Hydraulic calculation of water discharge and levels in river deltas

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SUMMARY: The method of hydraulic calculation of channel discharge and water levels in river deltas is discussed. The method is a further development of iteration techniques. It amounts to the determination of discharge in anabranches and water levels in the junction nodes. The method does not require the exact assumption of the flow direction in the anabranches. It allows an analytical calculation of the corrections to the stage in the junction nodes and it converges the solution for delta forks of any complexity. The calculation is readily automated.

On the basis of the discussed method, a programme of hydraulic calculation of complex braiding of river channels was worked out for an electronic computer of the Ural 2 type in the Arctic and Antarctic Research Institute, Leningrad. The programme provides for any number of anabranches of the delta with the water flow directed in and out of the channel braiding, with the given stage. The relationship between channel discharge and water level in the zone of tidal and surge-back water was investigated by hydraulic theory and electronic computer. Bearing in mind the peculiarities of water regimes in deltas, the requirements for the original data in the hydraulic calculation were considered. A numerical check of the method was carried out on the deltas of the Ob River and the Kolyma. This check revealed the actual possibilities of using this method to calculate water regime elements in real deltas.

The methods of river hydraulics for the calculation of discharge through the delta anabranches and the gradient of the water surface are complicated by their time-consuming nature, partly due to the inadequate development of the technique for different delta conditions. Additional difficulties arise during the preparation of the initial data for calculation, due to surge and tide effects on these data.

With the electronic computer, the time consuming nature of the calculations is no longer an obstacle, but the methods should need to be adapted for calculation by computer. The initial data for the calculation can be obtained in periods when water flow
in the delta is approximately constant or when the natural flow regime is reduced to stationarity by means of filtering techniques or by averaging water level variations for a specific time interval. The discharge characteristics at the outlet should be nearly constant. This period for the most large river deltas equals to 2-5 days, while the strongest surge and tide variations at the sea margin of a delta have the period of 2 days [6].

The peculiarities of hydraulic techniques for the calculation of discharge through river deltas arise from forks in the delta, low surface gradient of the flow, comparatively small velocities and slow changes of anabranch beds.

For the initial information the system of equations for steady nonuniform flow is taken.

\[ I = \frac{dZ}{dS} = \frac{\nu^2}{C^2 h} + (1 + \phi) \frac{d}{dS} \left( \frac{\alpha \nu^2}{2g} \right) \]  

\[ \frac{dQ}{dS} = 0 \]  

where:

- \( Z \) height of the available surface of the flow (level marks);
- \( S \) transverse curvilinear ordinate (distance counted along the geometric axis of the river bed);
- \( V \) mean velocity of the flow;
- \( C \) bed roughness coefficient;
- \( h \) mean cross-sectional depth of the flow;
- \( \alpha \) correction to mean velocity;
- \( g \) acceleration due to gravity;
- \( Q \) discharge.

The work amounts to the determination of water level (surface water level) in the nodes of delta forks where a knowledge of the discharge through the anabranches is required.

In calculation of this kind [2, 5, 7], the following assumptions are made: lengths of the sounding places are equal to those of the anabranches, water discharge in the anabranches between the adjacent nodes is constant, differences of kinetic energy of the flow in the first and last cross-sections at the junctions of the anabranches are negligible.

On the basis of the above assumptions the water flow in any single anabranch of the delta fork will be determined by two out of the three following quantities: 2 level marks in the last sections and the water discharge value through the anabranch. If we know two of these quantities, the third may be determined by the integration of the differential equation of nonuniform water flow. In our particular case the equation describing the steady water flow in a separate anabranch can be written as

\[ Q_i = \frac{K_i}{\sqrt{S_i}} \sqrt{Z_{k_1i} - Z_{k_2i}} = \sqrt{\frac{Z_{k_1i} - Z_{k_2i}}{F_i}} \]  

where:

- \( Q_i \) water discharge amount in the \( i \)-th anabranch;
- \( K \) modulus of discharge;
- \( i \) ordinal number of the anabranch;
- \( k \) ordinal number of the delta fork node;
- \( k_{2i} \) index of the node conjugated with the \( k \)-th node by means of the \( i \)-th anabranch;
- \( F_i \) modulus of bed resistance;

\( F_i \) modulus of bed resistance is supposed to be simply determined by an average
depth of the water surface in the anabranch, i.e.

\[ F_i = F_i \left( \frac{Z_{k1} + Z_{k2}}{2} \right) \]  

(4)

and it is calculated according to known hydraulic techniques.

