Analysis of infiltration into stratified soil columns

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ABSTRACT: A numerical solution of the flow equation for water in soil was obtained for a flow system consisting of a vertical column of stratified soil materials which had been drained from saturation to equilibrium with a water table. Water was assumed to be applied at the top of the column as either rainfall or ponded water. Infiltration into such a system involves hysteresis; i.e., the soil at each point wets along a different wetting scanning curve. This part of the hysteresis phenomena was taken into account in the solution of the problem. The solution of the equation depicts the time and depth distributions of water content and pressure head during the resulting infiltration as well as infiltration rate and accumulated water content.

The properties of the soil that influence the flow are the water capacity and hydraulic conductivity. Tables of $\kappa$, a dimensionless conductivity, and $\gamma$, a dimensionless water capacity, were calculated from tables of $\Theta$, a dimensionless water content, and $\varphi$, a dimensionless pressure head, for a set of moisture characteristic curves. These tables were fed into a digital computer which solved the problem.

Several hypothetical cases were examined. These were coarse-textured soils overlying fine-textured soils, fine-textured lenses in coarse-textured soils, and coarse-textured lenses in fine-textured soils.

Comparisons were made between these cases and also nonstratified cases on the basis of pressure head-time profiles, moisture content-time profiles, infiltration rate changes, and accumulated water content changes. The numerical solution predicts a hold up effect on the wetting

REFERENCES


front as it goes from a fine-textured layer into a coarse-textured one. As one would expect, the rate of advance of the fonts is faster for ponded infiltration than for rainfall infiltration.

RéSUMÉ : Une solution numérique de l’équation du mouvement de l’eau dans le sol est obtenue pour le cas d’un système consistant en un prisme vertical de sol stratifié qui est drainé de la saturation à l’équilibre avec nappe phréatique. L’eau est supposée être introduite au sommet du prisme comme dans le cas de la pluie recouvrant le sol. L’infiltration dans un tel système implique un hystérésis, c’est-à-dire que le sol en chacun de ses points se mouille suivant différentes courbes. Cette partie du phénomène d’hystérésis est envisagée dans la solution du problème. La solution de l’équation dépeint les distributions du temps et des distributions de teneur en eau et de pressions en profondeur pendant l’infiltration ainsi que les taux d’infiltration et la teneur d’eau accumulée.

Les propriétés du sol qui influencent le mouvement sont la capillarité de l’eau et la conductivité hydraulique. Des tables de $\alpha$, une conductivité sans dimension et de $\gamma$, une capacité en eau sans dimension ont été calculées en partant de tables de $\Theta$, une teneur en eau sans dimension et de $\theta$, une pression sans dimension; pour une série de courbes caractéristiques d’humidité. Ces tables ont alimenté une calculatrice digitale qui a résolu le problème.

Quelques cas hypothétiques ont été examinés : des sols à texture grossière recouvrant des sols à texture fine, des lentilles à texture fine dans des sols à texture grossière ainsi que le cas inverse. Des comparaisons ont été faites entre ces cas et aussi des cas non stratifiés sur la base de profils pressions-temps, de profils teneur en humidité-temps, de modifications du taux d’infiltration et de changements de la teneur en eau accumulée. La solution numérique prédit un effet de retard du front d’humidification quand il passe d’une couche à fine texture dans une couche à texture grossière. Comme on pouvait s’y attendre le taux d’avance des fronts est plus rapide pour une infiltration avec couverture superficielle d’eau que pour une infiltration par la pluie.

I. INTRODUCTION

The analysis of unsaturated flow systems has been considerably expedited by the development of high speed, large capacity digital computers. These have made it possible to obtain numerical solutions of the non-linear partial differential equation of flow which is valid at least to a first approximation for unsaturated flow. In recent years a number of analyses of the unsaturated flow during infiltration have appeared in the literature (Gardner, 1960; Hanks and Bowers, 1962; Philip, 1957; Rubin and Steinhardt, 1963; Whisler and Klute, 1965a).

The initial analyses of infiltration dealt with ponded water, uniform deep soil, and constant initial water content or pressure head. Rubin (1963) has recently considered infiltration under rain. In order to more closely represent actual field infiltration situations an extension of infiltration analysis to more complex boundary and initial conditions is desirable. The effect of non-uniform initial water content, or pressure head, and non-uniform soil infiltration is analyzed in this paper.

An arbitrary choice of non-uniform initial water content for infiltration may lead to a flow behavior in which hysteresis is involved, i.e. the water content in some parts of the system may at first decrease and then increase. This would introduce complexities which have only recently begun to yield to analysis (Rubin, 1965).

