Numerical analysis of ponded rainfall infiltration

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ABSTRACT: The preponding and ponded stages of rainfall infiltration were studied by solving numerically the partial differential equation of downward moisture flow in unsaturated soils. To make such a study possible, the previously developed numerical method for rain infiltration analysis was appropriately modified.

The modified method was used for computing moisture content and pressure head profiles during preponding and ponded rainfall infiltration, as well as the theoretically expected rainfall infiltration rates. In particular, the dependence of the above profiles and rates upon rain intensity was studied.

One of the conclusions reached was that the theoretical relations between ponded rainfall infiltration rates and infiltration duration, computed for rains of various intensities, cannot be expressed by a single curve. These relations also differ significantly from the flood-water infiltration capacity-time function.

I. INTRODUCTION

The moment of appearance of an enduring free-water cover upon the soil surface, often observed during sufficiently intense rainfalls, has been defined (Rubin and Steinhardt, 1964) as incipient ponding. In this paper, the term ponded rainfall infiltration, or in short, rainpond infiltration, will refer to the stage of rainfall entry into the soil which commences with incipient ponding. Ponded water is in evidence throughout this state, the prevalent infiltration capacities being exceeded by the rate of rainwater supply to the land surface.

Ponded rainfall infiltration is of considerable theoretical and practical interest, particularly because of its role in determining the relations between rainfall intensities and runoff rates. Nevertheless, it seems that as yet no quantitative theoretical treatment of this process has been attempted.

In the recent past, developments in computer science and in the theory of water flow through unsaturated porous media have been utilized in order to analyze a number of phenomena closely related to rainpond infiltration. Thus, several computer studies of infiltration due to flooding have been published (e.g. Hanks and Bowers, 1962). Nonponding infiltration of low intensity rains also has been investigated with the aid of numerical
methods, as was the cumulative rainwater uptake at incipient ponding (Rubin and Steinhardt, 1963 and 1964). However, in no previous investigation was an incipient ponding moisture profile actually computed. In fact, the numerical methods used till now in infiltration studies were not general enough to yield such a computation for a great many, commonly met soils (cf. below). With no knowledge of the incipient ponding profile, a theoretical treatment of rainpond infiltration was impossible.

This paper describes a rather general method for computing soil moisture content profiles of incipient ponding and of rainpond infiltration. In addition it presents several examples of results obtained with the aid of this method. These results are utilized for drawing certain conclusions about the theoretically expected rates of rainpond infiltration.

II. THE FLOW EQUATION AND ITS NUMERICAL SOLUTION

It is assumed in this study that vertical infiltration of rainwater and its subsequent downward movement within a semi-infinite soil profile are described by the following set of equations:

\[
\frac{\partial w}{\partial t} = \frac{\partial}{\partial x} \left[ K(H) \frac{\partial H}{\partial x} - K(H) \right] \quad x \geq 0 \quad t \geq 0
\]

\( H = H_i \quad x \geq 0 \quad t = 0 \)

\( H = H_i \quad x = \infty \quad t \geq 0 \)

\[
\text{Flux} = -K(H) \frac{\partial H}{\partial x} + K(H) = R \quad x = 0 \quad t_p \geq t > 0
\]

\( H = H_u \geq 0 \quad x = 0 \quad t \geq t_p
\]

In the above, \( w \) is the volumetric moisture content (cm\(^3\)/cm\(^3\)); \( x \) is soil depth (cm); \( t \) is elapsed infiltration time (sec); \( H \) is soil water pressure head (cm of water); \( K(H) \) is the soil's hydraulic conductivity (cm/sec). \( K(H) \) and \( u \) are assumed to be single-valued, non-decreasing functions of \( H \). The constant \( H_i \) is the initial soil moisture pressure head while the non-negative constant \( H_u \) represents the surface pressure head during rainpond infiltration. \( H_i \) is so low that \( K(H_i) \) is essentially zero. Without loss of generality, in this study \( H_u \) will be taken as equal to zero. \( R \) represents a constant rain intensity, while \( t_p \) is the occurrence time of incipient ponding.

The characteristics of the physical model implied by the above system of equations need not be considered here, since they have been described previously by several authors (e.g. Rubin and Steinhart, 1963 and 1964).

