Determination of capillary conductivity and diffusivity of soil in situ

R. Koitzsch

ABSTRACT: The paper gives the theory of an instrument consisting of a tensiometer with a cylindrical cup for measuring capillary conductivity and diffusivity of soil in situ. The theory starts from a problem of heat conduction, in which a perfect conductor and the surrounding material are initially at different temperatures. It takes into account the influence of the finite hydraulic conductivity of the cup wall and the influence of an additional contact resistance which may sometimes exist. Suggestions are made for the design of the instrument and for reducing the number of observations. An overall reduction is possible when the influence of the cup wall can be neglected and when an additional contact resistance does not occur. The results of the experiments yield the capillary conductivity as well as the diffusivity, the latter with a smaller accuracy. In the other case—with the exception of two instances—only the capillary conductivity can be determined at present.

I. INTRODUCTION

In recent years the methods of determination of the capillary conductivity and diffusivity in disturbed or undisturbed soil samples have continuously been improved. But there are hardly any publications on the corresponding measurements in situ, for which Richards, Russel and Neal (1937) suggested the use of tensiometers. Indeed, a simple experimental method of measuring in situ the capillary conductivity and diffusivity of the soil surrounding the cup is offered by the balance which takes place after opening the air trap of a tensiometer for a short time.

Approximate values giving the time relation of the balancing process under special conditions have been recorded by Gardner (1961) and have been used for the determination of capillary conductivity. For a cylindrical tensiometer cup, the length of which is large compared to the radius, a theory of this balancing process will be outlined in the following sections, taking into account the influence of the tensiometer cup wall. From the theory ideas can be obtained as to the construction of the instrument and the evaluation of the measurements.

II. THE TENSIOmeter

A. CONSTRUCTION

A tensiometer consists of a porous, permeable ceramic cup connected by a water-filled system to a manometer. The manometer measures the negative pressure or tension in the
water in the cup which is in contact with the water in the surrounding soil. We examine a cylindrical cup, the length of which will be large compared to the radius.

**B. WATER TRANSFER THROUGH THE WALL OF THE Tensiometer CUP**

The transfer of water through unit length of the cup wall is given by Darcy’s law

\[ v = -2\pi k^* r \frac{dh}{dr} \]  

where:

- \( v \) is the amount of water in \( \text{cm}^2/\text{sec}^{-1} \) transferred per unit of time and length of the cup;
- \( k^* \) the hydraulic conductivity of the cup material in \( \text{cm/sec}^{-1} \);
- \( r \) the radius in cm;
- \( h \) pressure head in cm.

The materials for tensiometer cups are selected in such a way as to keep the amount of water in the cup wall constant, in space and time, within the range of application of such instruments. Then \( v \) is constant everywhere in the wall, and with constant hydraulic conductivity we get from eq. (1):

\[ r \frac{dh}{dr} = \text{constant} = \frac{h^* - H}{\ln (R/R_1)} \]  

Where:

- \( h^* \) is the pressure head on the outer wall \( r = R \) of the cup and
- \( H \) the pressure head on the inner wall \( r = R_1 \),

which is equal to the pressure head in the water in the cup and which is indicated on the manometer as tension \(-H\).

Thus eq. (1) yields

\[ v = -2\pi R \frac{k^*}{\ln (R/R_1)} (h^* - H) = -f\alpha_1 (h^* - H) \]  

with

\[ f = 2\pi R \quad \text{and} \quad \alpha_1 = k^*/R \ln (R/R_1). \]  

In eq. (4) \( f \) is the area per unit length of the cylinder with the radius \( R \) and \( \alpha_1 \) with the dimension \( \text{sec}^{-1} \) is a constant depending on the hydraulic conductivity of the cup material and on the radius of the cup. We call this constant the transfer index. The amount of water transferred through the wall of the cup per unit time and unit length is determined by the variation in time of the manometer reading. We write:

\[ v = -S \frac{dH}{dt} \]  

and understand by \( S \) the amount of water transferred from the manometer to unit length of the cup with a decrease in the pressure head \( H \) in the water of the cup by \(-1\) cm (increase in the indicated tension \(-H\) by \(+1\) cm). Let \( S \) be the water capacity per unit length of the cup; this quantity has the dimension cm. Thus we can represent the complete tensiometer by the cylinder \( r = R \), which consists of a perfect water conductor (everywhere
Determination of capillary conductivity and diffusivity of soil in situ

within the cylinder the pressure head is \( H \), whose water capacity per unit length is \( S \).

