is satisfactorily described by the equation derived from the equation of mixing waters applied to temperature and the water discharge.

So, the variation of all main physical and chemical characteristics at the mouth depends on the vertical velocity gradient in the river and on thermal and other mixing processes. Theoretical and empirical relations permit approximate qualitative estimation of variations in properties from the variation in the most easily observed characteristic. At the same time, during natural and artificial variations of river discharge, one can estimate the variations in the main characteristics of the area of mixing at the mouth, from a knowledge of the mean velocity at the outlet and salinity of the sea water.

Water quality modeling of estuaries

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SUMMARY: A set of mathematical models which simulate the hydrodynamic and water quality behavior of a variety of estuaries is presented. The general approach consists of discretizing both embayments and delta areas as a network of elements for which numerical solutions to the equations of continuity, motion, mass transport and dispersion are obtained.

Three separate models are discussed:

1. the Tidal Hydrodynamics Model, which produces the time-varying flows and heads throughout the estuary as it responds to tidal action, unsteady inflows, evaporation, and wind.
2. the Dynamic Water Quality Model, which simulates the distribution and movement of pollutants, both conservative and nonconservative, as they vary tidally and on a long term basis;
3. the Steady State Water Quality Model, a simplified model which uses the dispersion concept to reduce the dynamic tidal problem to one steady state and mean values of flow and head.

Specific applications to the San Francisco Bay-Delta are shown. Particular emphasis is given to the control of sea-water intrusion in the Delta and to assimilation of wastes in that region. Brief treatment is given to studies of two other estuaries.

RÉSUMÉ : Un ensemble de modèles mathématiques simulant le comportement hydrodynamique et la qualité des eaux de divers estuaires est présenté. La méthode générale consiste à discrétiser à la fois haies et delta en un réseau d'éléments pour lesquels la solution numérique des équations de continuité, de l'impulsion, des échanges de masses et de dispersion peut être obtenue.

Trois modèles différents sont discutés :

1. le modèle hydrodynamique des marées, qui calcule le débit et la charge dans tout l'estuaire suivant l'action de la marée, des apports d'eau non stationnaires, de l'évaporation et du vent.

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INTRODUCTION

One of the major estuaries in the United States is San Francisco Bay and its tributary waters. The Bay itself has a surface area of over 400 square miles and varies in depth from 400 feet at the Golden Gate, the entry to the ocean, to only a few feet in the extensive tide flat areas. The primary sources of fresh water inflow to San Francisco Bay are the Sacramento River and San Joaquin River which enter the Bay through Carquinez Straits. Upstream from the Straits, these two rivers converge in one of the most complex delta areas in the world. More than 700 miles of interconnected tidally affected channels comprise the Sacramento-San Joaquin Delta. Roughly triangular in shape, it extends from Carquinez Straits on the west to the city of Sacramento on the north to Stockton on the south. The predominant use of the Delta region is agriculture. The Delta islands, many of which are below sea level, are composed of a rich peat soil which is excellent for a number of crops. A map of the Bay-Delta region is shown as background in figure 1.

Irrigation water is diverted from the channels throughout the Delta region. The high quality required by agriculture is in conflict with a number of pollution sources. These include:

1. Salinity intrusion due to tidal action;
2. Municipal and industrial waste inflows;
3. Agricultural drainage wastes from the San Joaquin Valley;
4. Agricultural drainage waste within the delta, and
5. Export of fresh-water inflows which formerly entered the Delta.

The purpose of this paper is to discuss the mathematical modeling techniques which have been applied to the San Francisco Bay-Delta region, with particular emphasis on the salinity intrusion problem as it is affected by the diversion of fresh water inflows. Three separate models are discussed:

1. The Tidal Hydrodynamics Model which produces time-varying flows and heads throughout the estuary due to tidal action, unsteady inflows, evaporation and wind;
2. The Dynamic Water Quality Model which simulates the spatial and temporal distribution of pollutants, and
3. The Steady-State Water Quality Model which uses the dispersion concept to reduce the dynamic tidal problem to one of steady-state.

HYDROLOGIC-WATER QUALITY MODELS

TIDAL HYDRODYNAMICS MODEL

Water quality problems cannot be studied effectively unless an adequate representation of the tidal flow problem is obtained. We make the assumption that the flow problem can
Figure 1. San Francisco bay-delta system mathematical model network
be separated from the quality problem. The general technique used in this study was described by Shubinski, et al. [1] and will only be referenced here. It consists basically of discretizing the continuous system and the subsequent integration of the governing equations of continuity and motion. Although our primary interest here is the Delta, all modeling studies included the Bay as well. A schematic of the network used is shown in figure 1. In finite form, the system was assumed to consist of a network of one dimensional channels. This idealization was used both in the river-like channels of the Delta and in the open water of the Bay. The number of one dimensional elements required is somewhat arbitrary, but the system used contained about 325 channel elements interconnected at about 225 nodal points and was believed to be a satisfactory representation.