The set of equations (1) and (5) contains two dependent variables. In this study, a well known transformation (Carslow and Jaeger, 1959) was utilized in order to change this set into a system containing a single dependent variable. The transformation involves a new variable \( v \), defined by:

\[
v = v(H) = \frac{1}{V} \int_{H_{max}}^{H} K(h) \, dh
\]

\[
V = \int_{H_{max}}^{H_i} K(h) \, dh
\]
In the above, both $H$ and $h$ represent the pressure head variable. The constants $H_{\text{max}}$ and $H_{i}$, respectively, are upper and lower bounds of the pressure heads in the porous medium under consideration.

Assuming that the inverse of $v(H)$ exists, using the relation

$$\frac{\partial H}{\partial v} = \frac{V}{K(H)}$$

which follows from (6), and employing the chain rule of calculus, it can be shown easily, that the system (1) to (5) is equivalent to the following set of equations:

$$Y(v)\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} - Z(v)\frac{\partial y}{\partial x} \quad x \geq 0 \quad t \geq 0$$

$$v = v(H_i) = 1.0 \quad x \geq 0 \quad t = 0$$

$$v = v(H_u) = 1.0 \quad x = \infty \quad t > 0$$

$$\frac{\partial v}{\partial x} = \frac{K[H(v)] - R}{V} \quad x = 0 \quad t_p \geq t > 0$$

$$v = v(H_u) = a \quad x = 0 \quad t \geq t_p$$

$$Y(v) = \frac{1}{V} \frac{\partial w}{\partial H} = \frac{1}{V} \frac{\partial w}{\partial v} = \frac{1}{V} \frac{\partial H}{K[H(v)]} \quad x \geq 0 \quad t \geq 0$$

$$Z(v) = \frac{1}{V} \frac{\partial K[H(v)]}{\partial H} = \frac{1}{V} \frac{\partial K[H(v)]}{\partial v} = \frac{\partial \ln K(H)}{\partial H} \quad x \geq 0 \quad t \geq 0$$

where $a$ is a non-negative constant, equal to 0.0, if $H_u = H_{\text{max}}$.

Note that in saturated layers of finite thickness, if such occur within the system under consideration, $H$ varies with depth, but $w$ and $K$ are constant. It follows that in such layers $\partial w/\partial H$ and $\partial K/\partial H$ are equal to zero, and hence $Y(v) = Z(v) = 0$. Thus, equations (9) and (12) become

$$\frac{\partial^2 v}{\partial x^2} = 0 \quad x \geq 0 \quad t \geq 0$$

$$\frac{\partial v}{\partial x} = \text{constant} = \frac{K_{\text{sat}} - R}{V} \quad x = 0 \quad t_p \geq t > 0$$

where $K_{\text{sat}}$ represents hydraulic conductivity of the saturated soil.

There are two reasons for using the above $v$-based approach, rather than the usual transformations which either lead to a flow equation involving only $H$ or to one involving only $w$ (the latter one being based on the water diffusivity concept).

First, for many soils, especially the more sandy ones, it seems that the rather versatile and useful difference methods sometimes do not yield accurate solutions, when used in conjunction with the $H$-based flow equation and its parameters alone (Hanks and Bowers, 1962; Rubin and Steinhardt 1964).

Secondly, during preponding infiltration into soils with non-zero air entry suctions, a saturated layer arises just below the land surface. It persists until incipient ponding as
Numerical analysis of ponded rainfall infiltration (Philip, 1958; Rubin and Steinhardt, 1964). Such a layer also exists during ponded infiltration into soils with zero air entry suction provided the value of the surface pressure head, \( H_u \), exceeds zero (Philip, 1957). Since in saturated layers \( \partial w/\partial H = 0 \), the soil moisture diffusivity of these layers defined as \( K/\partial H \), is infinite. Hence, the sole use of the diffusivity-based equation is impossible, whenever the flow process analyzed involves saturated soil layers of finite thickness.

Perhaps because of these limitations of the ordinarily used \( w \)-based and \( H \)-based approaches, the numerical difference methods were not employed until now in order to compute, from \( K(H) \) and \( H(w) \) data alone, the infiltration profiles involving both saturated and unsaturated soil layers.

