ANALYTICAL METHODS FOR PREDICTING SUBSIDENCE

Keshavan NAIR
Woodward-Clyde & Associates Oakland, California, U.S.A.

ABSTRACT
The paper discusses the need for predictive methods in subsidence problems. The importance of correctly formulating the problem is emphasized. The various steps in formulating a problem are discussed. The finite element technique is considered to be the most useful technique for application to subsidence problems. The results of the analysis of an example problem using this technique are presented.

INTRODUCTION
Subsidence of the ground surface can occur for a variety of reasons. Removal of fluids (e.g. oil, water) from the ground and the creation of underground openings can cause the ground surface to subside. This paper deals solely with the problem of subsidence which is a consequence of the creation of underground openings.

These underground openings are constructed during the course of mining operations using conventional or solution techniques. In the U.S.A., until recently, the great majority of the mined areas were located away from the population centers. However, the expansion in the populated areas, together with the limited number of suitable mining sites, has resulted in the location of underground openings under buildings and other structures. Consequently, a great deal more attention is being given to the prediction and control of subsidence above underground openings.

There are two general approaches to the problem of predicting subsidence, (a) experimental (b) analytical. The major disadvantage of the purely observational (experimental) approach is that it is not applicable to conditions outside the range of experimentation. A typical example of this approach has been developed in the U.K. [1]. The major advantage of the analytical methods is that they are predictive. However, it should be recognized that all analytical methods are based on a number of assumptions. These assumptions must be evaluated within the context of the real problem. Analytical methods based on the principles of solid mechanics and placed within the framework of a good understanding of the physical nature of the problem have proved to be the most useful means for developing predictive techniques for engineering problems. The prediction of subsidence using such methods requires the formulation and solution of appropriate boundary value problems. Solution techniques are developed in general terms by workers in solid and structural mechanics. The formulation of the problem and the application of these solution techniques fall within the scope of the engineers directly concerned with the subsidence problem.
FORMULATION OF PROBLEM AND METHODS OF SOLUTION

In order to formulate a boundary value problem for analysis it is necessary to go through
the following steps:
(1) define the geometry of the problem;
(2) define the materials by suitable constitutive equations;
(3) define the loads, and
(4) define the boundary conditions.

The major variables that have to be included in formulating a problem for investigating
subsidence are:
(I) location of the underground opening;
(II) shape of the opening;
(III) the rock profile and geologic conditions;
(IV) the properties of the rock mass surrounding the opening, and
(V) the initial (insitu) state of stress in the rock.

In general it is necessary to use various idealizing assumptions to reduce the real
problem to an idealized boundary value problem which is amenable to solution. The
more powerful the solution techniques the fewer the assumptions that have to be made in
the formulation of the problem. There are no solution techniques of sufficient versatility
which can solve boundary value problems formulated to include all the above listed
variables in complete generality. There are two general techniques for obtaining solutions
to pertinent boundary value problems for investigating subsidence: (1) classical methods
which often result in closed form solutions and (2) numerical methods. The majority
of the previous work on subsidence [2, 3, 4, 5] has utilized results obtained by using
classical mathematical methods for obtaining solutions to boundary value problems in
linear elasticity [6, 7, 8, 9]. The inability of these existing solutions to predict subsidence
has been pointed out by Voight and Pariseau [10].

The most powerful of the numerical techniques is the finite element method which
is a modern computer oriented approach to the analysis of continuous structures. The
theoretical basis and the utilization of the method for a wide class of boundary value
problems has been discussed extensively in the literature [11, 12, 13]. Because of its
versatility, the method is ideally suited for problems in rock mechanics [14, 15, 16].
However, the application of the technique to the solution of subsidence problems has
been very limited. This section discusses the steps in formulating the problem, the
major variables that have to be included in the formulation, and the capability of the
finite element method to account for these variables.

PROBLEM GEOMETRY

The location of the cavity, its shape, and the rock profile are factors which have to be
considered in defining the geometry of the problem. While there is generally little ambiguity
with regard to the location of the opening it is often necessary to make various assumptions
with regard to shape. All real shapes are three dimensional; existing methods of analysis
can only account for plane, and axisymmetric three dimensional shapes. Many under-
ground openings are in bedded deposits. From the standpoint of analysis, this implies
that the material around the opening is non-homogeneous. Another factor to be conside-
ed is the continuity between layers in bedded deposits. In addition to bedding planes the
occurrence of faults and joints can be very influential in causing subsidence. The mapping
of these and their representation is an important aspect in formulating the problem. With
the finite element method, any location of the cavity can be included in the problem. Arbi-
tary plane or axisymmetric shapes can be investigated. With regard to rock profile, in the
plane analyses any system of bedding can be analyzed, however, in the axisymmetric
analyses only horizontal bedding can be investigated. The ability to analyse layered deposits is a very significant advance. In addition, there are various techniques which can be used to analyse joints and conditions of continuity between layers of rock [16, 17].