The governing equations of tidal flow in the one-dimensional channels are the equation of continuity,

$$\frac{\partial H}{\partial t} = \frac{1}{B} \frac{\partial}{\partial x} (V A)$$  \hspace{1cm} (1)

where:

- $H$ = tidal stage;
- $B$ = channel width;
- $V$ = velocity;
- $A$ = cross-sectional area;
- $x$ = distance along the channel, and
- $t$ = time,

and the equation of motion,

$$\frac{\partial V}{\partial t} = -V \frac{\partial V}{\partial x} - kV |V| - g \frac{\partial H}{\partial x}$$  \hspace{1cm} (2)

where:

- $g$ = acceleration of gravity, and
- $k$ = friction coefficient.

These equations assume that the wave length is much longer than the channel depth and that the effect of vertical acceleration is negligible. Moreover, for all runs made in the Bay-Delta system, the wind stress was assumed to be zero, although the model will accommodate constant or variable wind.

The application of these differential equations to the finite system and the techniques of integration applied to the resulting system of equations is described in the previously mentioned paper by Shubinski, et al. [2]. Certain minor modifications have been made but space does not permit a discussion of these.

In all mathematical modeling techniques it is necessary to compare predicted results with field data. Such a comparison at four locations within the Delta is shown in figure 2. Similar verification was obtained for flow quantities although prototype data for flow were scarce. It was found in general that both head and flow measurements predicted by the model agreed with prototype measurements to within 10 percent, although isolated points showed slightly larger error.

**Dynamic Water Quality Model**

The dispersion of water quality constituents within an estuary is a dynamic problem. The primary mixing mechanism is provided by the advective motion due to the tides. For simplicity the discussion presented here will treat the movement of a conservative
Figure 2. Tidal stage verification
pollutant. The extension to nonconservative substances is a straight-forward operation within this fundamental framework.

Once the time-history of flow is available, the water quality problem reduces to one of tracing the movement of pollutant through the system. Advective transport may be expressed either in the form

$$\frac{\partial C}{\partial t} = Q \frac{\partial C}{\partial x}$$  \hspace{1cm} (3)$$

or in the form

$$\frac{\partial M}{\partial t} = QC$$  \hspace{1cm} (4)$$

where:

- $C$ = constituent concentration;
- $Q$ = instantaneous flow;
- $M$ = mass transport;
- $x$ = distance, and
- $t$ = time.

Numerically, the form shown in equation 4 is more tractable because of considerations of mass continuity. The *Dynamic Water Quality Model* is based on the same geometric assumptions and discretizations used in the *Tidal Hydrodynamics Model*. A detailed description of the implications of the discretization process and the technique of timewise integration employed in the solution of the discrete equations is described by Orlob, *et al.*

It consists of a modified Euler approach, stepping forward through time, with values of constituent concentration obtained at rather short intervals, usually hourly.

Satisfactory prototype data for the testing and verification of the *Dynamic Water Quality Model* is difficult to obtain. It is necessary to have both hydrologic and water quality data over a period of at least several weeks at points throughout the system. For completeness these data should be available under varying hydrologic conditions, and they are not. However, figure 3 shows a comparison over the one-month period of July, 1963. The concentration profile is drawn from the boundary at the Golden Gate to a point some 70 miles upstream on the Sacramento River. There is a strong tendency on the part of the model to show higher values than those observed in the prototype. In that part of the system near the ocean, it is not known whether the error is due to incomplete or faulty prototype information or is inherent in the model. In the upper part of the system, near the trailing edge of the salinity wedge, it is known that the primary source of error is computational inaccuracy, which has been observed on numerous occasions, will produce higher values than observed during a period of intrusion. Evaluation of model results must reflect our knowledge of this error and, as shown in the section below, is taken into account whenever possible.

**Steady-State Water Quality Model**

The very fact that the *Dynamic Water Quality Model* is based on the tidally varying flows in the estuary often requires long and relatively expensive simulations on the computer. An alternative statement of the problem can be made which is similar in appearance to that of the Dynamic Program, but radically different in concept. It requires the replacement of equation 4 by the following:

$$\frac{\partial M}{\partial t} = CQ + EA \frac{\partial \bar{c}}{\partial x}$$  \hspace{1cm} (5)$$
COMPARISON OF HISTORICAL CHLORIDES TO COMPUTED CHLORIDES

HISTORICAL JULY 1, 1963
COMPUTED JULY 9, 1963

Figure 3. Water quality verification
Water quality modeling of estuaries

where:

\( E \) = dispersion coefficient, and
\( A \) = cross-sectional area.