It is the purpose of this paper to describe water infiltration into a column of stratified soil which has initially been drained from saturation to equilibrium with a water table at its base. Both the cases of ponded water and rainfall infiltration will be considered and the results will be compared to similar infiltration into columns of uniform soil (Whisler and Klute, 1965b).

II. THEORY

If the initial condition chosen is an equilibrium condition then the difficulties with hysteresis mentioned above can be handled. In this case the water content in all parts of
the system will increase monotonically during the infiltration. When the initial condition is that obtained by wetting a dry vertical soil column from a water table at its lower end to equilibrium under gravity, the same water content-tension curve applies to all parts of the column. When the initial condition for infiltration is established by gravity drainage of a saturated soil, a different water content-tension relationship applies at each depth in the soil column. In this case the initial pressure head $h$ will be a linear function of elevation $z$ (fig. 1A, left side), varying from zero at the bottom of the column to $-L$ at the top of the column. The water content $\theta$ will vary from saturation $\theta_S$ at the water table to some lower water content at the top of the column (fig. 1A, right side). The shape of the profile will be dictated by the nature of the soil material. All points in the column will possess corresponding pressure head and water content values given by the main drainage curve of the water content-pressure head function (curve d, fig. 1B). During the infiltration, all points of the column will wet up, i.e. the pressure head and water content at each point will increase monotonically with time during the flow. Furthermore, each point will wet along its own wetting scanning curve, such as curve $s$ of fig. 1B. The pressure head water content curve for wetting, applicable at the top of the column is curve $w$ of fig. 1B.

![Figure 1](image_url)

**Figure 1.**
(A) Equilibrium pressure head and water content versus depth
(B) Water content versus pressure head
(C) Hydraulic conductivity versus pressure head
(D) Water capacity versus pressure head

The soil properties required in the analysis of the flow are a) the conductivity-pressure head function, $K(h)$ and b) the water capacity-pressure head function $C(h)$. The water capacity is defined as $\frac{\partial \theta}{\partial h}$, i.e. the derivative of the water content-pressure head function. Fig. 1C shows the general nature of the conductivity function, including a main drainage curve $d$ and wetting scanning curves $s$ and $w$. Fig. 1D shows the nature of the water capacity function. During infiltration the appropriate conductivity and water capacity functions at each point will be those given by the particular wetting scanning curves which start at a pressure head equal to the negative of the elevation of the point above the bottom of the column. Thus at each point a different set of conductivity and water capacity curves is applicable,
and one may say that the conductivity and water capacity now depend on both \( h \) and \( z \), instead of \( h \) alone.

There are two forms of the flow equation, the pressure head form and the diffusivity form (Gardner, 1960). In the problem to be solved it will be advantageous to use the pressure head form, because part of the flow region may become saturated, and the diffusivity form then becomes useless.

The flow equation for water in unsaturated soil then becomes:

\[
C(h, z) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left( K(h, z) \frac{\partial h}{\partial z} \right) + \frac{\partial K(h, z)}{\partial z} \tag{1}
\]

The water table at the bottom of the column is described by the condition

\[
h(-L, t) = 0 \quad t \geq 0 \tag{2}
\]

The condition of rainfall infiltration at the top of the column is described by:

\[-K(h, 0) \left( \frac{\partial h}{\partial z} + 1 \right)_{z=0} = f_i \quad t > 0 \tag{3}
\]

where \( f_i \) is the flux of water crossing the plane \( z = 0 \). During the period of rainfall infiltration the flux \( f_i \) is equal to the negative of the rainfall rate \( R \), and if the rain is steady then both \( R \) and \( f_i \) are constants. When \( R \geq K_s \), the saturated conductivity, the condition expressed by equation (3) will apply until the pressure head at the top of the column increases to zero. In this analysis it is assumed that zero pressure head implies saturation. After the top of the column becomes saturated it is assumed that the pressure head thereafter remains constant at zero, which implies that the excess water is to free run off. Thus, in this case, (3) is replaced by:

\[
h(0, t) = h_b \tag{3'}
\]

When \( R < K_s \), condition (3) applies at all times. For the case of rainfall infiltration \( h_b = 0 \). When infiltration was caused by ponded water on top of the column, equation (3) applies at all times with \( h_b > 0 \). The linear initial pressure head condition is given by:

\[
h(z, 0) = -(L + z), \quad -L \leq z \leq 0 \tag{4}
\]