The problem of water flow determination in a delta with a large number of anabranches can be solved provided the water level marks in all the nodes of the delta fork are known at a given sea level as well as the amount of water flowing in the delta forks. In this case the equation system can be expressed by all the relations of water transport balance, flowing in and out through each of the delta nodes [5, 7].

\[ \sum_{j} Q_{i,k} - \sum_{i} Q_{i2,k} = 0 \quad k = 1, 2, ..., N-n \]  

(5)

where:

- \( l_k \) the number of anabranches which transport the water to the \( k \)-th node of the fork;
- \( m_k \) the number of anabranches, transporting water away from the \( k \)-th node;
- \( i_{1,k} \) anabranches transporting water to the \( k \)-th node;
- \( i_{2,k} \) anabranches transporting water away from the \( k \)-th node;
- \( N \) total number of fork nodes;
- \( n \) the number of nodes with fixed marks of water level.

The number of equations in the system, being equal to the number of nodes, usually corresponds to the number of unknown quantities, the water transport in separate anabranches can be estimated according to equation (4).

A number of methods were offered [2, 5, 7, 8] for the solution of this system of equations, their detailed review is given by S.S. Baydin [1, 2]. A more general approach is given by the graphical analysis Voinovich method [2] and by iteration methods of A.I. Mordukhay-Bolotovskiy [8], V.M. Makkaveyev [7] and K.V. Grishanin [5]. These methods were developed and compared with manual calculations. Iteration methods, particularly interesting to the present study may be subdivided into two groups. The first one includes Mordukhay-Bolotovskiy’s method [8], which implies exact distribution of discharge through anabranches in accordance with (eq. 5) and then corrections to the water marks are determined. This method in principle is similar to that used for the calculation on water supply pipes, but due to its complexity is not applied to the natural river beds. The second group includes methods of V.M. Makkaveyev and K.V. Grishanin in which height of the water surface at bed fork nodes are given and the corrections to discharge calculations are found. This approach to the system determination, suggested by Makkaveyev, suits the computer calculation better, and it was further developed. K.V. Grishanin suggests the use of relaxation method, which differs from the iteration method in that it involves a simpler calculation scheme.

When solving problems of this kind manually, the correction to height of the water surface at the fork nodes is assumed, and it may give rise to the largest error in the discharge modulus. With this method the final result is dependent on the professional skill and experience of a calculator.

The author suggests another calculation method, which is a further development of the second group of methods (Makkaveyev and Grishanin) but it differs from them in being a more general and objective calculation scheme, suited to both computer and manual calculation:

In accordance with above assumptions the length between sounding places should be equal to the length of the anabranch. If the general length of the anabranch is divided into
several sections, then at the boundaries of these sections additional nodes are introduced into the calculation scheme. The differences of the kinetic energies in the first and last cross-sections are negligible. The discharge values in the anabranches between the adjacent nodes are taken to be constant. On the basis of the above made assumptions and using the equation of movement as

$$O_i = \sqrt{|Z_k - Z_{k+1}|}$$  \hspace{1cm} (6)

$$F_i = P(Z_i) = \sum_{j=0}^{m} \alpha_j Z_i$$

resistance modulus of the $i$-th anabranch $Z_i$—mean height of the water in the $i$-th anabranch, for a complex fork with $N-n$ nodes (where $N$ is a general number of bed fork nodes, $n$—a number of bed fork nodes with the fixed marks of water level), we have a non-linear $N-n$ equation system with $N-n$ unknown values

$$\sum_j^I Q_{jk} + \sum_i^II \text{sign}(Z_k - Z_{k+1}) \sqrt{|Z_k - Z_{k+1}|} = 0$$  \hspace{1cm} (7)

$\sum_j^I$ means that summation is accomplished according to the $j$-numbers which correspond to the anabranches adjacent to the $k$-th node and contributing water to the delta fork.

$\sum_i^II$ means that summation is accomplished according to the $i$-numbers, which correspond to the inner anabranches, adjacent to the $k$-th node.

sign $(Z_k - Z_{k+1})$ defines the direction of the flow, that is inflow and outflow of the water through the $i$-th anabranch for the $k$-th node which in turn allows one to proceed without assuming flow direction in the anabranches. It should be noted here that $Z_{k+1}$ in other equations of the system is given as $Z_k$. If $Q_{jk}$ and water level marks $Z_k$ in the water basin are given, equation 7 is non-uniform and yields at least one solution ($N-n$ values of $Z_k$). The solution obtained permits the calculation by means of (eq. 6) of water transport through inner anabranches of the fork. The relaxation method is accepted for the solution of equation 7. According to this method the largest modulus error becomes zero, due to the change of the corresponding component at every step. The calculation is considered finished when all the errors of the last transformed system are zero with the given accuracy. In this case an error in water discharge $Q_i$ in the $y$-th node (a node with the largest error) is transformed to zero, correcting the mark by $\delta Z_{y}$, which in the general case is calculated analytically. To solve the equation system (7) the heights of the surface of the flow in the fork nodes are assumed to be close to zero and at least some of these heights should be not equal each other. Then substituting $Z^{(0)}_{k}$ in the (eq. 7) we obtain $N-n$ errors of $q^{(0)}_{y}$ discharge.