This equation, which resembles the classical diffusion equation in form, can be used to shift the time scale of the quality problem from the consideration of instantaneous flows to that of average flows over one tidal cycle. The barred quantities represent the average values of flow, area, and concentration over the tidal cycle, and \( E \), the dispersion coefficient, insures the equality in the mass flow rates in equations 4 and 5. In other words, the dispersion coefficient accounts for that part of the mixing process which is caused by the tidal oscillation about its mean value.

Determination of the correct values of \( E \) for the Bay-Delta system is a complex and difficult problem. Primary factors affecting the selection were: 1) prediction of mass transport in the Dynamic Quality Program, and 2) empirical recommendations by various observers. Moreover, in the western Delta region, where the Dynamic Quality Model had shown too much salinity and intrusion, values of \( E \) were adjusted to eliminate this error.

Once all values of \( E \) for the system are selected, it is possible to solve directly for the steady-state water quality concentrations of the system by enforcing \( \delta M/\delta t = 0 \) at each node.

For each problem, then, a set of simultaneous equations is obtained, one for each node, the solution of which is the concentration distribution throughout the system. Details of the formation of the equations and their solution are described by Orlob, et al. [3].

Again, we simply state that the extension to nonconservative pollutants is a simple one if proper rate constants can be determined. Analyses performed in other studies have treated biochemical oxygen demand, dissolved oxygen, and coliform problems but we will concern ourselves here only with conservative pollutants.

STUDY OF DELTA OPERATIONS

The Sacramento-San Joaquin Delta is a pivotal element in California water resource planning. It is the focal point for the transfer of waters to the areas of deficiency in southern California and for drainage entering the Bay-Delta system. The Delta itself is a prime agricultural area and a major recreational and fish and wildlife resource. Maintenance of satisfactory water quality in this complex estuarine environment is of vital importance to each of these functions.

The principal objective of the operations studies described here was to estimate the levels of net Delta outflow required to insure compliance with water quality relating to total dissolved solids (TDS). In addition to the quantity of flow, water quality in the Delta is also affected by the distribution of flows and the nature of water-use practices within the System.

A key feature of Delta operation is the proposed “peripheral canal” which skirts the eastern edge of the Delta, carrying water from the Sacramento to the southwestern edge of the Delta. Its primary purpose is to insure the availability of high quality water for shipment to the southern part of California, and it is hydraulically isolated from the southern and central portion of the Delta. A secondary use of the canal is for redistribution of Delta inflows. Some of the water diverted from the Sacramento River into the canal is released into the Delta for quality control at various points along the canal.

HYDROLOGIC-WATER QUALITY CONDITIONS

Aside from the effects of physical facilities, the hydraulic behavior of the Delta and its water quality are influenced by
1. flows which are delivered to or exported from the rim of the Delta, of which the Sacramento River constitutes the principal inflow;
2. Delta agricultural practices;
3. San Joaquin Valley agricultural drainage;
4. municipal and industrial waste waters, and
5. net Delta export to other systems.

It was necessary to develop certain basic data concerning these factors and to adopt appropriate assumptions as to future conditions of Delta operation. For the most part, the data utilized were developed within governmental agencies directly concerned with Delta operations.

A typical dry year hydrology (1931), regulated in accord with 1990 water project operation, was used to establish the rim flows in the Delta operation studies. The choice of 1931, which represents a set of hydrologic circumstances likely to occur about once every 20 years, is regarded as reasonably conservative. To examine the effect of seasonal variations in hydrology and agricultural practice on Delta water quality, it was decided to use data corresponding to the months of April, August, and December. These months were taken as representative of spring, summer, and winter, respectively. No attempt was made to simulate a complete annual cycle.

CRITERIA FOR DELTA EVALUATION

In order to provide protection to water quality and to insure required allocations of fresh water to the Delta, a number of interested agencies have agreed upon standards of TDS and chloride levels which are not to be exceeded. Model studies indicated that the governing criteria was 500 mg/l TDS at various control points designated in the Delta. These are shown as solid diamonds in figures 1, 4, and 5.

INVESTIGATIONS TO ESTIMATE REQUIRED NET OUTFLOW

Investigations were made to determine the levels of net Delta outflow that would meet the conditions stipulated above (500 mg/l TDS at specified locations). Simulations were performed for each of the three representative hydrologic periods—spring, summer, and winter—and under a variety of operating conditions for the peripheral canal. In general, the procedure followed consisted of beginning the mathematical model run with a Delta outflow less than that to achieve the desired objectives, and then increasing the flows introduced through the Sacramento River until water quality objectives for TDS were met throughout the Delta. When this state was reached, attention was given to refining the estimates of required flows by some manipulation of peripheral canal release patterns.