For convenience and economy of effort in computation it is convenient to introduce the following dimensionless pressure head, position and time variables:

\[
\varphi = \frac{h}{L} \tag{5}
\]

\[
\zeta = \frac{z}{L} \tag{6}
\]

\[
\tau = \frac{tK_s}{\theta_s L} \tag{7}
\]

Using these, equation (1) becomes:

\[
\gamma(\varphi, \zeta) \frac{\partial \varphi}{\partial \tau} = \frac{\partial}{\partial \zeta} \left[ \chi(\varphi, \zeta) \frac{\partial \varphi}{\partial \zeta} \right] + \frac{\partial \chi(\varphi, \zeta)}{\partial \zeta} \tag{8}
\]
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where:

\[ \gamma(\varphi, \zeta) = \frac{LC(h, z)}{\theta_s} = \frac{\partial \theta(\varphi, \zeta)}{\partial \varphi} \]  
(9)

\[ \chi(\varphi, \zeta) = \frac{K(h, z)}{K_s} \]  
(10)

\[ \Theta(\varphi, \zeta) = \frac{\theta(h, z)}{\theta_s} \]  
(11)

The boundary and initial conditions become:

\[ \varphi(-1, \tau) = 0 \quad \tau \geq 0 \]  
(12)

\[ -\chi(\varphi, 0) \left( \frac{\partial \varphi}{\partial \zeta} + 1 \right)_{\zeta=0} = F_i \]  
(13)

for

\[ \varphi(0, \tau) < 0 \]

For ponded water infiltration and for rainfall infiltration when \( \varphi(0, \tau) \) becomes equal to zero, condition (13) is replaced by:

\[ \varphi(0, \tau) = \varphi_b = \frac{h_b}{L} \quad \tau > \tau_c \]  
(13')

\[ \varphi(\zeta, 0) = -(1 + \zeta) \quad -1 < \zeta < 0 \]  
(14)

The dimensionless flux at the top of the column \( F_i \) is given by \( f_i / \kappa_s \), and \( \tau_c \) is the time at which \( \varphi(0, \tau) \) reaches zero when condition (13) is applied.

In the analysis described above it was assumed that the gas phase pressure remained constant at atmospheric pressure during the infiltration. In fact the pressure in a lens of air trapped between the ponded water and the water table will increase as the infiltration proceeds (Youngs and Peck, 1964). This will have an effect on the flow of the water. Further numerical analysis of the problem should incorporate this phenomenon.

In order to solve equation (8) numerically, subject to the above conditions, a grid of points was superimposed on the region \( \tau \geq 0, -1 \leq \zeta \leq 0 \). The \( \zeta \)-axis was divided into \( N-1 \) intervals. \( N \) in this case was chosen equal to 101. The mesh points are defined by:

\[ \tau_m = m \Delta \tau \quad m = 0, 1, 2 \ldots \]  
(15)

\[ \zeta_n = n \Delta \zeta \quad n = 0, 1, 2 \ldots N-1 \]  
(16)

\[ \Delta \zeta = \frac{1}{N-1} \]  
(17)

The partial derivatives in (8) were approximated by the finite differences:

\[ \frac{\partial}{\partial \zeta} \left[ \chi(\varphi, \zeta) \frac{\partial \varphi}{\partial \zeta} \right] \approx \frac{1}{2(\Delta \zeta)^2} \left[ \chi_{n+\frac{1}{2}, m-\frac{1}{2}}(\varphi_{n+1, m} + \varphi_{n+1, m-1} - \varphi_{n, m} - \varphi_{n, m-1}) - \chi_{n-\frac{1}{2}, m-\frac{1}{2}}(\varphi_{n, m} + \varphi_{n-1, m} - \varphi_{n-1, m-1} - \varphi_{n, m-1}) \right] \]  
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\[
\frac{\partial \phi}{\partial t} \approx \frac{\phi_{n,m} - \phi_{n,m-1}}{\Delta t} \tag{19}
\]

\[
\frac{\partial \kappa}{\partial \zeta} \approx \frac{\kappa_{n+\frac{1}{2},m-\frac{1}{2}} - \kappa_{n-\frac{1}{2},m-\frac{1}{2}}}{\Delta \zeta} \tag{20}
\]

where arbitrarily

\[
\kappa_{n+\frac{1}{2},m-\frac{1}{2}} = \frac{\kappa_{n,m-1} + \kappa_{n+1,m-1} + \kappa_{n+1,m} + \kappa_{n,m}}{4} \tag{21}
\]

and a similar definition is true for \(\kappa_{n-\frac{1}{2},m-\frac{1}{2}}\). This method of finite differencing is a modification of the Crank-Nicholson method discussed in Richtmeyer (1957).