At the surface \( r = R \) there is a contact resistance of \( 1/\alpha_1 \) per unit area.

If the tensiometer cup is not in perfect contact with the surrounding soil, this fact can be taken into account by a further contact resistance of \( 1/\alpha_2 \) per unit area at the surface \( r = R \). If \( h_R \) signifies the pressure head in the soil water at \( r = R \), eq. (3) may be written as:

\[
v = -f\alpha(h_R - H) \quad \text{with} \quad 1/\alpha = 1/\alpha_1 + 1/\alpha_2,
\]

and from equations (5) and (6) the tensiometer equation follows:

\[
S \frac{dH}{dt} = f\alpha(h_R - H)
\]

with the dimension sec is the response time \( T \) of the tensiometer according to Richards (1949).

III. THE SURROUNDING SOIL

A. FLOW OF WATER IN THE SURROUNDING SOIL

Neglecting gravity, the flow equation in the region \( r > R \) appropriate to the cylindrical geometry is:

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{1}{D} \frac{\partial h}{\partial t}, \quad r > R, \ t > 0.
\]

If we assume the changes in pressure head in the soil water to be so small the capillary conductivity \( k \) and the diffusivity

\[
D = \frac{k}{W}
\]

can be regarded as constant. The volumetric water capacity \( W \) is the absolute value of the rate of change of volumetric water content with soil water pressure and has the dimension \( \text{cm}^{-1} \).

B. BOUNDARY CONDITION

At the surface \( r = R \) the flux of water per unit length per unit time from the cup must be transferred by capillary conduction:

\[
2\pi R\alpha(h - h) = -2\pi Rk \frac{\partial h}{\partial r}, \quad r = R, \ t > 0
\]

IV. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

Initially let the constant pressure head everywhere in the soil water be \( H_0 \), which we use as zero-point for pressure measurement and which is indicated by the manometer as tension \( P_0 = -H_0 \).

At time \( t = 0 \) the pressure head in the tensiometer is increased by \( \Delta H \), i.e. the tension is decreased by \( \Delta H \), and the manometer reading for \( t > 0 \) has to be calculated, which for \( t \to \infty \) goes back to \( P_0 \). Here the tensiometer eq. (7) has to be integrated with the initial
condition $H = \Delta H$ for $t = 0$, $h_R$ being determined by the eq. (8) with the initial condition $h = 0$ for $t = 0$ and the boundary condition (10).

Mathematically—and with the assumptions made also physically—our problem of water transfer corresponds to the following problem of heat transfer: the region outside the circular cylinder $r = R$ consists of a solid with the thermal conductivity $k$, heat capacity $W$ and diffusivity $D$, initially at zero temperature. The region $r = R$ is occupied by a perfect conductor with heat capacity $S$ per unit length, having the initial temperature $\Delta H$. There is a thermal contact resistance $1/\alpha$ per unit area at the surface $r = R$ between the perfect conductor and the surrounding soil. This problem has been solved by Jaeger (1956). Using the three dimensionless parameters, namely:

$$\tau = \frac{Dt}{R^2}$$

$$\eta = 2\pi R^2 \frac{W}{S}$$

$$\chi = \frac{k}{R\alpha}$$

the pressure head indicated by the manometer is:

$$H = \Delta HF(\chi, \eta, \tau)$$

with

$$F(\chi, \eta, \tau) = \frac{4\eta}{\pi^2} \int_0^\infty \frac{e^{-x^2}}{uA(u)} \, du$$

$$\Delta(u) = [uJ_0(u)-(\eta-\chi^2)J_1(u)]^2 + [uN_0(u)-(\eta-\chi u^2)N_1(u)]^2,$$

where $J_0$ and $N_0$ are Bessel and Neumann functions of order $v$. For small values of $\tau$ and for large values of $\tau$ approximations of the complicated integral are useful (Jaeger, 1956), for medium values of $\tau$ the integral must be evaluated numerically. For small values of $\tau$ we get

$$F(\chi, \eta, \tau) = 1 - \eta\tau/\chi + \ldots$$

if $\chi \neq 0$;

for $\chi = 0$ it becomes:

$$F(0, \eta, \tau) = 1 - \frac{2\eta}{\sqrt{\pi}} \sqrt{\tau} + \eta \left( \eta - \frac{1}{2} \right) \tau + \ldots$$

