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ANALYSIS OF LAND SUBSIDENCE IN NIIGATA

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ABSTRACT

The land subsidence in the Niigata area was analyzed by means of consolidation theory. Pumping the underground water from the deep sandy layers for extracting methane gas caused a time-dependent loading to the clay layers. An analytical solution of consolidation under the load increasing linearly with time was obtained in terms of the vertical strain. A numerical method of analysis for the load decreasing linearly and then becoming constant was developed considering the difference in values of soil constants for consolidation and for rebound respectively. These two methods were combined for analyzing the subsidence in the Niigata area, and the results compared favourably with the observed settlement.

RéSUMÉ

L'affaissement du sol à Niigata a été analysé par le moyen de la théorie de consolidation. Le pompage de l'eau souterraine à partir de la couche profonde sableuse pour extraire le gaz méthane a causé la mise en charge, variable avec le temps, de la couche de l'argile. Une solution analytique de la consolidation sous la charge augmentant avec le temps est obtenue en ce qui concerne la déformation verticale. Une méthode numérique d'analyse de la charge augmentant d'abord linéairement et devenant ensuite constante est développée en tenant compte de la différence dans les valeurs des constantes du sol à la consolidation et au régonflement. Ces deux méthodes sont combinées pour analyser le tassement à Niigata, et les résultats ont été favorablement comparés avec les valeurs observées.

1. INTRODUCTION

Niigata City and the neighbouring area have suffered a severe land subsidence since the 1950's. The greatest rate of settlement of 53 cm per year was observed around the Port of Niigata in the period of 1958-1959 as shown in figure 1 (1st District Bureau for Port Construction et al., 1963). Many of the port and coast facilities, river and road embankments, and farms and factories had gone out of use.
Every possible reason of land subsidence was investigated, and finally it was concluded that the main source of the subsidence was the pumping of underground water for extracting methane gas. Natural methane gas in the Niigata area is dissolved in the water of deep sandy aquifers. The ratio of the volume of the dissolved gas to the water was one to one, and the greatest rate of the pumpage was 17,000,000 cubic meters per month. It caused the water heads of the aquifers to be below sea level by a maximum amount of 44 m, and resulted in a considerable consolidation of the clay strata.

While large scale countermeasures were rushed in the subsiding area, various investigations were carried out; leveling, tide observations, borings and soil tests, measurements of compression and water table in each stratum by means of observation wells, and the statistical survey of a quantity of the pumped water and gas. Subsidence analyses were made by means of the consolidation theory and others, and future settlement was predicted with satisfactory accuracy at each time.

Pumping restrictions, have been enforced four times from 1959 through 1962. It resulted in a marked recovery of the water heads in the aquifers, hence a new loading condition for the clay strata, as illustrated in figure 2. The writer developed a new method for analyzing consolidation under such time-dependent loading (Okumura and Moto, 1967), and applied it to the analysis of the land subsidence in the Niigata area.

2. THEORETICAL BASIS

2.1. CONSOLIDATION UNDER INCREASING LOAD

According to Mikasa's theory (1963), it is more convenient to analyze consolidation in terms of strain than in terms of excess pore water pressure, since the problem of pore pressure set-up in this case can be replaced by the problem of boundary conditions.

If the change in thickness of a clay layer and the influence of its own weight are neglected, and the coefficient of consolidation, $C_u$, is assumed constant, the fundamental differential equation of one dimensional consolidation in terms of compression strain, $\varepsilon$, is given as,

$$\frac{\partial \varepsilon}{\partial t} = C_u \frac{\partial^2 \varepsilon}{\partial z^2}$$

Assuming that the total stress distribution is linear with depth and the load increases at a constant rate up to time $t_1$, as shown in figure 2, the boundary and initial conditions become,

$$\begin{align*}
\varepsilon(0, t) &= m_v p_1 t/t_1 \\
\varepsilon(2H, t) &= m_v p_2 t/t_1 \\
\varepsilon(z, 0) &= 0
\end{align*}$$

where the coefficient of volume compressibility, $m_v$, is assumed constant. The solution of equation (1) under the condition of equation (2) can be obtained by the theory of conduction of heat (Carslaw and Jaeger, 1959) as follows,

$$\varepsilon = \frac{m_v}{T_1} \left[ T \left( p_1 + \left( p_2 - p_1 \right) \frac{z}{2H} \right) - \frac{p_1 + p_2}{2} F_1(T, z/2H) + \frac{p_2 - p_1}{2} F_2(T, z/2H) \right]$$
FIGURE 1. Extraordinary rate of subsidence in and around Niigata City