In order to solve the system of equations (9) to (15) numerically, partial differential equations (9) and (12) were approximated by difference equations. The latter equations involve a set of points in the \((x, t)\) plane (i.e., a so-called grid of points). This grid is given by \( x = j \Delta x \) and \( t = n \Delta t \), where \( \Delta x \) and \( \Delta t \) are arbitrarily small increments of variables \( x \) and \( t \), respectively; \( j = 0, 1, 2, \ldots J; n = 0, 1, 2, \ldots ; j = 1 \) corresponds to the soil surface; \( j = J \) corresponds to soil depth, which is large enough to be uninfluenced by infiltration during the time period under consideration; and \( n = 0 \) is the instant at which the infiltration commences. In the notation employed below, the value of variable \( v \) at point \((j \Delta x, n \Delta t)\) is denoted by the symbol \( v^n_j \).

For soil layers which at a given computation stage were unsaturated (except, possibly— at boundaries), the following difference equations were derived from (9) and (12), respectively, utilizing the Crank-Nicolson implicit scheme (Richtmeyer, 1957).

\[
Y(v_j^{n+\frac{1}{2}}) \frac{v_j^{n+1} - v_j^n}{\Delta t} = \frac{1}{2}(A_x^2 v_j^{n+1} + A_x^2 v_j^n) - Z(v_j^{n+\frac{1}{2}}) \frac{1}{2}(A_x v_j^{n+1} + A_x v_j^n)
\]

\[
A_x v_j^{n+1} = \frac{K(v_j^{n+1}) - R}{v_j^n} \tag{19}
\]

the difference symbols above being defined as follows:

\[
A_x^2 v_j^m = \frac{v_{j+1}^m - 2v_j^m + v_{j-1}^m}{\Delta x^2}
\]

\[
A_x v_j^m = \frac{v_{j+1}^m - 2v_j^m + v_{j-1}^m}{2\Delta x}
\]

\[
m = n \text{ or } (n+1)
\]

For saturated soil layers of finite depth, equation (9), which in such a case reduces to (16), was not approximated by (18). Instead, the following relation was used:

\[
A_x^2 v_j^{n+1} = 0 \tag{20}
\]

In (20), unlike in (18), variables \( v \) of time \( n \) do not appear, because equation (16) does not contain the time variable.

Following the suggestion of Douglas (1958), the arguments \( v_j^{n+\frac{1}{2}} \) of functions \( Y(v_j^{n+\frac{1}{2}}) \) and \( Z(v_j^{n+\frac{1}{2}}) \) in (18) were approximated by means of the following explicit difference equation (Richtmeyer, 1957), based on (9):

\[
Y(v_j^n) \frac{v_j^{n+\frac{1}{2}} - v_j^n}{\frac{1}{2} \Delta t} = A_x^2 v_j^n - Z(v_j^n) A_x v_j^n \tag{21}
\]
Equation (21) can be solved for \( v_j^{n+\frac{1}{2}} \), thus making it possible to evaluate \( Y(v_j^{n+\frac{1}{2}}) \) and \( Z(v_j^{n+\frac{1}{2}}) \), when the values of variables \( v \) at time \( n \) are known.

An analogous procedure was used to evaluate \( K(v_1^{n+1}) \) of equation (19). The pertinent explicit equation is:

\[
Y(v_1^n) \frac{v_1^{n+1} - v_1^n}{\Delta t} = 2v_1^n - Z(v_1^n) A_x v_1^n
\]  

(22)

The explicit equation method for estimating \( v_j^{n+\frac{1}{2}} \) was employed here instead of the previously used method of linear extrapolation from two earlier time levels (Rubin and Steinhardt, 1963). This was done because at a certain stage of the computation under consideration \( (cf. \text{ below}) \) there arises a need of changing repeatedly the value of \( \Delta t \). The linear extrapolation method cannot be used under such conditions.

