Flow over side-weirs

K. Subramanya and S. C. Awasthy
Civil Engineering Department, Indian Institute of Technology, Kanpur, India

Abstract. An experimental investigation of the flow over sharp-edged side-weirs in rectangular channels is reported in this paper. The parameters affecting the coefficient of discharge have been identified and an expression has been derived analytically for the variation of the discharge coefficient of a side-weir of zero height with the Froude number of upstream main channel flow. The experimental data show a very good agreement with this expression.

The discharge coefficient of side-weirs of finite height has been studied experimentally and is shown to be essentially the same for the corresponding side-weir of zero height.

ECOULEMENT SUR UN DEVERSOIR LATERAL

Résumé. Cet article présente une étude expérimentale d'écoulement sur barrage de cote de fil tranchant en canaux rectangulaires. Les paramètres affectant le coefficient d'écoulement sont identifiés et on a déduit une expression analytique pour vérifier les variations du coefficient d'écoulement d'un barrage de cote hauteur zéro en fonction du nombre de Froude de l'écoulement du canal principal à l'amont. Les résultats sont conformes à cette expression.

Les coefficients d'écoulement des barrages de cote de hauteur finie sont étudiés expérimentalement, et l'on montre qu'ils sont peu différents des valeurs obtenues pour le barrage de cote de hauteur zéro.

FLUJO SOBRE VERTEDEROS LATERALES

Resumen. En la presente comunicación, se expone una investigación experimental del flujo sobre vertederos laterales con borde afilado en cauces rectangulares. Los parámetros que afectan al coeficiente de caudal han sido identificados y se ha deducido una expresión analítica para la variación del coeficiente de caudal, de un vertedero lateral de altura cero, con el número de Froude del flujo del cauce principal aguas arriba. Los datos experimentales muestran estar muy en conformidad con dicha expresión.

El coeficiente de caudal de los vertederos laterales de altura finita ha sido estudiado experimentalmente y se ha visto que concuerda esencialmente con el correspondiente a vertedero lateral de altura cero.

ПОТОК ЧЕРЕЗ БОКОВЫЕ ВОДОСЛИВЫ

Резюме. Доклад посвящен экспериментальным исследованиям потока через тонкостенные боковые водосливы в прямоугольном русле. Были уточнены параметры, оказывающие влияние на коэффициент расхода, и получено аналитически математическое выражение для оценки изменения коэффициента расхода бокового водослива нулевой высоты с учетом числа Фруда основного руслового потока в верхнем бьефе. Экспериментальные данные показывают весьма хорошее соответствие с этим выражением.

Коэффициент расхода боковых водосливов ограниченной высоты был исследован экспериментально, причем оказалось, что он имеет в основном то же значение, что и для соответствующего бокового водослива нулевой высоты.

328
INTRODUCTION

A side-weir is defined as a free overflow weir set into the side of a channel with the purpose of allowing part of the liquid to spill over the side if the surface of flow in the channel rises above the weir crest. The applications of side-weirs can commonly be found in: (a) sanitary engineering practice, to separate storm flow in the combined sewer system; (b) irrigation engineering, to remove excess flow in a canal and as protection to flood embankments from overtopping.

The hydraulic behaviour of side-weirs has received considerable interest since the early twenties of this century. However, most of the early works were empirical in nature. Probably the first rational approach was made by De Marchi [ref. 3] in 1932. Even though there have been many other investigations on the problem of side-weirs, notably by Allen [2], Frazer [5] and Collinge [4], the effective contribution to knowledge is not significant. The present position in this subject is that no reliable method is available to predict the discharge over a side-weir or to predict the change in the water surface profile due to the resulting, spatially varied flow. This paper presents the results of an investigation on the flow over side-weirs (specifically the variation of De Marchi coefficient) undertaken at the Indian Institute of Technology, Kanpur, India.

DE MARCHI EQUATION FOR SIDE-WEIRS

The general differential equation of the spatially varied flow modified for a rectangular horizontal channel ($S_0 = 0$) of zero friction (i.e., $S_f = 0$) becomes [3]:

$$\frac{dy}{dx} = \frac{Qy \left( -\frac{dQ}{dx} \right)}{gB^2y^3 - Q^2}$$

(1)

where $y$ = the depth of flow at any distance $x$ from the beginning of the side-weir; $B$ = width of the channel; $Q$ = discharge flowing in the channel at a given section.

(See definition sketch, Figure 1.)

The discharge over the side-weir per unit length ($q_s$) is assumed as

$$q_s = \frac{dQ_s}{dx} = -\frac{dQ}{dx} = \frac{2}{3} C_M \cdot \sqrt{2g} \cdot (y - s)^{1.5}$$

(2)

where $s$ = height of the side-weir; $C_M$ = a discharge coefficient, called here the De Marchi coefficient.