The finite difference approximations (19) through (22) were substituted in equation (8) to obtain a set of \(N-2\) algebraic equations of the form:

\[
A_n \phi_{n-1,m} - B_n \phi_{n,m} + C_n \phi_{n+1,m} = -H_n \tag{22}
\]

where:

\[
A_n = \kappa_{n-\frac{1}{2},m-\frac{1}{2}} \tag{23}
\]

\[
B_n = \frac{2\gamma_{n, m-\frac{1}{2}}}{r} + \kappa_{n+\frac{1}{2}, m-\frac{1}{2}} + \kappa_{n-\frac{1}{2}, m-\frac{1}{2}} \tag{24}
\]

\[
C_n = \kappa_{n+\frac{1}{2},m-\frac{1}{2}} \tag{25}
\]

\[
H_n = \kappa_{n+\frac{1}{2}, m-\frac{1}{2}} (\phi_{n+1,m-1} - \phi_{n,m-1}) - \kappa_{n-\frac{1}{2}, m-\frac{1}{2}} (\phi_{n,m-1} - \phi_{n-1,m-1}) + \\
+ (\kappa_{n+\frac{1}{2}, m-\frac{1}{2}} - \kappa_{n-\frac{1}{2}, m-\frac{1}{2}}) 2\Delta \zeta + 2\gamma_{n, m-\frac{1}{2}} \frac{\phi_{n,m-1}}{r} \tag{26}
\]

where:

\[
r = \frac{\Delta t}{(\Delta \zeta)^2} \tag{27}
\]

and

\[
\gamma_{n, m-\frac{1}{2}} = \frac{\gamma_{n, m-1} + \gamma_{n, m}}{2} \tag{28}
\]

When equation (22) is applied at \(n = 1\) and the boundary condition (12) is invoked the result is:

\[
B_1 \phi_{1,m} + C_1 \phi_{2,m} = -H_1 \tag{29}
\]

At \(n = N-2\) the combination of the finite difference form of (13) and equation (22) yields:

\[
A_{N-2} \phi_{N-3,m} - (B_{N-2} - C_{N-2}) \phi_{N-2,m} = \\
- \left[ H_{N-2} - C_{N-2} (2\Delta \zeta - \phi_{N-2,m-1} + \phi_{N-1,m-1}) - 2 \Delta \zeta F_b \right] \tag{30}
\]

When condition (13') is applicable, equation (30) is replaced by the following:

\[
A_{N-2} \phi_{N-3,m} - B_{N-2} \phi_{N-2,m} = -H_{N-2} - C_{N-2} \phi_b \tag{30'}
\]
Equations (22), (29) and (30), or (30') constitute a set of $N-2$ algebraic equations in $N-2$ unknowns.

The coefficients $A_n$, $B_n$ and $C_n$ and the term $H_n$ as defined above depend on values of $\varphi_{n,m}$ and thus the equations are non-linear algebraic equations. To remove this non-linearity an iteration process was used in which an initial estimate of the $\varphi_{n,m}$ values at each point was made and used to determine the coefficients $A_n$, $B_n$, $C_n$ and $H_n$.

The solution of the problem proceeds in steps of $d\tau$ along the $\tau$-axis. The initial condition is used as the first set of $\varphi_{n,m\_1}$ values and these were also used as the first estimate of the $\varphi_{n,m}$-values for the purpose of evaluating $A_n$, $B_n$, $C_n$ and $H_n$. The resulting set of linear algebraic equations was solved by an algorithm (Richtmeyer, 1957) for an improved estimate of the $\varphi_{n,m}$-values. This improved estimate of $\varphi_{n,m}$ was then in turn used to get new values of $A_n$, $B_n$, $C_n$ and $H_n$ and a second improved estimate of the $\varphi_{n,m}$'s was obtained from equations (22), (29) and (30). When this iteration process had converged to a satisfactory point, the last estimate of the $\varphi_{n,m}$-values was retained, and considered to apply at $\tau_m = \Delta \tau + \tau_{m\_1}$. The iteration process described above was then repeated for successive time steps to build up the time and space dependence of $\varphi$.