For large values of $\tau$ the approximation:

$$F(\chi, \eta, \tau) = \frac{1}{2\eta\tau} \left[ \left( 4\chi^2 - 2 \right) - \left( \eta - 2 \right) \ln \left( \frac{4\tau}{C} \right) \right] + \ldots$$

holds, where:

$$C = 1.7811 \ldots = e^\gamma, \gamma = 0.5722 \ldots,$$

is Euler's constant.
Jaeger (1956) published values of the function $F(0, \eta, \tau)$ with three decimals for $\eta = 0.5(0.5) 2(2) 8$ and $\tau = 0.2(0.1) 1(1) 10(5) 20$ and Bullard (1954) values with 4 decimals for $\eta = 2(2) 8$ and $\tau = 0.2(0.2) 3(1) 10$. For $x \neq 0$ Jaeger gives two graphs of the function $F(x, 1, \tau)$ and $F(x, 2, \tau)$ for $x = 0.0(0.5) 1(1) 5, 7, 10$ and 20 for $0.2 \leq \tau \leq 20$ in a double logarithmic graph.

V. SIZE OF PARAMETERS

The size of the parameters is essential for instrument design and for reducing the number of observations. Their influence will be discussed in connection with three soils, Chino (silty clay loam), Indio and Pachappa (sandy loam), for which the dependence of the water content and capillary conductivity on tension are known, according to measurements by Gardner (1959). The capillary conductivity $k$ for the tensions $P_0 = 0.2$ bar and $P_0 = 0.8$ bar and the water capacity $W$, the latter obtained by differentiating graphically the tension curves, have been taken from Richards' (1961) representation of the results. From both $k$ and $W$ the diffusivity $D$ has been calculated. The data are given in table 1.

### Table 1. Data of the three soils

<table>
<thead>
<tr>
<th>Soil</th>
<th>$10^3 k$</th>
<th>$10^3 W$</th>
<th>$D$</th>
<th>$10^3 k$</th>
<th>$10^3 W$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cm min$^{-1}$</td>
<td>cm$^{-1}$</td>
<td>cm$^2$ min$^{-1}$</td>
<td>cm min$^{-1}$</td>
<td>cm$^{-1}$</td>
<td>cm$^2$ min$^{-1}$</td>
</tr>
<tr>
<td>Chino</td>
<td>2.8</td>
<td>4.2</td>
<td>0.067</td>
<td>6.9</td>
<td>0.5</td>
<td>0.014</td>
</tr>
<tr>
<td>Indio</td>
<td>7.0</td>
<td>8.5</td>
<td>0.082</td>
<td>6.9</td>
<td>1.0</td>
<td>0.007</td>
</tr>
<tr>
<td>Pachappa</td>
<td>2.8</td>
<td>3.9</td>
<td>0.072</td>
<td>3.5</td>
<td>0.5</td>
<td>0.007</td>
</tr>
</tbody>
</table>

$P_0 = 0.2$ bar $P_0 = 0.8$ bar

A. THE PARAMETER $\tau$

In the three soils the diffusivity within the given range of tension changes by about a factor of 10, and it is to be expected that we will get

$$0.01 \leq D \leq 0.1 \text{ cm}^2 \text{ min}^{-1}$$

in the experiments. A tensiometer cup with a diameter of 10 mm seems to be a practical proposition. Then the parameter $\tau = Dt/R^2$ in the time $t = 30$ sec to $t = 30$ min passes through the values 0.2 to 12 in moist soil ($D = 0.1 \text{ cm}^2 \text{ min}^{-1}$) and 0.02 to 1.2 in drier

### Table 2. Values of the function $F(0, \eta, \tau)$

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\eta = 0.5$</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.920</td>
<td>0.850</td>
<td>0.791</td>
<td>0.741</td>
<td>0.642</td>
</tr>
<tr>
<td>0.2</td>
<td>0.757</td>
<td>0.595</td>
<td>0.482</td>
<td>0.401</td>
<td>0.229</td>
</tr>
<tr>
<td>1</td>
<td>0.514</td>
<td>0.317</td>
<td>0.221</td>
<td>0.166</td>
<td>0.080</td>
</tr>
<tr>
<td>2</td>
<td>0.381</td>
<td>0.206</td>
<td>0.135</td>
<td>0.099</td>
<td>0.047</td>
</tr>
<tr>
<td>10</td>
<td>0.117</td>
<td>0.052</td>
<td>0.033</td>
<td>0.024</td>
<td>0.011</td>
</tr>
<tr>
<td>20</td>
<td>0.059</td>
<td>0.026</td>
<td>0.017</td>
<td>0.012</td>
<td>0.006</td>
</tr>
</tbody>
</table>

soil ($D = 0.01$/cm$^2$/min$^{-1}$). For $x = 0$ table 2 contains values of $F(0, \eta, \tau)$ for $0.02 \leq \tau \leq 20$ as a function of the parameter $\eta$. The values given in the table represent the relation
$H/\Delta H$ when the influence of the cup wall on water flow can be neglected and no contact resistance occurs on the outer wall of the cup.