FIGURE 2. Typical loading condition
in which $T$ is the time factor ($T = C_v t / H^2$), $T_1$ is the time factor at time $t_1$, and $F (T, z/2H)$ is referred to the coefficient of strain and expressed in the forms,

$$F_1(T, z/2H) = \frac{16}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} \left(1 - e^{-n^2 \pi^2 T/4}\right) \sin \frac{n \pi z}{2H} \quad (4a)$$

$$F_2(T, z/2H) = \frac{16}{\pi^3} \sum_{n=2, 4, 6, \ldots} \frac{1}{n^3} \left(1 - e^{-n^2 \pi^2 T/4}\right) \sin \frac{n \pi z}{2H} \quad (4b)$$

The relationship between the strain and the excess pore water pressure, $u$, is,

$$\varepsilon = m_v (p - u)$$

$$= m_v \left[ \frac{t}{t_1} \left( p_1 + (p_2 - p_1) \frac{z}{2H} \right) - u \right] \quad (5)$$

where $p$ is the total pressure at the depth and time under consideration. The solution in terms of excess pore pressure is then written as,

$$u = \frac{1}{T_1} \left[ \frac{p_1 + p_2}{2} F_1(T, z/2H) - \frac{p_2 - p_1}{2} F_2(T_1, z/2H) \right] \quad (6)$$

This corresponds to Terzaghi-Frölich's solution (1936). And the condition of $p_1 = p_2$ gives,

$$u = \frac{p_1}{T_1} F_1(T_1, z/2H) \quad (7)$$

This corresponds to Schiffman's solution (1963).

Denoting the degree of consolidation, $U$, as the ratio of the mean effective pressure to the mean total pressure at that time, the expression for the degree of consolidation is,

$$U = 1 - \frac{1}{T} \int_0^{2H} \frac{u \, dz}{\int_0^{2H} p \, dz}$$

$$= 1 - \frac{1}{T} \left(1 - \frac{32}{\pi^4} \sum_{n=1, 3, 5, \ldots}^{\infty} \frac{1}{n^4} e^{-n^2 \pi^2 T/4}\right)$$

$$= U_0(T) \quad (8)$$

in which $U_0(T)$ shall be called the coefficient of degree of consolidation and is independent of the magnitude of pressures $p_1$ and $p_2$ respectively. Using the coefficient of degree of consolidation, the expression for the settlement, $S$, becomes,
It may be said that the settlement in this case is expressed as the product of the thickness of the clay layer, the coefficient of volume compressibility, the mean total pressure at that time, and the degree of consolidation.

2.2. CONSOLIDATION UNDER DECREASING LOAD

When a load decreases before a corresponding consolidation of a clay layer is completed the following situation may occur: the effective pressure in a part of the layer continues to increase, whereas it decreases in the other part, causing this part of the soil to swell or rebound. In such a case, the compound phenomenon of consolidation and rebound should be considered.

The coefficient of volume expansibility, \( m_{ov} \), of the clay will be smaller than the coefficient of volume compressibility by the factor of about 10 (fig. 6), and it will be dependent on the magnitude of the preconsolidation load and the overconsolidation ratio. As far as the writer is aware, however, no established relationship between the above factors has been reported. Similarly much ambiguity exists in the coefficient of rebound, \( C_{or} \), which is a counterpart of the coefficient of consolidation, \( C_v \). If the coefficient of permeability does not change, however, the coefficient of rebound will become \( m_v/m_{ov} \) times the coefficient of consolidation. Therefore, in this section it may be assumed,

\[
m_{ov} = m_v/r \quad \text{and} \quad C_{or} = rC_v
\]

where \( r \) is a constant for a particular soil.