At the end of each time step the flux at the bottom of the column $F_0$ was calculated from:

$$F_0 = -A_1 \left( \frac{\varphi_1 - \varphi_0}{\Delta \zeta} + 1 \right) = \frac{f_0}{K_s} \quad (31)$$

When (30') was applicable, the flux at the top of the column $F_i$ was calculated from:

$$F_i = -C_{N\_2} \left( \frac{\varphi_{N\_1} - \varphi_{N\_2}}{\Delta \zeta} + 1 \right) = \frac{f_i}{K_s} \quad (32)$$

It should be noted that the infiltration rate, as usually defined, is the negative of the flux $f_i$. Likewise, the outflow rate will be the negative of $f_o$. Both $f_i$ and $f_o$ will be negative for downward flow.

The amount of water which has accumulated an the column due to infiltration, hereafter referred to as cumulative infiltration $\sigma$ is:

$$\sigma = \int_{-1}^{0} \theta(\zeta, \tau) d\zeta - \int_{-1}^{0} \theta(\zeta, 0) d\zeta \quad (33)$$

It is seen that $\sigma = 0$ for $\tau = 0$ and approaches a limit depending upon the initial distribution. Cumulative infiltration was evaluated at each time step by numerical integration of equation (33).

III. RESULTS AND DISCUSSION

The calculations described above can and should be performed with measured $\theta(h,z)$- and $K(h,z)$-data for real soils. Such data must include the main drainage curve and the wetting-scanning curves. Due to the scarcity of such data, hypothetical $\theta(h,z)$-curves were constructed to represent various types of soil materials. In constructing these curves the authors were guided by their own experience, and the data available in the literature on scanning curve behavior. Fig. 2 and 3 show the dimensionless water content-pressure head relations $\Theta(\varphi)$, for two hypothetical soils, designated as soil 3 and soil 4. Infiltration into columns of soil 1 and soil 2, two medium textured soils, is discussed elsewhere (Whisler and Klute, 1965a; Whisler and Klute, 1965b). Soil 3 might be considered as a clay or clay loam while soil 4 might be considered as a sand or fine sand if the
The value of $L$ used to reduce the real variables is approximately 500 cm. Each wetting scanning curve can be identified with a depth $z$ in the column, which is equal to the pressure head at which the wetting scanning curve intersects the main drainage curve. Thus the family of wetting scanning curves in fig. 2

![FIGURE 2. Dimensionless water content $\Theta$ as a function of dimensionless pressure head $\varphi$ showing the main drainage curve and wetting scanning curves for soil 3. The value of $\zeta$ which applies to each scanning curve is equal to the value of $- (\varphi + 1)$ where the scanning curve leaves the drainage curve](image)

represents the $\Theta(\varphi, \zeta)$ function for this 'soil'. These $\Theta(\varphi, \zeta)$ curves may be considered to represent any combination of $\theta(h, z)$, $L$ and $\partial_s$ which might happen to reduce to these particular dimensionless curves.

After choosing the $\theta(h, z)$-function one is not at liberty to choose just any conductivity-pressure head curve. A $K(h, z)$-or $\kappa(\varphi, \zeta)$-curve must be selected that is physically consistent with the chosen $\theta(h, z)$ curve. If measured $\theta(h, z)$ and $K(h, z)$ data were available this matter would be taken care of automatically. In order to make a reasonable choice of the $\kappa(\varphi, \zeta)$-function for a given $\Theta(\varphi, \zeta)$-function, the conductivity was calculated from the $\Theta(\varphi, \zeta)$ curves (Whisler and Klute, 1965a) using procedures developed by Millington and...
Quirk (1960) and by Childs and Collis-George (1950). The results of such calculation are shown in fig. 4 and 5. The water capacity function \( \gamma(\varphi, \zeta) \) was obtained by numerical differentiation of the \( \Theta(\varphi, \zeta) \) functions in fig. 2 and 3. Tables of data relating \( \Theta, \varphi \) and \( \gamma \) to \( \varphi \) and \( \zeta \) were then prepared for the computations.

Three cases of textural stratification will be considered, viz. a) coarse over fine; b) coarse over fine over coarse; and c) fine over coarse over fine. Soil 4 was considered the coarse textured material, soil 3 the fine textured material. The position of a layer in the column and the linear initial pressure head distribution determined the segment of the drainage curve and the appropriate wetting scanning curves to be used.