**B. The parameter $\eta$**

Table 2 shows that the manometer reading reaches the initial pressure head $H_0$ the faster the larger the value of the parameter $\eta$ is. For $\eta = 0$ the difference in pressure head $\Delta H$ is maintained for all times $t$ (probe with constant difference in pressure head), and the amount of water transferred would have to be measured as a function of time, if the capillary conductivity of the soil surrounding the probe is to be determined.

On the other hand with $\eta \to \infty$ the initial pressure head returns immediately. We would then have an ideal tensiometer, which, however, is unsuitable for the task in view. Let us assume that 30 sec after starting measurements it is to be:

$$H/\Delta H \approx 0.75$$

then in our example, according to table 2, $\eta$ must be about 0.5 in moist and about 2 in drier soil. Thirty minutes after start in both cases $\Delta H$ is diminished by about 90 per cent.

It is to be noted that the amount of water transferred to the soil by the measurement increases with decreasing $\eta$ and constant $\Delta H$. To avoid changing essentially the soil moisture content in the vicinity of the tensiometer cup, $\eta$ should not be smaller than is necessary for the determination of capillary conductivity and diffusivity with satisfactory accuracy. This calls for smaller values of $\eta$ in moist soils than in drier soils.

Fig. 1 shows for the three soils the values of $W$ in cm$^{-1}$ in the whole tension range which comes into consideration for measurements with tensiometers. Above all with small initial tensions $P_0$ (high water content) $W$ varies considerably, reaches a maximum with $0 < P_0 < 0.15$ bar and quickly decreases with increasing tension (decreasing water content). The theory presupposes that $W$ is constant. This condition is much better fulfilled within the range $0.2 < P_0 < 1$ bar than for $P_0 < 0.2$ bar. For this reason this value has been regarded as the lower limit for the employment of a tensiometer for measuring capillary conductivity and diffusivity.

Using a tensiometer cup with length $L$ and a mercury manometer, the tube of which has the diameter $2r^*$, we get:

$$S = \frac{r^*^2 \pi}{\rho_{Hg}L} \rho_w,$$

where $\rho_{Hg}$: density of mercury

$\rho_w$: density of water

**Figure 1.** Specific water capacity of the 3 soils (according to measurements by Gardner)
and
\[ \eta = 27.1 L \left( \frac{R}{r^*} \right)^2 W \] (21)

The length \( L \) must be sufficiently large to maintain radial flow during the time of experiment. We assume a length of \( L = 25 \text{ cm} \) and get finally
\[ \eta = 0.678 \left( \frac{R}{r^*} \right)^2 10^3 W \] (22)

This relation is plotted in fig. 2 for different ratios \( R/r^* \). From the figure it is evident that parameter \( \eta \) within the given range of \( W \) varies all the more the larger the ratio \( R/r^* \) is, \( \eta \) always remaining proportional to \( W \). This is an obstacle to the solution of the problem, for our argument had the result that \( \eta \) must decrease with increasing \( W \) (decreasing initial tension \( P_0 \)) when during the experiment no more water shall be passed on to the soil than is absolutely necessary. \( \eta \) can only be adjusted to the respective conditions by changing the diameter of the manometer tube or the diameter of the cup. Therefore a whole set of tensiometers would be required to take measurements within the range \( 0.05 \leq 10^3 W \leq 1 \text{ cm}^{-1} \) under optimal conditions. As a compromise, with a cup of 10 mm diameter and a length of 25 cm, two manometers can be used with \( R/r^* = 1.5 \) for the range \( 0.3 \leq 10^3 W \leq 1 \text{ cm}^{-1} \) and \( R/r^* = 3 \) for \( 0.1 \leq 10^3 W < 0.3 \text{ cm}^{-1} \).