Equations (18) and (20) can be transformed into a more useful form by elementary algebraic manipulations which yield the following:

\[
-A_j v_{j+1}^{n+1} + B_j v_j^{n+1} - C_j v_j^{n+1} = D_j
\]  

(23)

where:

\[
A_j = 2r - r A_x Z(v_j^{n+\frac{1}{2}})
\]  

(24)

\[
B_j = 4Y(v_j^{n+\frac{1}{2}}) + 4r
\]  

(25)

\[
C_j = 2r + r A_x Z(v_j^{n+\frac{1}{2}})
\]  

(26)

\[
D_j = A_j v_{j+1}^n + (B_j - 8r) v_j^n + C_j v_j^{n-1},
\]  

(27)

when (18) is applicable

\[
D_j = 0,
\]  

(28)

when (20) is applicable

\[
r = \Delta t / A_x^2
\]  

(29)

Note that because of (21) and (22), the parameters \( A_j, B_j, C_j \) and \( D_j \) as well as the right-hand side of (19) are known, if the values of variables \( v \) at time \( n \) are known.

For each value of \( j \), except for \( j = 0 \) and \( j = J \), there exists an equation given by (23). The lower and upper boundary conditions [(11) and either (12) or (13)] yield equations which involve \( v_j^{n+1} \) and \( v_j^{n+\frac{1}{2}} \). One of these equations is (19), the form of the other two being obvious. Thus, the above considerations lead to a system of \( J \) equations, which are linear in the \( J \) advanced-time variables, \( v_j^{n+1} \). Since the values of the variables \( v \) at \( n = 0 \) are known, the above system can be solved for the variables corresponding to time \( n = 1 \). Subsequently, this procedure can be repeated using the newly calculated \( v_j \) values as a basis. By employing this procedure repeatedly one can compute \( v \) values for any time.

In the course of calculations a continuous check must be kept as to whether the pertinent variable \( v_j^{n+\frac{1}{2}} \) or the estimated variable \( v_j^{n+\frac{1}{2}} \) indicates that the soil at a given point is saturated. This check makes it possible to choose at a given stage of the computation either (27) or (28) as the proper equation for \( D_j \).

The calculations commence with all variables \( v_j^0 = 1.0 \) and is carried out for as many \( \Delta t \) cycles as necessary, using the upper boundary condition described by (19). This series of computations yields data on preponding infiltration. As these computations proceed, the calculated value \( v_j^{n+1} \) continuously decreases. At a certain stage of computation this value usually becomes such that the \( H \) value corresponding to it exceeds zero. When this happens, the last \( v_j^{n+1} \) results are discarded and the computation is repeated, using again
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the same \( v \) values, but with \( \Delta t \) value decreased threefold. This recomputation procedure with its \( \Delta t \) diminution is repeated whenever \( H(0, t) \) exceeds zero and until the \( v^{t+1} \) value indicates that \( H(0, t) \) corresponding to it is in the range \(-0.00001\) to zero cm. The soil moisture profile corresponding to the latter situation is considered as the incipient ponding profile. At this stage of the computation the upper boundary condition (19) is replaced by condition (13), the value of \( \Delta t \) is increased to its original level and the rainpond infiltration stage of calculation is started. During this stage the added water contents of each profile and infiltration rates are computed periodically, using the trapezoidal rule for the integration needed.

In order to employ the above described difference method, empirical data on the soil’s retention and hydraulic conductivity functions must be converted into algorithms for evaluating the functions \( Y(v) \) and \( Z(v) \) and for converting the output from its \( v \)-basis to \( H \)- or \( w \)-basis.

Two cases of constructing such algorithms must be distinguished: one in which the available experimental data involve functions of \( H \), and second in which they involve functions of \( w \).

In the former case, the experimental data usually can be expressed by empirical equations which have the following general form:

\[
w = w_{sat} \quad K = K_{sat} \quad H > H_A
\]

\[
w = w(H) \quad K = K(H) \quad H \leq H_A
\]

where \( w_{sat} \) is the saturated moisture content, \( H_A \) is the air entry pressure head and \( K_{sat} \), as before, represents the hydraulic conductivity of the saturated soil.