**Material Properties**

In subsidence problems the properties of the rock mass surrounding the opening have to be defined. It is necessary to select constitutive equations to represent the various materials. The numerical magnitude of various parameters which are necessary to quantify the selected constitutive equations have to be determined. It is well known that rocks in general are not isotropic and because of their different behavior in tension and compression are non linear. In addition, yielding (e.g. elasto-plastic) and time dependent (creep) behavior may also be present. The present knowledge on constitutive equations and the ability to solve boundary value problems does not permit us to completely model the complex behavior of rock. Therefore, it becomes necessary to select more than one constitutive equation to represent the materials in order to obtain bounds on the subsidence that might occur. It is important to recognize that the results of laboratory tests cannot be assumed to represent the properties of the rock mass. Various correlative techniques to relate laboratory values with rock mass properties are available [18]. Approximate methods for including nonlinear, bilinear, elasto plastic, and time dependent material properties are currently available [13]. Furthermore, methods to include progressive failure in the rock mass which could lead to additional subsidence have also been developed [19].

**Definition of Loads**

The existing equilibrium state of stress often termed the “initial” or “insitu” stress is disturbed by the creation of an opening. In addition, the weight of the material (i.e., gravity load) is acting to close the opening. The initial state of stress may be entirely due to the weight of the material or in some cases stresses may exist in the rock due to tectonic history. The initial state of stress is usually defined by the vertical stress and the ratio \( k \) of the horizontal to vertical stress; for subsurface problems these are the principal stresses. In order to compute the gravity loads it is necessary to know the unit weight of the overlying materials. The inclusion of gravity loads in the analysis is a routine matter with the finite element method technique. Arbitrary initial stresses can be easily introduced by varying the pressure applied to the external boundaries. In addition, any pressure applied to the internal face of the cavity can be readily included in the analysis.

**Boundary Conditions**

There are two boundaries in the problem, the ground surface and the interior face of the opening. Both these surface are stress free and have no restrictions on their displacement. In some cases it might be required to determine the surface subsidence due to a certain percentage of closure in the opening. Under these circumstances displacement boundary conditions have to be imposed on the interior surface of the cavity. In the real physical problem (i.e., a half space) the vertical boundaries and the lower horizontal boundary are at an infinite distance in horizontal direction and vertical direction respectively. For purposes of making the problem amenable to solution, it is often necessary to locate these boundaries at a finite distance from the opening. These boundaries should be located at a sufficient distance away from the opening so that their influence on the conditions at the opening is negligible and vice versa. Therefore, the conditions at these boundaries can be assumed to be those that existed prior to the creation of the opening. All the boundary conditions required by the problem can be
Analytical methods for predicting subsidence

easily accommodated. Conditions of no horizontal displacement or any practical form of pressure distribution for representing the initial stress state can be applied to the vertical boundaries. Displacement boundary conditions at the lower horizontal boundary and at the interior face of the cavity can also be incorporated into the analyses. Since the finite element technique is developed for use with a digital computer, a range of conditions e.g. material properties and initial stress states can be easily investigated. This is of considerable significance in subsidence problems where because of the uncertainty about geologic factors it is prudent to investigate various possible conditions.

It can also be seen from the above discussion that the finite element technique has capabilities of solving realistic boundary value problems as they apply to the prediction of subsidence, and provides greater capacities for analyses than were heretofore available. Its use in analysing subsidence problems should increase greatly in the next few years.

EXAMPLE PROBLEM

The engineer evaluating the possibility of subsidence is cast in the role of a user of the finite element method. Digital computer programs based on the finite element method are available for solving appropriate boundary value problems. It is the use of these programs that the practicing engineer is concerned with. Using an existing operational program can be accomplished by following certain set instructions and requires a minimum understanding of the method. However, new and improved programs are always being developed and the engineer should try to keep informed of these developments.

The application of the method is best illustrated through the use of an example. The example chosen is relatively simple but it does serve to illustrate some of the capabilities of the method. An axisymmetric analysis is chosen for this example, as a plane analysis has been published in the literature [20].

The steps followed in formulating the problem were those discussed earlier in this paper. As applied to this specific case they are as follows.

PROBLEM GEOMETRY

The opening under consideration was 25 feet high and was assumed of circular cross section with a diameter 1272 feet located at a depth of 4225 feet below the surface. The rock profile consisted of surface sand and gravel overlying approximately 2000 feet of interspersed layers of sandstone, shale and siltstone. Underlying this there is limestone, dolomite, anhydrite and salt, below the salt is limestone. The opening is in the salt. Material properties had to be obtained from three dimensional velocity logs. The rock profile is idealized into seven layers as shown in figure 1.