C. The Parameter \( \chi \)

All considerations made so far have neglected the influence of the cup wall and that of an additional contact resistance at \( r = R(\chi = 0) \). First we examine the influence of the finite hydraulic conductivity of the cup material and put in equation (6) \( 1/\chi = 0 \), so that
\[ \chi = \frac{k}{k^*} \ln(R/R_1). \] (23)

In the range \( 0 \leq P_0 \leq 0.9 \text{ bar} \) Schott's sintered glass G 5 fine is a suitable cup material. It has a maximum pore size of less than 1\( \mu \) and a hydraulic conductivity of 0.3 to 0.4 cm day\(^{-1}\). Let us assume that \( R = 0.5 \text{ cm} \) and the thickness of the wall is 2 mm; we get
\[ \frac{k^*}{\ln(R/R_1)} \approx 0.7 \text{ cm day}^{-1} = 50 \cdot 10^{-5} \text{ cm min}^{-1}. \]

A ceramic bacteria filter available here consists of a material with an estimated maximum pore size of 1.8\( \mu \).

A tensiometer cup made from it should be usable up to \( P_0 = 0.6 \text{ bar} \).

In such a cup
\[ \frac{k^*}{\ln(R/R_1)} = 2.0 \text{ cm day}^{-1} = 150 \cdot 10^{-5} \text{ cm min}^{-1} \]

has been measured. For the three soils table 3 contains values of the parameter \( \chi \) with \( P_0 = 0.2 \text{ bar} \) and \( P_0 = 0.8 \text{ bar} \), index 1 referring to the cup of sintered glass, index 2 to the ceramic cup.

With high initial tensions (\( P_0 > 0.5 \text{ bar} \)) the influence of the hydraulic conductivity of the two cup materials on the flow of water can be neglected with safety and we may put \( \chi = 0 \). Also with \( P_0 = 0.2 \text{ bar} \) the influence is insignificant when the ceramic cup is
TABLE 3. Values of parameter $x$ for two tensiometer cups

<table>
<thead>
<tr>
<th>Soil</th>
<th>$10^5 k$ cm min$^{-1}$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$10^7 k$ cm min$^{-1}$</th>
<th>$x_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chino</td>
<td>2.8</td>
<td>0.06</td>
<td>0.02</td>
<td>6.9</td>
<td>0.001</td>
</tr>
<tr>
<td>Indio</td>
<td>7.0</td>
<td>0.14</td>
<td>0.05</td>
<td>6.9</td>
<td>0.001</td>
</tr>
<tr>
<td>Pachappa</td>
<td>2.8</td>
<td>0.06</td>
<td>0.02</td>
<td>3.5</td>
<td>0.001</td>
</tr>
</tbody>
</table>

$P_0 = 0.2$ bar

used. The above mentioned possibility of adjusting the parameter $\eta$ to the respective conditions by appropriately choosing the ratio $R/r*$ has to be extended in so far as with $R/r*$ the cup material is also changed to get $x = 0$. If one succeeds in fitting the cup into the soil without creating an additional contact resistance, the function $F(0, \eta, \tau)$ can be used for reducing the observations. No data are available in scientific literature on the extent of such a contact resistance. If it is large enough to be taken into account or if the influence of the cup material cannot be neglected, we must resort to the function $F(x, \eta, \tau)$.

VI. REDUCTION OF EXPERIMENTS

The determination of capillary conductivity and diffusivity from the experimental time curves $H(t)$ is possible with all values of $\tau$ by means of the numerical values of the function $F(x, \eta, \tau)$, published by Jaeger and Bullard, if either the influence of the cup wall can be neglected ($x = 0$) or if with $x > 0$ the parameter $\eta$ has the values 1 or 2. According to fig. 2 the last condition will only occasionally be fulfilled in the measurements. In all other cases we use the approximations (18) or (19).

A. LARGE VALUES OF $\tau$

For large values of $\tau$ $F(x, \eta, \tau)$ can be represented by $f(\eta, \tau) = \frac{1}{2}\eta\tau$, and we get to a first approximation:

$$H/\Delta H = \frac{S}{4\pi kt}.$$  \hspace{1cm} (24)

Plotting $H(t)$ against $t^{-1}$ we get a straight line through the origin, from the slope of which $k$ can be calculated (Gardner, 1961; Klute and Gardner, 1962). If systematic errors up to 10% due to insufficient curve-fitting are permitted in the determination of capillary conductivity, equation (24) can be used from the moment onwards when the parameter $\tau$ goes beyond the values given in the left-hand half of table 4, then $H/\Delta H$ being equal to or smaller than the value given in the right-hand half of the table.