Using (6), (7), (30) and (31) one obtains:

\[
H > H_A \quad v = \left[ K_{sat}(H-H_{max}) \right] / V
\]

\[
H \leq H_A \quad v = \left[ K_{sat}(H_A-H_{max}) + \int_{H_A}^{H} K(h) \, dh \right] / V
\]

\[
V = K_{sat}(H_A-H_{max}) + \int_{H_A}^{H} K(h) \, dh
\]

Equations (32), (33) and (34) are used for preparing a table of corresponding \( v \) and \( H \) values, the integrals of (33) and (34) being evaluated analytically, if possible, or numerically. From such a table, for any given \( v \) it is possible to obtain, by extrapolation, a corresponding value of \( H \). This table is employed for converting the \( v \)-based output into \( H \)-based one. It is also used for computing \( Y(v) \) and \( Z(v) \) in the following manner: for a given \( v \) obtain the corresponding \( H \); for this \( H \) compute from (30) and (31) the values of \( K(H) \), \( \partial w / \partial H \) and \( \partial \ln K(H) / \partial H \); with the aid of the latter values compute \( Y(v) \) and \( Z(v) \), by substitution into (14) and (15).

When the available data on soil moisture properties involve functions of \( w \), a procedure analogous to the above is used with the parametric functions:

\[
H > H_A \quad K = K_{sat} \quad w = w_{sat}
\]

\[
H_A \geq H = H(w) \quad K = K(w) \quad w \leq w_{sat}
\]

In order to compute the required table of the corresponding \( w \) and \( v \) values, (33) and (34) are used. The respective integrals of (33) and (34), \( I(H) \) and \( I(H_i) \) are converted into \( w \) basis as follows:

\[
I(H) = I[H(w)] = \int_{w(H, A)}^{w} K[h(w)] \frac{dh(w)}{dw} \, dw
\]

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An analogous equation can easily be written for \( I(H_t) \). Both \( K[h(w)] \) and \( dh(w)/dw \) can be computed for any given \( w \), using (36). Hence, for any \( w \), the integrals under consideration can be computed (numerically, if necessary).

\( Y(v) \) and \( Z(v) \) functions are evaluated for any given \( v \) by employing the table of the corresponding \( w \) and \( v \) values as well as equations (14) and (15), recast into \( w \)-basis as follows:

\[
Y(v) = \frac{1}{\partial H/\partial w} \frac{1}{K[H(w)]} \\
Z(v) = \frac{1}{K[H(w)]} (\partial K[H(w)]/\partial w)
\]

(38) (39)

III. RESULTS AND DISCUSSION

The numerical method described above was tested by computing with its aid preponding and rainpond infiltration profiles of several coarse and fine textured soils, at two initial moisture contents and with three ponded-water depths. The results presented here involve only the coarsest of these soils, Rehovot sand, one initial moisture content (air dryness) and zero ponded-water depth. These results are typical of all the results obtained.

Previously published data (Rubin and Steinhardt, 1964) on the retention and hydraulic conductivity functions of Rehovot sand were utilized. The second of these two functions was slightly modified in order to take into account the effects of water vapor transfer at low moisture contents. The fitted empirical equations of these functions had the following form [compare with (35) and (36)]:

\[
\begin{align*}
H > H_A &= -12.0 \text{ cm} \\
-12.0 \geq H &= -11.5 - (3.18/w) + \exp(9.92w - 1.79) + \exp(16.3 - 575w) \\
K &= K(w) = \exp\left\{18.6w - 11.5 - 2.30 \left[\exp(3.00 - 51w) - \exp(2.85 - 62w)\right]\right\}
\end{align*}
\]

(40) (41) (42)

It follows from (42) that \( K_{sat} \) of Rehovot sand is equal to 0.0133 cm/sec. In the computations reported here the initial moisture content of the soil was assumed uniform and equal to 0.005 cm\(^3\)/cm\(^3\). The study involved six constant relative rain intensities. The ratio of these rain intensities to the hydraulic conductivity of the saturated soil (ratios, which in this paper will be referred to as relative rain intensities) ranged from 1.50 to 5.25.