In each of the cases of stratification to be examined it was necessary to construct the composite appropriate \( \Theta(\varphi, \zeta) \)- and \( \chi(\varphi, \zeta) \)-functions for the soils as they were placed in the column. The same values of \( \theta_s \) and \( K_s \) must be used as scaling parameters throughout the column. In the analyses presented here, the scaling was considered to have been done with \( \theta_s \) and \( K_s \) values appropriate to the coarse textured material (soil 4). Thus, the
\( \Theta(\phi, \zeta) \)-data for soil 3 (fig. 2) must be rescaled. In fig. 2, \( \Theta \) is given by \( \theta/\theta_s^3 \) where \( \theta_s^3 \) is the saturation water content of soil 3. This data may be rescaled on the basis of \( \theta_s^4 \), the saturation water content of soil 4, by multiplying all the values of \( \Theta \) in fig. 2 by the ratio \( \theta_s^3/\theta_s^4 \). A similar conversion applies to the conductivity data.

Fig. 6 shows the drainage and main wetting curves of the \( \Theta(\phi, \zeta) \)-function constructed in the above manner for a soil column composed of a layer of coarse over fine soil with the interface between the layers at \( \zeta = -0.2 \) (designated as case 2). Case 1 was a combination similar to case 2 where \( \theta_s^3/\theta_s^4 = 1.0 \). The results were similar to those discussed here for case 2. The data used to construct the curve in fig. 6 were the drainage curve for soil 3 between \(-0.8 \leq \phi \leq 0\) and the wetting scanning curves associated with these \( \phi \)'s, and the drainage curve for soil 4 between \(-1.0 \leq \phi < 0.8\) and the wetting scanning curves for these \( \phi \)'s. The \( \Theta \) values for soil 3, from fig. 2, were multiplied by \( \theta_s^3/\theta_s^4 = 0.75 \), and the \( \chi \) values from fig. 4 were multiplied by \( K_s^3/K_s^4 = 0.2 \).

The direct result of the computations using equations (22), (29) and (30) was the pressure head-depth-time function, \( \phi(\zeta, \tau) \). One of these is shown in fig. 7 for ponded infiltration where \( \dot{\phi}_b = 0.01 \). The initial linear pressure head-depth relation is shown by the dashed line. As the infiltration proceeds, a pressure head front moves downward into the soil. When the front reaches the fine layer it quickly moves further down the column before the top layer becomes saturated and is less steep in the fine layer.

The pressure head distribution in the steady state limit of flow for this column may be calculated from Darcy's law using the saturated conductivities of the layers because the entire column will be saturated. The right hand curve in fig. 7 is the pressure head distribution calculated in this manner.

Fig. 8 shows the dimensionless water content-depth-time curves, \( \Theta(\zeta, \tau) \) which were constructed from the pressure head profiles of fig. 7 by using the \( \Theta(\phi, \zeta) \) relationship for this column, shown in fig. 6. Observe that the lower layer wets to saturation, \( \Theta = 0.75 \), before the upper layer becomes completely saturated.

For the cases of infiltration from a given rainfall rate \( R \), one may define a dimensionless rainfall rate, \( \rho = R/K_s \). Fig. 9 shows the resulting pressure head distributions in the same soil column for \( \rho = 0.9 \). The results appear similar to those in fig. 7 except for the behavior of \( \phi \) at \( \zeta = 0 \). In contrast to the ponded case the pressure head at the top of the column took a non-zero but finite time to reach zero. Since this rainfall rate \( \rho = 0.9 \) is greater than the saturated conductivity \( \chi = 0.2 \) of the lower layer, then one would expect ponding to
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occur and it does. Because the pressure head at the soil surface did not immediately reach zero, the time required for the front to reach the interface between layers was somewhat longer under rainfall than under ponded infiltration (compare fig. 7 and 9). The results for the dimensionless water contents are shown in fig. 10 and similar comparisons can be made.

For \( \rho = 0.1 \) which is less than the saturated conductivity of the fine layer, it can be seen that the pressure head front moves even more slowly down the column and is not as steep, fig. 11. For this rainfall rate the pressure at the top of the column does not reach zero nor does the water content reach saturation. Instead both tend toward steady state limits which were about \(-0.02\) for \( \phi \) and about 0.84 for \( \Theta \) (fig. 12).

The effect of ponding and rainfall rate on the variation of water content with time at a fixed depth, \( \zeta = -0.18 \), is shown in fig. 13. Increasing the rainfall rate decreased the time required for the front to reach the selected depth, with ponded infiltration requiring the smallest time. When \( \rho > 0.2 \) the layer of soil at \( \zeta = -0.18 \) will become saturated in the steady state limit. The dashed portions of the curves are estimates of where they should be.