MATERIAL PROPERTIES

It was assumed for purposes of this analysis that the materials surrounding the cavity would be treated as linear, isotropic and elastic. Material properties were obtained from three dimensional velocity logs. Conservative estimates of the modulus of elasticity (E) were utilized and are shown in figure 1. For illustrative purposes it may be assumed that the geophysical data indicated that there might be a weak zone in the rock in layers II and IV. The worst condition could be that layers II and IV were composed entirely of this weak material.

LOADS

An average unit weight of 144 pcf was used in the analysis. The initial state of stress was assumed to be hydrostatic.
**BOUNDARY CONDITIONS**

The surface and interior of the cavity was assumed stress free. The displacement boundary conditions on the other boundaries are shown on figure 1. The boundaries are at a sufficient distance from the cavity so that their influence on the results of the analysis are not significant.

The problem having been formulated it is necessary to construct a finite element mesh. The mesh should be constructed so that the boundaries between different materials are also boundaries of various elements.

The data which includes the problem geometry material properties, loads, boundary conditions and the finite elements mesh are input for the computer program. The output consists of the stress and displacement field in the rock mass. The surface displacement is of course the subsidence. In determining the subsidence due to the creation of a cavity it is important to exclude the displacement of the rock mass due to gravity loads and initial stress prior to the creation of cavity.

The results of these analyses in the form of surface subsidence profiles is shown in figure 2. These results indicate the ability to analyse a bedded deposit using the finite element technique. The results indicate that the weak layers as located in this problem have little effect on the surface subsidence but do cause an increase in the surface displacement of the openings. Discussion of this behavior and the possibility of rock failure...
resulting in greater subsidence is outside the scope of this paper. It also shows how uncertainties in the rock profile can be investigated by analysing a number of cases. It is difficult to establish trends as the results are greatly influenced by the particular geologic conditions. However, the ability of the finite element technique to account for these conditions is a significant advantage.
FINAL REMARKS

It is evident that using the finite element technique permits the solution of boundary value problems which are realistically representative conditions under which subsidence is likely to occur. It is also possible to investigate a range of conditions with relatively little effort. Care must be exercised in formulating the problem. Comparison of predicted and observed phenomena is the only way in which confidence can be developed in the formulation of the problem and in the methods of solution. Analysing phenomena which has already occurred and obtaining good agreement is always suspect.

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REFERENCES

AN EXAMPLE OF GROUND SUBSIDENCE ESTIMATION

Masami FUKUOKA
Director, Public Works Research Institute, Ministry of Construction

ABSTRACT

The Kōtō delta area of the Tokyo Metropolis is a low alluvial land of triangle shape, extending for about 5 km from east to west, about 10 km from south to north, with an area of approximately 40 km², bordered by the River Sumida on the west, and by the Arakawa canal on the east. The external embankment was planned for the purpose of defending this area from the calamity of floodtide. Before carrying out this undertaking, the estimation of future sinking has been found necessary.

The group involved in the investigation including the author, having been engaged in the undertaking, found that the data serviceable for the estimation were not sufficient. Accordingly, the sinking amount for about 20 years after 1955 was estimated by the calculation on a very bold assumption, based upon technical decision. About 15 years having elapsed thereafter, it became possible to compare facts with estimation, which was proved to be satisfactory on the whole. In the present paper, explanation has been made of the method of estimating the ground subsidence, taking the Minami-Sunamachi, of the Kōtō district, as an example.

RÉSUMÉ

La zone détaillé de Kōtō à Tokyo est une région basse de forme triangulaire d'alluvions, dont la surface couvre environ 40 kilomètres carrés, s'étendant sur environ 5 kilomètres de l'est à l'ouest, et environ 10 kilomètres du sud au nord. Cette surface se situe entre le Fleuve Sumida à l'ouest et le Canal de fuite d'Arakawa à l'est.

Une digue d'enceinte a été prévue pour que la zone puisse être préservée du désastre des grandes marées. Avant que cette entreprise ait démarré, il fallait prévoir le volume futur de tassement. Le groupe de recherche dont faisait partie l'auteur a constaté que les données qui servaient à l'établissement des pronostics n'étaient pas suffisantes.

Par conséquent, des pronostics ont été faits en se basant sur une hypothèse ancienne basée elle-même sur un jugement technologique, et avec une hypothèse bien audacieuse, le volume d'affaissement de 20 ans après 1955 a été calculé. Environ 15 ans ayant passé depuis, nous avons pu comparer le pronostic avec la réalité, et nous avons trouvé que la comparaison est à peu près satisfaisante.