Relative cost factors dictated that the steady-state water quality model be used throughout these studies. This introduces the assumption of steady-state behavior of the system—a phenomenon which has not occurred in historical records. However, it is believed that the control operating conditions after project development will result in controlled steady releases to the Delta through most of the hydrologic year.

In all, the evaluation of Delta outflow requirements was made on the basis of nine or more model runs similar to those illustrated below. The range of outflows examined was 2,000 cfs to 10,000 cfs. Figures 4 and 5 are representative of model run results and the kinds of comparisons which were made.

Figure 4 shows the complete set of contours for 3,000 cfs net Delta outflow under summer conditions. It is apparent that this outflow level met the water quality requirements and, in fact, it was found that lesser outflow could not meet the requirement regardless of the geographic distribution of the inflows. This figure is typical of the results obtained in a single model run and indicates the kind and detail of information available for comparison of various alternative hydrologic or water quality conditions.
Figure 5 shows the location of the same set of TDS contours under spring hydrology. It should be noted that the net Delta outflow required to satisfy the water quality objectives increased from 3,000 cfs under summer conditions to 4,000 cfs. This increase is largely attributable to the changes in agricultural practices and to changes in the quantity and quality of San Joaquin River inflow. Irrigation during the summer season generally results in deposition of salt on the land which is leached out into the streams during the spring months.

The principal findings related to the Delta outflow problem were:

1. Outflow requirements to meet TDS criteria were: spring, 4,000 cfs; summer, 3,000 cfs; winter, 5,000 cfs.

2. The quality of northern Delta waters is influenced to a high degree by the flow allowed to pass the point of diversion on the Sacramento River to the peripheral canal. It was found that because inflow quality varied only slightly in the northern Delta tributaries, it was necessary to maintain a net flow in the Sacramento River of approximately 2,200-2,400 cfs regardless of the season—summer, spring, or winter—in order to meet the criteria.
The quality of water in the southern Delta was particularly sensitive to the combined effects of agricultural waste water return flows and rim flows supplied to this region. Because the rim flows were generally lower, and the returns of salt from leaching of farm land were higher during the winter season, this period became the most critical. In order to maintain a favorable hydrologic balance, and to provide for dilution and flushing of accumulated salts, it was found necessary to supply a larger quantity of imported water through the southernmost distribution points along the peripheral canal. The result of this operational requirement was an overall increase in flow needed for quality control in the winter and a consequent increase in the net outflow required for the entire Delta.

RELIABILITY OF ESTIMATES

It has been pointed out that application of the Hydrologic-Water Quality Models and interpretation of their performance requires an appreciation of limitations imposed by the character of the data used and the nature of the models themselves. Accordingly,
it is appropriate to qualify the values cited above attaching to them some reasonable bounds of reliability. Considering the sources of possible error, and the relative magnitudes and influence on each on estimates on net Delta outflow, it may be expected that actual values would most probably fall within limits corresponding to 80-130 percent of predicted value. Hence, the required outflows for the three seasonal conditions examined would be expected to fall within the following ranges:

- Spring: 3,200-5,200 cfs,
- Summer: 2,400-3,900 cfs, and
- Winter: 4,000-6,500 cfs.

It is especially important, however, in assessing the results of the Delta operation studies to recognize that the differences between estimated values are much less subject to error than are absolute values. For example, the differential flow requirement of 2,000 cfs between summer and winter conditions is a highly significant quantity—probably reliable within ±10 percent. This would lead to a general conclusion that excess flow requirements for winter over summer conditions are most likely to be within the comparatively narrow range of 1,800-2,200 cfs.

OTHER APPLICATIONS

Space does not permit a description of other applications of these techniques which have been made. It must suffice to say that they include San Diego Bay (California), Elkhorn Slough (California), Sydney Harbor (Australia) and Port Phillip Bay (Australia). A study of Humboldt Bay (California) is underway. Each of these cases involves certain unique hydrodynamic or hydrologic features, but the general techniques are those discussed here.

CONCLUSIONS

Three related hydrologic-water quality models were used to analyze the water quality response of the San Francisco Bay-Delta system to varying hydrologic conditions. Both the models' ability to reproduce existing prototype conditions and their use as predictive tools were demonstrated. An estimate of levels of accuracy provided the means of judging and applying results.

It can be concluded that a general mathematical modeling technique for the analysis of estuarine water quality problems has been developed. In the hands of experienced and careful users, it can predict future behavior and provide meaningful inputs for the formulation of engineering decisions.

REFERENCES