 16.68 = 4.81$ inches per year. Greater rates of groundwater discharge would give rise to surface runoff.
The water budget of the lake itself is presently under study but as yet, insufficient data are available. The methods outlined in this paper should prove valuable in estimating the subsurface inflow and possible outflow.

4. CONCLUSIONS, AND LIMITATIONS OF METHOD

4.1 Conclusions

1. It is possible to simulate steady-state regional groundwater flow patterns in a three dimensional, non-homogeneous, anisotropic groundwater basin by obtaining solutions to an appropriate mathematical model.

2. The natural basin yield is a measure of the dynamic flow through a groundwater basin. It can be determined by a quantitative analysis of regional groundwater flow patterns. It represents a unique property of a basin and one which can be of use in estimating the groundwater terms in the “average annual” water balance equation.

3. The method has proven to be an aid to preliminary water balance studies in the Good Spirit Lake basin, Saskatchewan, Canada.

4.2 Limitations

1. The steady-state assumption inherent in the mathematical development limits the application of the method to certain hydrologic environments. In particular, the water table must have the following properties:
   (a) The amplitude of the water table fluctuations must be small in comparison with the total saturated thickness of the groundwater basin.
   (b) The relative configuration of the water table must remain the same throughout the cycle of fluctuations; that is, the high points must remain the highest and the low points the lowest.

2. The general application of the mathematical model method of simulating regional groundwater flow patterns is at present (1967) restricted by computer limitations. The method requires a very fast computer with large internal storage and high precision. For more complex geological configurations than that of Good Spirit basin, computer time for a three-dimensional model can become prohibitive.

3. The results are only as good as the data. The stratigraphy must be well understood and the permeability values accurate to the correct order of magnitude. The lack of availability of suitable methods for measuring or estimating in-situ values of anisotropy is at present a limiting factor to the range of applicability of the method.

4. The Good Spirit study is in the early stages. Further field measurement and interpretation will allow improvements in the mathematical model and the refinement of quantitative results. The present reconnaissance model ignores the possible effects of local flow systems in the vicinity of sloughs.

5. The “average annual” approach ignores seasonal phenomena, in particular the effects of the rather severe prairie winter. While the method provides the overall quantitative result, it offers no evidence of the mechanism of groundwater recharge and discharge. This latter aspect is presently being investigated in the Good Spirit basin with the aid of instrumentation in the unsaturated zone.
ACKNOWLEDGEMENTS

Dr. P.A. Witherspoon of the University of California kindly provided access to the Berkeley computer centre. He also provided stimulation through our many discussions.

The field measurements were carried out with the help of Joel Grice, Richard Dunphy, and Herb Carlson, the latter of whom braved the winter's cold in the interests of science.

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