Since $S_f = 0$, $E$ = specific energy, constant along the channel,

$$Q = By \cdot \sqrt{2g \cdot (E - y)}.$$  

(3)

From equations 1, 2 and 3

$$\frac{dy}{dx} = \frac{4C_M}{3B} \cdot \frac{\sqrt{(E - y) \cdot (y - s)^3}}{3y - 2E}.$$  

(4)
Integrating

\[ x = \frac{3B}{2C_M} \cdot \phi \left( \frac{y}{E} \right) + \text{constant} \]  

where:

\[ \phi \left( \frac{y}{E} \right) = \frac{2E - 3S}{E - S} \cdot \sqrt{\frac{E - y}{y - S}} - 3 \sin^{-1} \left( \frac{E - y}{E - S} \right) \]  

\[ \phi = \text{varied flow function of De Marchi.} \]

For a side-weir of length \( L \),

\[ L = \frac{3}{2} \frac{B}{C_M} \cdot [\phi_2 - \phi_1] \]  

where suffixes 1 and 2 refer to the beginning and end of the weir.

If conditions at section 1 are known (i.e. \( Q_1, y_1 \)), \( \phi_2 \) and hence \( Q_2, y_2 \) can be found, provided \( C_M \) is known. The total discharge over the side-weir \( Q_s \) would then be

\[ Q_s = Q_1 - Q_2 \]  

DE MARCHI COEFFICIENT, \( C_M \)

Reliable information on the value of \( C_M \) corresponding to various flow conditions is lacking. Ackers \([1]\) suggests a value of \( C_M = 0.625 \) if \( y \) is measured at a distance remote from the plane of the weir and \( C_M = 0.725 \) if \( y \) is measured at the plane of the weir. Apparently he assumes \( C_M \) to be constant. Collinge \([4]\) finds that \( C \) varies with the mean velocity of the main channel.

Dimensional analysis indicates that

\[ C_M = \text{fn} \left( F_1 = \frac{V_1}{\sqrt{g y_1}}, \frac{L}{B}, \frac{y_1}{L}, \frac{S}{y_1} \right). \]  

It should be expected that \( F_1 \) would be a significant parameter affecting the value of \( C_M \) and the remaining parameters, representing the geometrical configurations of the flow, would have small effects, if any.

For a side-weir of zero height (i.e. \( S = 0 \))

\[ C_M = \text{fn} \left( F_1, \frac{L}{B}, \frac{y_1}{L} \right). \]  

Assuming \( L/B \) and \( y_1/L \) have very insignificant effects, an expression for the variation of \( C_M \) with \( F_1 \) for a weir of zero height is derived as follows.

VARIATION OF \( C_M \) WITH \( F_1 \) FOR A SIDE-WEIR OF ZERO HEIGHT

The flow over the side-weir can be considered as a deflected jet, as in Figure 1. Consider an elemental length \( dx \) of the weir. According to equation 2
Flow over side-weirs

\[ Q'_{s} = q_{s} \cdot dx = \frac{2}{3} C_{M} \sqrt{2g} \cdot dx \cdot y^{3/2} \]  \hspace{1cm} (\because S = 0). \tag{11}

Since the effective width of the jet through this element is reduced such that \((dx)_{e} = dx \cdot \sin \theta\), the discharge \(Q'_{s}\) can be rewritten as

\[ Q'_{s} = \frac{2}{3} C_{M}^{*} \cdot \sqrt{2g} \cdot dx \sin \theta \cdot y^{3/2} \]  \hspace{1cm} \tag{12}

where \(C_{M}^{*}\) is a constant coefficient. Also, as a first approximation in subcritical flow

\[ \sin \theta = \sqrt{1 - \left(\frac{V_{1}}{V_{j}}\right)^{2}}. \]  \hspace{1cm} \tag{13}

Now it is assumed that the critical depth corresponding to \(q_{s}\) occurs at the plane of the side-weir of zero height such that

\[ \frac{V_{c}^{2}}{2g} = \frac{1}{3} \cdot E \]  \hspace{1cm} \tag{14}

where

\[ E = y_{1} + \frac{V_{1}^{2}}{2g}. \]  \hspace{1cm} \tag{15}

Substituting \(V_{c} = V_{j}\) and noting \(F_{1}^{2} = \frac{V_{1}^{2}}{gy_{1}}\),

\[ \sin \theta = \sqrt{1 - \frac{3F_{1}^{2}}{F_{1}^{2} + 2}}. \]  \hspace{1cm} \tag{16}

From equations 11, 12 and 16

\[ C_{M} = C_{M}^{*} \cdot \sqrt{1 - \frac{3F_{1}^{2}}{F_{1}^{2} + 2}}. \]  \hspace{1cm} \tag{17}

The value of \(C_{M}^{*}\) can be assumed to be 0.611 as it represents the outflow from a constriction with \(F_{1} \to 0\). With this

\[ C_{M} = 0.611 \cdot \sqrt{1 - \frac{3F_{1}^{2}}{F_{1}^{2} + 2}}. \]  \hspace{1cm} \tag{18}

Equation 18 could be expected to give the variation of \(C_{M}\) with \(F_{1}\) for subcritical flow over a side-weir of zero height.