Fig. 14 shows the effect of ponding and rainfall rate on the fluxes \( F_t \) and \( F_0 \), in the top and out the bottom of the column, respectively. All fluxes are negative because the flow is downward in the negative \( z \) direction. Again the dashed portions of the curves are extensions or estimates of where they should be. When the rainfall rate \( \rho \) is less than the
saturated conductivity of the fine layer, \( i.e. \rho < 0.2 \), the flux across the top of the column will always be equal to \( \rho \). When \( \rho \) exceeds the saturated conductivity of the fine layer but is less than the saturated conductivity of the coarse layer, there will be a period of constant flux during which \( F_1 \) equals \( \rho \). When the pressure head at the interface between layers becomes zero, \( F_1 \) increases rather rapidly to \( -\bar{x} = -0.2381 \) where \( \bar{x} \) is calculated from:

\[
\frac{1}{\bar{x}} = \frac{\text{depth of top layer}}{x_s \text{ of top layer}} + \frac{\text{depth of bottom layer}}{x_s \text{ of bottom layer}}
\]

(34)

where \( x_s \) is the value of \( x \) at saturation. When \( \rho \) exceeds the saturated conductivity of the coarse layer, there is an initial period of constant flux during which \( F_1 \) equals \( -\rho \). At a time \( t_1 \) when the pressure head at the top of the column becomes zero, \( F_1 \) increases to \( -x_s \) of the upper coarse layer. This is then followed by a period of constant flux until the pressure head at the interface becomes zero. \( F_1 \) then increases to the limit \( -\bar{x} \). For ponded infiltration the initial period is one of increasing \( F_1 \) until it reaches \( -x_s \) of the coarse top layer and then \( F_1 \) behaves in manner similar to that described for rainfall infiltration with \( \rho > x_s \) of the coarse layer.

Another case of stratification, designated as case 3, of interest would be a layer of fine material in a coarser textured soil, \( i.e. \) the plow pan or fragipan case. To represent this
case a layer of soil 3 was assumed to be located in the region \(-0.2 > \zeta > -0.3\) in a column of soil 4. The ratio \(K_{s3}/K_{s4}\) was chosen as 0.2 and \(\theta_{s3}/\theta_{s4}\) was assumed to be unity. The composite \(\Theta(\wp, \zeta)\) function for this case is shown in fig. 15. A similar composite \(x(\wp, \zeta)\) function was prepared but is not shown.

The pressure head depth-time distribution for infiltration from ponded water \((\wp_b = 0.01)\) is shown in fig. 16. The distribution at small times, before the front enters the fine layer, is the same as that shown in fig. 7. for case 2.

Observations of infiltration into air dry stratified soil, e.g. Miller and Gardner (1962), have indicated that when the wetting front encounters an interface between layers of different texture the rate of advance of the front and the infiltration rate are decreased. This holdup effect is predicted in the analyses of infiltration given in this paper. For example, in case 3 when the wetting front encountered the interface between the fine and coarse materials at \(\zeta = -0.3\) there was a rather abrupt decrease in the rate of advance of a given water content and pressure head (see fig. 16 and 17). On the other hand, when the wetting front reached the boundary between the coarse and fine materials at \(\zeta = -0.2\) there was not a very noticeable holdup effect on the water content and pressure head. This may be due to the fact that the water was entering a fine material at a higher initial water content than that of the coarse layer and which had a conductivity at saturation which was of the same order of magnitude (1 to 5 ratio) as that of the upper layer. There was a decrease in infiltration rate when the front reached \(\zeta = -0.2\) as shown by the increase in the
flux $F_i$ for case 3 at $t$ equal to approximately 0.13 (see fig. 23). A similar behavior of the infiltration rate was obtained in case 2 (see fig. 14).

**Figure 13.** Dimensionless water content versus log $t$ at a fixed depth ($\zeta = -0.18$) for ponded infiltration and four rainfall rates, Case 2

**Figure 14.** Fluxes in the top and out the bottom of the column as a function of time for ponded infiltration, and four rainfall rates, Case 2

The infiltration in case 3 will continue until a steady state limit is reached. An analysis using Darcy’s law for each layer shows that the pressure head will be negative at the fine-coarse interface when $\phi_h = 0.01$. Therefore the steady state limit will not be one of saturation throughout the profile but the steady state profile will have a zone of unsaturation in the vicinity of the fine-coarse interface.
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Figure 15. Composite, rescaled $\Theta(\Phi, \zeta)$ function for Case 3, a layer of soil 3 inserted into a column of soil 4. Only the drainage and main wetting curves are shown.