In the consolidation phenomenon, including rebound, the strain depends upon the stress history. Thus the choice of the strain as the dependent variable may not be relevant. The fundamental differential equation of consolidation including rebound, under an assumption of linear stress distribution, is represented as a function of excess pore water pressure,

\[
\frac{\partial u}{\partial t} = \begin{cases} C_v \frac{\partial^2 u}{\partial z^2} + \frac{\partial p}{\partial t} & \text{for } \frac{\partial \bar{p}}{\partial t} \geq 0 \quad \text{i.e. } \frac{\partial^2 u}{\partial z^2} \leq 0 \\ rC_v \frac{\partial u}{\partial t} > 0 \quad \text{for } \frac{\partial \bar{p}}{\partial t} < 0 \quad \text{i.e. } \frac{\partial u}{\partial z} > 0 \end{cases}
\]

The choice of the coefficients in braces depends on whether the effective stress, \( \bar{p} \), increases or decreases with time, and may be written as follows,
Under the loading condition shown in figure 2 \((t_1 \leq t \leq t_1 + t_2)\), the fundamental equation, the boundary conditions, and the initial condition are represented respectively as follows,

\[
\frac{\partial u}{\partial t} = \left\{ \frac{C_v}{rC_v} \right\} \frac{\partial^2 u}{\partial z^2} - \frac{1}{t_2} \left( \frac{p_3 + (p_4 - p_3) z}{2H} \right)
\]

(13)

\[
u(0, t) = 0
\]

\[
u(2H, t) = 0
\]

\[
u(z, 0) = \frac{1}{T_1} \left( \frac{p_1 + p_2}{2} F_1(T_1, z/2H) - \frac{p_2 - p_1}{2} F_2(T_1, z/2H) \right)
\]

(14)

Since the above equation cannot be solved analytically, a numerical method has to be used. Dividing the whole layer of clay into \(\alpha\) slices as shown in figure 3, equation (13) can be written in the form of a finite difference equation,

\[
\Delta u_i = \left\{ \frac{C_v}{rC_v} \right\} \frac{u_{i-1} + u_{i+1} - 2u_i}{\Delta z^2} - \frac{1}{t_2} \left( \frac{p_3 + (p_4 - p_3)}{\alpha} \right)
\]

(15)

The excess pore pressure at time \((t + \Delta t)\) can then be computed by the following equation, using the excess pore pressures at time \(t\),

\[
u_i^{t+1} = u_i^t + \left\{ \frac{\beta}{r} \right\} (u_{i-1}^t + u_{i+1}^t - 2u_i^t) - \frac{1}{t_2} \left( \frac{p_3 + (p_4 - p_3)}{\alpha} \right) \Delta t
\]

(16)

in which

\[
\beta = rC_v \Delta t/\Delta z^2
\]

(17)

\[
u_0^t = u_\alpha^t = 0
\]

(18)

\[
u_i^0 = \frac{1}{T_1} \left( \frac{p_1 + p_2}{2} F_1(T_1, i/\alpha) - \frac{p_2 - p_1}{2} F_2(T_1, i/\alpha) \right)
\]

In some cases it will be probable that the effective stress in some slices decreases at first and then increases. Such situations may be encountered when the rate of loading changes after a period of time, as shown in figure 2 \((t > t_1 + t_2)\). In this case equation (12) alone is not sufficient. Assuming that the same consolidation parameters are applicable to both rebound and recompression, as far as the effective pressure does not exceed the preconsolidation pressure, the choice of the coefficients in equation (16) may be extended approximately as,

\[
\beta/r \ldots \text{ for } F \leq 0 \text{ and } 3\bar{p}_i^t - \bar{p}_i^{t-1} \geq 2\bar{p}_{\text{max}}
\]

(19a)
\begin{align*}
\beta \quad \text{for } F \leq 0 \text{ and } \quad 3 \bar{p}_i^j - \bar{p}_i^{j-1} < 2 \bar{p}_{\text{max}} & \quad (19b) \\
\beta \quad \text{for } F > 0 & \quad (19c)
\end{align*}

in which
\begin{equation}
F = u_{i-1}^j + u_{i+1}^j - 2u_i^j
\end{equation}

\begin{equation}
\bar{p}_i^j = p_i^j - u_i^j = \frac{p_1 - p_3}{t_2} \frac{j \Delta t}{t_2} + \left( \frac{p_2 - p_1}{t_2} (p_4 - p_3) \right) t^j - u_i^j
\end{equation}

and \( \bar{p}_{\text{max}} \) is the maximum value of \( \bar{p}_i^j \) in \( t \leq t_1 + \Delta t (j - 1) \).