In supercritical flow, however, the discharge coefficient \(C_{M}^{*}\) could be assumed to be essentially independent of the Froude number as the zone of influence of the side-weir will be confined to its immediate neighbourhood only.
EXPERIMENTAL STUDY

An experimental investigation was undertaken at the Hydraulic Laboratories of the Indian Institute of Technology, Kanpur: (a) to verify equation 18 giving $C_M$ for side-weirs of zero height; (b) to find the effect of other parameters on $C_M$; and (c) to find $C_M$ for supercritical flow. The experiments were conducted in two horizontal channels:

Flume A: width 610 mm, length 9.0 m, bed: cement plaster.
Flume B: width 248 and 124 mm, length 3.0 m, bed: aluminium.

The ranges of variables studied are shown in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>0.02–4.3</td>
</tr>
<tr>
<td>$L/B$</td>
<td>0.2–1.0</td>
</tr>
<tr>
<td>$S/y_1$</td>
<td>0.2–0.96</td>
</tr>
<tr>
<td>$y_1/L$</td>
<td>0.1–2.4</td>
</tr>
</tbody>
</table>

The weirs were sharp-edged and were fully aerated on the down-stream side. The depth $y_1$ was measured on the centreline at section 1, (i.e. $x = 0$). A total of 200 experiments were conducted and the results are shown in Figures 2, 3 and 4.

Subcritical flow

Figure 2 shows a plot of $C_M$ calculated from experimental data for side-weirs of zero height, plotted against the Froude number of the flow. Only data for the subcritical region are shown in this plot. Also plotted are the variation of $C_M$ given by equation 18. It can be seen that all the plotted points follow the variation given by equation 18 very well. It can be seen that there is no effect of the parameter $L/B$. A further examination revealed a similar insignificant effect of the parameter $y_1/L$. Thus it is concluded that the expression given by equation 18 correctly predicts the variation of $C_M$ with Froude number for subcritical flow in side-weirs of zero height.

Figure 3 represents the plot of the experimental data on subcritical flow over side-weirs of finite height. A very good fit of the experimental data with the line given by equation 18 is indicated. It is interesting to note the absence of any significant effect of the parameter $s/y_1$ and also of the two parameters $L/B$ and $y_1/L$. A slight observable deviation of data at a Froude number of 0.7 can be attributed to experimental errors.

Supercritical flow

Figure 4 shows a plot of $C_M$ versus $F_1$ for supercritical flow over side-weirs of zero height. It is once again seen that there is no effect of the parameters $L/B$ and $y_1/L$. Also, the effect of Froude number $F_1$ itself is very small, the variation being expressible as

$$C_M = 0.36 - 0.8F_1.$$ (19)
Flow over side-weirs

Figure 1. Definition sketch

Figure 2. Variation of $C_M$ with $F_1$ for side weirs of zero height
\[
L = \frac{B}{2/3 C_M} \left[ \phi_2 - \phi_1 \right]
\]

**NOTATION**

<table>
<thead>
<tr>
<th>CMS</th>
<th>SYMBOL</th>
<th>RANGE OF ( \gamma / \gamma_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>○</td>
<td>0.78 TO 0.96</td>
</tr>
<tr>
<td>58</td>
<td>△</td>
<td>0.65 TO 0.95</td>
</tr>
<tr>
<td>25</td>
<td>□</td>
<td>0.21 TO 0.96</td>
</tr>
<tr>
<td>11</td>
<td>●</td>
<td>0.40 TO 0.93</td>
</tr>
<tr>
<td>21</td>
<td>△</td>
<td>0.58 TO 0.77</td>
</tr>
<tr>
<td>8</td>
<td>□</td>
<td>0.35 TO 0.64</td>
</tr>
</tbody>
</table>

**Figure 3.** Variation of \( C_M \) with \( F_1 \) for side-weirs of finite height

\[
C_M = 0.36 - 0.08 F_1 \quad (F_1 > 2.0)
\]

**Figure 4.** Variation of \( C_M \) with \( F_1 \) in supercritical flow for side-weir of zero height
Flow over side-weirs

It is possible that in an ideal case $C_M$ would be independent of $F_1$. The small effect of $F_1$, seen in equation 19, is probably due to friction effects.

CONCLUSION

As a result of the investigation of the variation of the De Marchi discharge coefficient $C_M$, the following conclusions are drawn:

1. Equation 18 adequately represents the variation of $C_M$ with $F_1$ in subcritical flow as confirmed in Figure 2.
2. The discharge coefficient $C_M$ for side-weirs of finite height is essentially the same as that for side-weirs of zero height.
3. For supercritical flow, the discharge coefficient varies very slowly with the Froude number $F_1$, as indicated in Figure 4.

BIBLIOGRAPHY / BIBLIOGRAPHIE


DISCUSSION

Markovic: What is the physical reason for the same discharge coefficient at zero and finite height of the side-weir?

Subramanya: The physical explanation is most probably that the side-weir acts as a skimming weir in the sense that the velocity of the flow over the weir is predominant, unlike the case of an ordinary weir. In this case only that part of the flow which is above the side weir is important in deflecting the jet.