Then supposing that a certain step $\mu = 0, 1, 2\ldots$ the equation system 7 is not satisfied i.e.

$$\sum_j^I O_{jk} + \sum_i^II \text{sign}(Z_k^{(\mu)} - Z_{k+1}^{(\mu)}) \sqrt{|Z_{k}^{(\mu)} - Z_{k+1}^{(\mu)}|} = q_k^{(\mu)}$$  \hspace{1cm} (8)

The largest modulus error is chosen from the obtained values of $q_k^{(\mu)}$

$$|q_k^{(\mu)}| = \max |q_k^{(\mu)}|$$  \hspace{1cm} (9)

Considering that $q_k \neq 0$ due to the fact that the height of the water surface $Z_{y}^{(\mu)}$ at the $y$-th node has a $\delta Z_{y}^{(\mu+1)}$ error.
Calculating the discharge of an intricate river bed, two cases may arise:

(a) when in all the anabranches, adjacent to the v-th node

\[ Q_v \neq 0, \text{i.e. } \Delta Z_v = \Delta Z - Z_{v+1} \neq 0 \]

(b) when, excepting the above case (a), the anabranches, adjacent to the node in the \( \mu \)-th approximation may yield \( Q_v = 0 \), i.e. \( \Delta Z_v = 0 \). Considering these cases and assuming that \( F_i \) with the level correction by \( \delta Z \) does not change, we obtain for the v-th node

\[
\sum_j Q_{iv} + \sum_i \text{sign} (Z_v^{(\mu)} - Z_{v+1}^{(\mu)}) \sqrt{Q_{iv}^{(\mu)^2} + \frac{\left| \delta Z_v^{(\mu+1)} \right|}{F_{iv}^{(\mu)}}} + \sum_i \frac{\delta Z_v^{(\mu+1)}}{F_{iv}^{(\mu)}} = 0 \quad (10)
\]

where:

- \( \sum_{i}^{\text{III discharge sum, corresponding to case (a);}} \)
- \( \sum_{i}^{\text{IV additional term, corresponding case (b).}} \)

Equation 10 gives a square equation

\[
\frac{1}{2} \left| \delta Z_v^{(\mu+1)} \right| A_v^{(\mu)} + \sqrt{\left| \delta Z_v^{(\mu+1)} \right|} B_v^{(\mu)} - |q_v^{(\mu)}| = 0 \quad (11)
\]

where:

\[
A_v^{(\mu)} = \sum_i^{\text{III}} \frac{1}{Q_{iv}^{(\mu)} F_{iv}^{(\mu)}}, \quad B_v^{(\mu)} = \sum_i^{(\mu)} \frac{1}{\sqrt{F_{iv}^{(\mu)}}}
\]

The we are solving equation 11 relative to \( \sqrt{(\delta Z_v^{(\mu+1)})} \). Considering only positive squares, provided that the errors have the sign, which corresponds to the sign of \( q_v \), we finally obtain the formula of the correction of \( \delta Z_v^{(\mu+1)} \)

\[
\delta Z_v^{(\mu+1)} = \text{sign} q_v^{(\mu)} \left[ \frac{-B_v^{(\mu)} + \sqrt{B_v^{(\mu)^2} + 2 A_v^{(\mu)} |q_v^{(\mu)}|}}{A_v^{(\mu)}} \right]^2 \quad (12)
\]

where \( \delta Z_v^{(\mu+1)} \) is the correction to the water surface heights in the node with the largest discharge error.

In the case of simple fork calculations (without transverse arms), when, in all the anabranches adjacent to the v-th node, (the condition \( Q_v \neq 0 \) holds) the formula \( \delta Z_v \) correction is simpler.

\[
\delta Z_v^{(\mu+1)} = \text{sign} q_v^{(\mu)} \frac{2 |q_v^{(\mu)}|}{A_v^{(\mu)}} \quad (13)
\]

Calculating the correction to \( \delta Z_v^{(\mu+1)} \) either by (12) or by (13), we correct the available surface height in the v-th node. Then all the discharge values for all the anabranches which come together in the v-th node are recalculated by means of (eq. 6). The calculations are repeated in the same manner as for \( \mu \)-th approximation, until all the errors of the last transformed system are less than the given accuracy for the calculation. Thus, solving equation 7 with any given water discharge values and any sea levels we obtain \( Z_k \), the height of the surface of the flow. Simultaneously, with the same initial data, we obtain discharge value in delta anabranches. The method does not require any definite direction of the flow and the solution implies any number of anabranches with water inflow and water outflow, with fixed marks of available surface. On the basis of this method a program of hydraulic calculations of water regime elements in deltas was worked out for the Ural 2 Computer.
A numerical check on the calculations using data from the Kolyma Delta verified the practical use of this method.