Examples of the results obtained for preponding infiltration are shown in fig. 1 and 2. The computed profiles of these figures correspond to relative rain intensity of 1.50. Infiltration durations, in seconds, are indicated next to the appropriate curves. The lowermost curve of each figure represents the profile at incipient ponding. The figures in question show that during the very early stages of preponding infiltration the soil surface is unsaturated and all the moisture and pressure gradients of the wetted zone are very steep. The computations indicate that during these early stages, the surface moisture content and pressure head continuously increase with time, while moisture and pressure head gradients of any given wetted depth become less steep. It is demonstrated by fig. 1 and 2 that in the particular case considered, water saturation and a pressure head equal to the air entry value were reached at the soil surface after 110 seconds of infiltration. From this instant on, it appears that the wetted profile consists of two parts: an uppermost, saturated part and a lower, unsaturated, wetted part. Furthermore, fig. 2 shows that in the pre-incipient ponding profiles in question the pressure heads of the saturated layer are
negative. Thus, this region corresponds to Philip's (1958) 'tension-saturated layer', the lower boundary of which exhibits the air entry pressure head. The results obtained indicate that the depth of the saturated layer continuously increases with time. At incipient ponding the depth of this layer appeared to be between 23 and 24 cm.

The changes in the preponding infiltration profile described above are in complete qualitative agreement with the analytical conclusions published elsewhere (Rubin and Steinhardt, 1964). Note that, according to one of these conclusions, at incipient ponding,
the depth of saturated layer, $B_{\text{POND}}$, should be given by the substitution of $H_0 = \text{surface pressure head} = 0$ into the following equation:

$$B_{\text{POND}} = \frac{-H_A + H_0}{R} - \frac{1}{K_{\text{sat}}}$$

(43)

Since in the case under consideration $H_A = -12.0$ and $R/K_{\text{sat}} = 1.5$, the expected $B_{\text{POND}} = 24.0$. This analytically derived value agrees well with the results of the numerical procedure reported above.

The influence of rain intensity upon the incipient ponding moisture content profile is demonstrated by fig. 3. The two profiles shown involve relative rain intensities of 1.50 and 5.25. Their elapsed times are 581.7 and 21.7 seconds, respectively, the corresponding cumulative water uptakes being 11.6 and 1.52 cm$^3$/cm$^2$. Fig. 3 demonstrates clearly that the higher the intensity of rainfall, the shallower is the saturated layer at incipient ponding. This relation is consistent with equation (43) into which the incipient ponding condition, $H_0 = 0$, was substituted. Fig. 3 also shows that the higher the rain intensity, the steeper are the moisture content gradients of the wetted, unsaturated part of the incipient ponding profile. The correctness of this statement can be proven analytically, at least for the gradient at the upper boundary of the unsaturated layer, because it follows from (4) and from the relation $(\partial w/\partial x) = (\partial w/\partial H)(\partial H/\partial x)$ that at this boundary

$$\frac{\partial w}{\partial x} = \left(\frac{\partial w}{\partial H}\right)\left[1 - \frac{R}{K(H)}\right]$$

where $K(H)$ and $(\partial w/\partial H)$ are evaluated at $H_A$ and hence are constant.

The above rain-intensity dependent features of incipient ponding profiles explain the rapid decrease of incipient ponding water uptakes with increasing rain intensity. It should be noted that the corresponding effect involving a decrease in the incipient ponding time always must be even more pronounced, since this time is equal to the ratio of incipient ponding uptake to rain intensity.
Transformations of moisture content profile during rainpond infiltration are demonstrated in fig. 4. The profiles shown involve rain with relative intensity of 1.50. The infiltration durations are indicated next to appropriate curves. The uppermost curve represents the profile at incipient ponding. It is shown here again, as a reference state.

**Figure 4.** Soil moisture content profiles during rainpond infiltration into initially air-dry Rehovot sand. The uppermost profile represents conditions at incipient ponding. The numbers next to the curves indicate infiltration durations, in seconds.

The generalizations proven analytically for infiltration due to flooding must be applicable to rainpond infiltration, since at least according to the theory employed here, the only distinctive feature of the latter type of ponded infiltration is its initial state (i.e. the incipient ponding profile). In particular, one of these generalizations (Philip, 1957 and 1958) is confirmed by the continuous increase in the depth of the 'tension-saturated layer', shown in fig. 4. Since the pressure heads at the upper and lower boundaries of this layer remain constant throughout rainpond infiltration, the increase in the layer's depth is equivalent to a continuous decrease in its pressure gradient. This, of course, is associated with the expected decrease in the rates of rainpond infiltration. Note that the above changes in gradient are in contrast to the situation in the preponding saturated zone (cf. the two lower curves of fig. 2, which clearly obey equation (43), with $H_0$ appropriately adjusting itself with time).