Applying the trapezoid formula, the settlement, \( S^j \), of the layer is represented as follows,
\begin{equation}
S^j = \frac{1}{2} (\varepsilon_0^j + \varepsilon_2^j) \Delta z + \sum_{i=1}^{a-1} \varepsilon_i^j \Delta z
\end{equation}

\begin{equation}
= m_v \Delta z \left[ \frac{p_1 + p_2}{2} - \frac{j \Delta t}{2r t_2} (p_3 + p_4) + \sum_{i=1}^{a-1} \left( \frac{p_i^j}{(1 - \frac{1}{r}) \bar{p}_{\text{max}} + \frac{\bar{p}_i^j}{r}} \right) \right]
\end{equation}

\textbf{FIGURE 3. Key sketch to numerical analysis}

The third term of the right hand side of equation (22) should be,
\begin{equation}
\sum_{i=1}^{a-1} \bar{p}_i^j \quad \cdots \text{for condition of equation (19a)}
\end{equation}

\begin{equation}
\sum_{i=1}^{a-1} \left[ \left( 1 - \frac{1}{r} \right) \bar{p}_{\text{max}} + \frac{\bar{p}_i^j}{r} \right] \quad \cdots \text{for condition of equations (19b) and (19c)}
\end{equation}

The degree of consolidation, defined in the same way as that for increasing load, is represented by the trapezoid formula as follows,
When the negative excess pore pressure is prevalent within the layer, the degree of consolidation may become more than 100 percent.

After the load has become constant as shown in figure 2 \((t > t_1 + t_2)\), the differential equation of consolidation including rebound is represented,

\[
\frac{\partial u}{\partial t} = \begin{cases} \frac{C_v}{r} \frac{\partial^2 u}{\partial z^2} \\ \frac{C_v}{r} \frac{\partial^2 u}{\partial z^2} \end{cases}
\]

And the fundamental equation for numerical analysis becomes,

\[
u_i^{j+1} = u_i^j + \left\{ \frac{\beta |r|}{\beta} \left( u_{i-1}^j + u_{i+1}^j - 2u_i^j \right) \right\}
\]

in which

\[
u_0^j = u_x^j = 0
\]

\[
u_i^{j+1} : \text{from equation (16)}
\]

\(i:\) from 1 to \((\alpha - 1)\);

\(j:\) from \((t_2/\Delta t)\) to infinity.

The effective pressure, the settlement and the degree of consolidation of the layer become,

\[
\bar{p}_i^j = p_1 - p_3 + (p_2 - p_1 - p_4 + p_3) \frac{1}{\alpha} - u_i^j
\]

\[
S_i^j = m_v \Delta z \left[ \frac{p_1 + p_2}{2} - \frac{p_3 + p_4}{2} + \sum_{i=1}^{\alpha-1} \left\{ \bar{p}_i^j \left( \frac{1 - \frac{1}{r}}{r} \right) \bar{p}_{max} + \frac{\bar{p}_i^j}{r} \right\} \right]
\]

\[
U_i^j = 1 - \frac{2}{\alpha(p_1 + p_2 - p_3 - p_4)} \sum_{i=1}^{\alpha-1} u_i^j
\]

The choice of the terms in \(\{\}\) in equations (25), (26) and (28) should be made after equations (19) and (23).

3. SOIL CONSTANTS AND LOADING CONDITIONS

In order to measure the water head of the aquifers and the compression of the clay layers, several observation wells were installed in the Yamanoshita area. The wells consist of an outer steel pipe with a filter tip through which water in the aquifer may enter freely, and of an inner pipe supported by the frictionless centralizer through the outer pipe and based on the aquifer. The water head of the aquifer was recorded automatically by measuring the water table in between both pipes, and the compression settlement down to the aquifer was automatically recorded by the movement of the inner pipe relative to the ground surface.
Figure 4. Observed change in underground water level

Niigata Earthquake

Voluntary Pumping Restriction
1st
2nd
3rd
Figure 4 shows the underground water head of each aquifer in meters below the sea level, which was supplemented with the record taken from the industrial wells for gas. These decreases in water head may lead to additional loads both for the clay layers and the sandy aquifers. An example of the settlement record by the well is shown in figure 8.