Hydraulic calculations make it possible to decrease the volume of hydrological observations required for the comprehensive study of water regime in a delta, and, what is more important, they permit regime changes due to hydrotechnical constructions and natural water-way improvements to be predicted.

However, a certain minimum of initial data is required. Long term data variations of water discharge and water level between the delta head and at the sea edge in a single
system are of prime importance. Also of importance are data showing relationships be­tween discharge and water level of the delta head. If the sea effect spreads beyond the delta head, long-term characteristics of level variations and water discharge in the channel control the sea effect are required. With such data available the boundary conditions of the calculations can be established.

The above mentioned base station and control section data should be supplemented by hydrometrical and hydrographic data which are necessary to calculate bed resistance modulus. The data pattern and methods of observations are dependent on the method of calculation. In one of these methods $F_i$ is found from data on channel cross-sections and roughness parameters. According to another (hydrometrical) method developed by A.N. Rakhmanov [9] this value is found on the basis of hydrometrical data only:

$$ F_i = \frac{Z_{k+1} - Z_{k-1}}{Q_i^2} \quad (18) $$

Since the data coverage and the network of level control in the anabranches are not everywhere sufficient, both methods should be employed for resistance modulus calculations. Preference should be given to Rakhmanov's method and the other one may be used for secondary anabranches of the inner delta, with poor hydrometrical data coverage. For river deltas with generally inadequate data coverage the second method is preferable. The method of aerodynamic modelling, widely used in water regime studies [4, 3 and others] can be applied to the river deltas which have not yet been sufficiently studied while they have been hydrographically surveyed to obtain the $F_i = F_i(Z_i)$ relations in the anabranches. A single run of the model gives heights of available surface of the flow and water discharge through the anabranches as well as the other data. This permits (eq. 6) to be used to calculate $F_i$ values for all the anabranches. Making measurements on the model during several runs for every regime, $F_i$ can be obtained and then the $F_i = F_i(Z_i)$ relations for every anabranch can be developed. Satisfactory agreement between the field and model data [4, 3 and others] confirm the reliability of modulus resistance values calculated by model measurements.

It should be noted, that model data can be used for an approximate calculation of roughness coefficients in deltas, with poor data coverage.

Summing up, it can be noted that the present paper suggest methods of hydraulic calculation of water discharge and levels in deltas which give an opportunity:

1. To decrease the number of hydrometrical observations needed for sufficiently complete delineation of water regime in intricate river deltas
2. To obtain spatial characteristics of the surface of the flow and discharge distribution through delta anabranches for any given values both of water discharge in the river and sea level, as well as any constructional requirements.
3. To predict changes of the surface of the flow and water discharge distribution through the anabranches during construction works for waterway improvement and to predict the effect of these works.
4. To calculate discharge and the relation between sea level and the level in the delta areas at any moment.
5. To calculate water discharge values in the anabranches at any moment, that is to calculate water discharge through delta anabranches which are situated in the zone of tide and surge influences.

The present method should also be used for discharge and level calculations in deltas in winter periods, when rivers are covered with an ice layer and during steady wind-induced water level changes.
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Considérations théoriques sur la dispersion d'un courant liquide de densité réduite et à niveau libre, dans un bassin contenant un liquide d'une plus grande densité

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ABSTRACT: Several characteristics concerning the way the river waters contact the sea waters in situ (at the Sulina mouth) are presented in the paper.

On a mathematical hydraulic model which presents schematically the contact of two non-mixable liquids with different density it is developed the theory of the river jet depth and width variation in the sea.

The relations obtained are expressed either by the equations (17) and (24) or by the equations (29 and 30).

The theoretical results approach closely those given by observations and measurement in situ.

RÉSUMÉ : On présente quelques caractéristiques du processus in situ (à l'embouchure du canal de Sulina) du contact des eaux fluviales et marines.

Sur un modèle hydraulique on a élaboré le schéma du contact de deux liquides non miscibles et à densités différentes, et on a développé la théorie de la variation de l'épaisseur et de la largeur dans la mer du jet fluvial.

Finalement on obtient les fonctions respectives exprimées soit par les équations (17) et (24), soit par (29 et 30).

Les résultats obtenus théoriquement sont très proches des résultats issus d'observations et mesures in situ.