Water uptake rates and their change with time are of particular theoretical and practical interest. The continuous-line curves of fig. 5 demonstrate the dependence of the computed preponding and rainpond infiltration rates upon infiltration time. Results for three constant relative rain intensities (2.25, 3.00 and 4.50) are shown. For comparison purposes the corresponding calculated flood-water infiltration-capacity graph (broken-line curve) is also included.

Each one of the rainfall infiltration-rate curves consists of two distinct parts: a horizontal, constant infiltration-rate portion, which corresponds to preponding stage of infiltration; and a descending part, which corresponds to the rainpond stage. All three rainfall infiltration curves shown have similar general shapes and seem to be approaching the
same limiting infiltration rate. However, a comparison of their descending parts indicates that they do not constitute horizontally displaced segments of a single curve. The data of fig. 5 also demonstrate clearly that each one of these descending parts differs considerably from the corresponding portion of the flood infiltration curve.

**Figure 5. Relations between infiltration rates and infiltration duration during water uptake by initially air-dry Rehovot sand. The continuous-line curves represent relations during rainfall infiltration. The numbers labeling them indicate the relative rain intensities. The broken-line curve represents relations during infiltration due to flooding.**

The reason for the above differences between ponded infiltration curves must involve the moisture profile conditions at the commencement of water uptake. In particular, the dependence of incipient ponding profile characteristics upon rain intensity explains the differences between the rainpond infiltration curves of fig. 5.

In the past engineering and agricultural practice, information on the time dependence of rainpond infiltration rates often was read off infiltration capacity-time graphs obtained from experimental plots which were subjected either to instantaneous flooding or to high-intensity artificial rain. The results shown in fig. 5 indicate, that in the cases to which the theory under consideration is applicable, such practice might result in inaccurate estimates of the time-rainpond infiltration rate relation for a given rain intensity.

It has been pointed out above, that the incipient ponding time varies more with rain intensity than the incipient ponding water uptake. Possibly, this and the crossing-over of the fig. 5 curves explain the observed fact, that the discrepancies between these curves are larger than those based on the same data, but plotted as infiltration rate-cumulative water uptake graphs. It follows from this observation, that using this latter type of graph for deriving certain rainpond infiltration information from flooded plots often might result in smaller inaccuracy than the employment of rate-time curves.

The numerical analysis of rainpond infiltration presented in this paper and the conclusions implied by some of its results are applicable only to soil-water systems which obey the set of equations (1) to (5). The accuracy of the descriptions afforded by these equations and the conditions which limit their use are still being actively investigated. The methods outlined in this paper might help in such studies by increasing the range of the phenomena and the number of predictions which can be investigated experimentally. If the future work will further confirm the existing data on the approximate applicability of (1) and (5) to certain field phenomena, the analysis of this paper might also help in using the theoretical principles involved for solving some of the applied problems of rainfall uptake by soils.
REFERENCES


Analysis of infiltration into stratified soil columns

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ABSTRACT: A numerical solution of the flow equation for water in soil was obtained for a flow system consisting of a vertical column of stratified soil materials which had been drained from saturation to equilibrium with a water table. Water was assumed to be applied at the top of the column as either rainfall or ponded water. Infiltration into such a system involves hysteresis; i.e. the soil at each point wets along a different wetting scanning curve. This part of the hysteresis phenomena was taken into account in the solution of the problem. The solution of the equation depicts the time and depth distributions of water content and pressure head during the resulting infiltration as well as infiltration rate and accumulated water content.

The properties of the soil that influence the flow are the water capacity and hydraulic conductivity. Tables of $\kappa$, a dimensionless conductivity, and $\gamma$, a dimensionless water capacity, were calculated from tables of $\Theta$, a dimensionless water content, and $\varphi$, a dimensionless pressure head, for a set of moisture characteristic curves. These tables were fed into a digital computer which solved the problem.

Several hypothetical cases were examined. These were coarse-textured soils overlying fine-textured soils, fine-textured lenses in coarse-textured soils, and coarse-textured lenses in fine-textured soils.

Comparisons were made between these cases and also nonstratified cases on the basis of pressure head-time profiles, moisture content-time profiles, infiltration rate changes, and accumulated water content changes. The numerical solution predicts a hold up effect on the wetting

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