Figure 5. Simplified soil profile and loading condition for calculation in Yamanoshita
In connection with installing the observation wells several borings were carried out down to the depth of 1200 m, and undisturbed clay samples were taken with the tin-wall fixed-piston sampler from the clay layer above 500 m depth, as well as some disturbed samples from the layer above 1200 m depth. Consolidation tests, unconfined compression tests, and classification tests were performed on these samples.

The soil profile is shown in simplified form in figure 5. Except for the thin surface
layer, alternate silty-clay and gravelly-sand strata down to the depth of about 600 m are considered to be diluvial deposits.

A large capacity oedometer, with maximum load intensity of 200 kg/cm², was used to investigate the consolidation properties of the clay samples. Test results in figure 6 and 7 show that, in spite of extremely high overburden pressure, the coefficient of volume compressibility and the coefficient of consolidation are not so different from those of typical alluvial soils in Japan. The preconsolidation pressure determined by Casagrande's method was slightly larger than the estimated overburden pressure, but it was not so clear.

**Figure 7. Coefficient of volume compressibility and volume expansibility (Virgin compression)**
Rebound tests in which the pressure was released from the highest consolidation pressure gave the ratio of the coefficients of volume compressibility to expansibility to be 10 to 50 as shown in Figure 6.

\[ \frac{\text{Coefficient of volume compressibility}}{\text{Coefficient of volume expansibility}} = 10 \]
4. COMPARISON OF CALCULATED RESULTS WITH THE OBSERVED SETTLEMENT

For simplicity of analysis it is assumed that consolidation loads change linearly with time as shown in figure 4, and that they are applied to each layer as shown in figure 5. After a number of trial calculations, the soil constants considered to be relevant to the analysis are as follows,

\[ m_r (\text{clay}) = 8 \times 10^{-3} \text{cm}^2/\text{kg} \]
\[ m_r (\text{sand}) = 4 \times 10^{-4} \text{cm}^2/\text{kg} \]
\[ m_r (\text{clay}) = 1.6 \times 10^{-4} \text{cm}^2/\text{kg} \]
\[ m_r (\text{sand}) = 0 \]
\[ C_r (\text{clay}) = 0.1 \text{ cm}^2/\text{min} \text{ for increasing load} \]
\[ C_r (\text{sand}) = 0.2 \text{ cm}^2/\text{min} \text{ for decreasing load} \]
\[ C_r (\text{clay}) = 0 \]
\[ C_r (\text{sand}) = 10 \text{ cm}^2/\text{min} \]
\[ \text{Overconsolidation pressure} = 1 \text{ kg/cm}^2 \]

Three kinds of settlement records are available, as shown in figure 8. The record of leveling survey will be most reliable, but there is no detailed record until 1957. Harmonic analysis of the tide level gives a smooth curve of relative settlement, but it may be somewhat overestimated in comparison with that obtained by the leveling. The rise in the inner pipe founded at 1200 m shows the least settlement, which may be to some extent due to the friction between the pipe and the soil. Therefore, none of these three records is satisfactorily reliable. Moreover, a considerable earth crust movement by the Niigata Earthquake complicated the settlement record.

The calculated settlement is shown in figure 8. As compared with the observed settlement record, the rate of calculated settlement is found to be greater in the period until 1956, and smaller in the subsequent years. This difference in the rate of settlement may be partly due to an ambiguity in assessing the loading condition in the early period. (See fig. 4) However, the calculated result compares favourably with the observed settlement, as a whole, and may be used for estimating future settlement with satisfactory accuracy.

5. CONCLUSION

A newly developed method for analysis of consolidation under time-dependent loading was applied to the analysis of land subsidence in the Niigata area. Although the soil constants, the loading conditions, and even the observed settlement were not fully reliable, the calculated result compared favourably with the observed settlement as a whole. The present method may be applicable to the analysis of consolidation in which the applied load is partly removed before the consolidation is completed, e.g., in the pre-loading method of a road embankment.

REFERENCES