TABLE 4. Values for the use of eq. (24)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\eta$</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td></td>
<td>0.21</td>
<td>0.15</td>
<td>0.10</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>0.5</td>
<td>1.1</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.41</td>
<td>0.41</td>
<td>0.31</td>
<td>0.31</td>
<td></td>
</tr>
</tbody>
</table>

$\tau_{\text{min}}$ $(H/\Delta H)_{\text{max}}$
Where higher accuracy is demanded \( \tau_{\text{min}} \) quickly increases and \( (H/\Delta H)_{\text{max}} \) becomes accordingly smaller.

**Figure 2.** Parameter \( \eta \) with \( L = 25 \text{ cm} \). The numbers at the straight lines are the values of \( R/r^* \)

For \( x = 0 \) this method of reduction is only suitable with small values of \( \eta (\eta < 1) \). More satisfactory results are to be expected with \( x = 0.5 \), for here even with \( \eta = 2 \) the last third of the curve \( H(t) \) can be used to reduce the experimental results. From eq. (24) it follows that with large values of \( \tau \) only the capillary conductivity can be determined, but not the diffusivity.

**B. SMALL VALUES OF \( \tau \)**

For \( x > 0 \) from eq. (17) we get:

\[
\frac{H}{\Delta H} = 1 - \frac{2\pi R\alpha}{S} t
\]

(25)

Here only the water capacity per unit length of the tensiometer cup and the transfer index determine the time graph of the manometer readings. Eq. (25) may serve to determine the contact resistance

\[
1/\alpha = 1/\alpha_1 + 1/\alpha_2 = \frac{R \ln (R/R_1)}{k^*} + 1/\alpha_2
\]

in situ.

For \( x = 0 \) in a first approximation we get according to eq. (18)

\[
1 - \frac{H}{\Delta H} = 4 \frac{\sqrt{\pi R}}{S} \cdot \frac{k}{\sqrt{D}} \sqrt{t}.
\]

(26)

Plotting \( 1 - H/\Delta H \) against \( \sqrt{t} \), a straight line through the origin is obtained, from the slope of which \( k/\sqrt{D} \) is determined. In connection with \( k \) found in the case of large values of \( \tau \), the diffusivity \( D \) is now known, though with a relative accuracy which is only half that of the capillary conductivity.
C. Graphic Reduction

As a consequence of the approximations (24) and (26) only a part of the experimental curve \( H(t) \) has been used to find \( k \) and \( D \). Graphic reduction permits all values of \( H(t) \) to be considered.

From eq. (14) it follows that

\[
\log H = \log(\Delta H) + \log F(\kappa, \eta, \tau). \tag{27}
\]

A representation of \( \log F(\kappa, \eta, \tau) \) against \( \log \tau \) for a given value of \( \kappa \) or \( \eta \) is plotted in an evaluation sheet as a system of curves with \( \eta \) or \( \kappa \) as a parameter. The experimental values of \( \log H \) are plotted against \( \log t \) on transparent paper having the same coordinate-scales as the evaluation sheet. The paper is then shifted parallel to the axes on the evaluation sheet until the best fit is obtained. The evaluation curve chosen provides \( \eta_2 \) or \( \kappa_2 \), from the displacements of the axes we get \( \log(D/R^2) \) and \( \log(\Delta H) \). Then \( \Delta H \) itself need not be measured; this will be favourable for practical application. Evaluation sheets can be designed with the existing values of \( F(0, \eta, \tau) \), \( F(\kappa, 1, \tau) \) and \( F(\kappa, 2, \tau) \).

In practice the requirement \( \kappa = 0 \) can be maintained (table 3). When the gradation of the \( \eta \)-values is precise enough on the evaluation sheet, it is theoretically possible to determine simultaneously the diffusivity \( D \) from \( \log(D/R^2) \) as well as the capillary conductivity from \( D/R^2 \cdot \eta = 2\pi k/S \). There are limits to the gradation of \( \eta \), moreover it is generally possible, because of unavoidable errors, to fit the experimental values to several curves with different parameter \( \eta \), without being able to decide by eye which of the fits is the best. Then the diffusivity thus obtained will vary much more than the capillary conductivity, for the value \( D/R^2 \cdot \eta \) remains nearly constant (Beck, Jaeger and Newstead, 1956).

With \( \kappa > 0 \) graphic reduction is at present restricted to the two special cases of \( \eta = 1 \) and \( \eta = 2 \). The selection of the parameter \( \kappa \) in the evaluation sheet is undertaken after the fit with small values of \( \tau \).

REFERENCES