Figure 16. Dimensionless pressure head versus $\zeta$, with $\tau$ as a parameter, Case 3 ponded infiltration.
Figure 17. Dimensionless water content versus $\zeta$ with $\tau$ as a parameter, Case 3 ponded infiltration

Figure 18. Composite, rescaled $\Theta(\varphi, \zeta)$ function for Case 4, a layer of soil 4 inserted in a column of soil 3. Only the drainage and main wetting curves are shown.
The reverse of case 3, viz., a layer of coarse soil in a fine soil, was studied and designated as case 4. The same $K_s$ and $\theta_s$ ratios and boundary conditions were used as in case 3. The composite $\theta(\phi, \zeta)$-curve for this case is shown in fig. 18. The calculated pressure head distributions are shown in fig. 19. The water content profiles are shown in fig. 20.

![Figure 19](image)

**Figure 19.** Dimensionless pressure head versus $\zeta$, with $\tau$ as a parameter, for Case 4, ponded infiltration

Again there is a holdup effect at the fine-coarse interface at $\zeta = -0.2$. It may be seen from the dashed curve in fig. 20, which is the initial water content distribution, that case 4 represents infiltration into a wetter soil than does case 3. Therefore as is observed the fronts move more quickly down to the coarse layer than they did in case 3, even though the $K_s$ of soil 3 is less than that of soil 4. Analysis also shows that in case 4 at the fine-coarse interface ($\zeta = -0.2$) at steady state the pressure head will be slightly negative and the column will not be completely saturated with these boundary conditions and soil materials.

The holdup phenomenon is shown in terms of the position of a given $\phi$ and $\theta$ in figs. 21 and 22. In fig. 21 the position of $\phi = -0.2$ was plotted as a function of $\tau$ for cases 3 and 4. It is seen that in case 4 the position of this $\phi$ lagged behind that for case 3, but as the front reached the fine-coarse interface ($\zeta = -0.2$) the $\phi$ position in case 4 advanced quite rapidly until it moved into the coarse layer ($\tau \approx 0.6$). It then slows down with slight fluctuations in its rate of advance for the rest of the time. In fig. 22 the position of $\theta = 0.94$ is plotted as a function of $\tau$. The results are similar to those for $\phi$ shown in fig. 21 except that the selected water content always advanced more rapidly in case 4 than in case 3.

Fig. 23 shows the time dependence of the flux for ponded infiltration for cases 2, 3 and 4. For cases 2 and 3 there is an intermediate period of constant infiltration rate and then a
final period of constant steady state infiltration rate. Case 4 did not have this intermediate period of constant infiltration rate. It is possible that if the coarse layer had been at a greater depth that the intermediate constant rate period would have been demonstrated.

Figure 20. Dimensionless water content versus $\zeta$, with $r$ as a parameter for Case 4, ponded infiltration

Figure 21. Position of $\phi = -0.2$ versus log $r$ for Cases 3 and 4

The steady state limit on the flux for cases 3 and 4 are only approximate in that they were calculated assuming the column to be completely saturated, which was not true. The actual limits would occur at somewhat higher values of flux than shown. The fact that the steady state flux for case 3 was less than that for case 4, *i.e.* the limiting infiltration rate was
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Figure 22. Position of $\Theta = 0.94$ versus $\log \tau$ for Cases 3 and 4

Figure 23. Flux at the top $f_1$ and at the bottom of the column $f_0$ versus $\log \tau$ for Cases 2, 3 and 4

Figure 24. Dimensionless cumulative infiltration $\sigma$ versus $\log \tau$ for Cases 2, 3 and 4, and a uniform column of soil 4
greater, is expected, since case 3 is mostly a column of coarse soil with a higher saturated conductivity than case 4, which is mostly a column of fine soil.

The cumulative infiltration is also affected by stratification as is shown in fig. 24. In this figure the dimensionless cumulative amounts are plotted against $\tau$. A comparison is made between cases 2, 3 and 4 and a uniform column of soil 4. For $\tau$-values between 0.004 and 0.05 case 4 has a greater cumulative infiltration than cases 2 or 3. The value $\tau = 0.05$ corresponds roughly to the time at which the wetting front passes the fine-coarse interface ($\zeta = -0.2$) in case 4. From then on case 4 was the lowest in cumulative infiltration. This is to be expected since it was initially wetter and it did not need as much water to fill it. For $\tau$ values greater than 0.14 the column of uniform soil 4 had greater cumulative infiltration than any other and it had the highest limit at steady state. Again this is to be expected since it was the driest initially.

REFERENCES


