Mathematical model of the river-bed erosion below a dam

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Abstract. The principles of choosing a mathematical model of river-bed erosion are discussed. According to these principles a mathematical model based on the gradually varied flow equation and on (1) Meyer-Peter, (2) Meyer-Peter–Müller, (3) Gontcharoff formulae for the bed-load are proposed. The models are checked on the experimental data for the Upper Vistula River. The second one is found to have the best correspondence with measured values.

Résumé. On a analysé les principes du choix d'un modèle mathématique d'erosion des lits alluvionaires. Après ces principes un modèle basé sur l'équation d'écaulment graduellement varié et les loi de charriage de (1) Meyer-Peter, (2) Meyer-Peter–Müller et (3) Gontcharoff, est proposé. Le modèle est vérifié sur les données experimentalles pour la Haute Vistule et on a trouvé que le second a donné les meilleurs résultats.

INTRODUCTION

The river-bed erosion below a dam is a problem known to hydraulic engineers and was treated by many authors. The existing methods for computation of the bed-degradation values can be divided into three groups:


(3) Mathematical models based on unsteady flow equations and the Meyer-Peter bed-load formula (Daubert et al., 1966; Daubert and Lebreton, 1967; de Vries, 1965).

The above-mentioned methods unfortunately do not give the limits within which they are applicable. Their validity is in fact very limited.

In this paper some factors are defined which should determine the choice of a model. In accordance with this principle the proper model for the Upper Vistula River is put forward. It is hoped that an approach similar to this may be useful in the construction of mathematical models valid for different conditions.

FACTORS TO BE CONSIDERED IN CHOOSING THE PROPER MODEL

We shall try to define the basic factors governing the flow and sediment transportation in the case of the perturbation of the initial equilibrium state by the construction of a dam.

(1) Geological factors -- the structure of the layers just below the river bed can differ from the river sediment. The degradation may uncover the lower layers that could change the empirical relations, coefficients and constants used for a model.

(2) Hydrological characteristics of a river, especially hydrographs, type of the channel (single or divided), the characteristics of the berms and flood observations.
(3) Characteristics of the channel control—weirs, sluices, etc. The dam either stops the bed-load completely or during floods the upstream sediment passes through channel controls. Moreover the channel controls change the flow characteristics.

(4) The choice of bed-load formulae is crucial for the results of bed erosion computations. All existing formulae are either purely empirical, e.g. Meyer-Peter, or have a semi-theoretical background, e.g. Einstein. The range of sediment discharge computed by different formulae is very broad. As Herbertson (1969) showed this non-uniqueness of results is caused by the incompleteness of these equations. The only way to avoid the bigger errors is either to perform the field measurements of bed-load transport, or to select the formula which was derived and justified in analogous conditions. Moreover some formulae have a form not easily adaptable to mathematical operations, e.g. Einstein and Colby's.

(5) Additional factors—in particular cases the following factors can influence the bed erosion: the heating of water by an electric-power station, heavy pollution of water, exploitation of bed material, work of dredgers and so on.

TYPE OF THE MATHEMATICAL MODEL CHOSEN FOR THE UPPER VISTULA RIVER

For the Upper Vistula River the author used the field measurements, more or less all the data demanded in the previous section [some of the data are discussed below and in this section; a more complete discussion is in the paper by Witkowska et al. (1971)]:

1. The layer of bed-material alluvia is of thickness from 4 to 10 m below the initial river bed.
2. The channel is divided into a main channel, an approximate rectangle of width about 70 m and depth about 3 m, and broad berms up to 300 m each. The bank full discharge is about 200 m³/s and represents a duration of 4 to 5 days per year and has a steady flow characteristic. The river is regulated and no bank erosion is observed; the erosion takes place only in the main channel. The berms are covered by grass and plants.
3. The bed-load is completely arrested by the dam during the steady flows, but the opening of weirs and sluices permits the passing of the bed-load in times of flood.
4. The Meyer-Peter (Müller, 1943) formula and Meyer-Peter—Müller formula were selected as the most appropriate, but because of the frequent use for the Vistula of the Gontcharoff formula (Gontcharoff, 1954) this was also considered.
5. The water was highly polluted, the bed-material was exploited by dredgers (the total amount $2.1 \times 10^6$ m³).

The above cited factors and the fact that the river-bed slope was different from one cross-section to another—in some intervals it was even negative—suggested that a mathematical model based on the gradually varied flow equation should be discussed.

MATHEMATICAL MODEL OF BED EROSION

Basic assumptions
The basic assumptions can be divided into two groups. The first contains assumptions which have to be fulfilled always. The second contains ones which by the use of some numerical methods might be omitted.

The first group includes:

(a) river cross-section approximates a rectangle; the mean depth can be used but the width may change;
(b) river-bed is regulated, banks not erodible, meandering does not occur;
(c) no vegetation growth.
The second group includes:
(a) sediment transport is completely arrested;
(b) discharge is not varied;
(c) no tributaries occur;
(d) wash load and suspended load is neglected;
(e) layer below the river bed is the same as sediment;
(f) the influence of changes in bed-forms on the bed-resistance is neglected.

FIGURE 1. $I_o$ – initial energy slope, $i_o$ – initial bed slope, $y$ – river depth, $B$ – river width, $x, z$ – coordinates.

The equations
(a) The equation of continuity for sediment transport:
$$\frac{\partial Z}{\partial t} + \frac{1}{\gamma_s} \frac{\partial q_s}{\partial x} = 0$$

where
$\gamma_s$ is the specific weight of sediment;
$q_s$ is the rate of sediment transport per unit time and unit width.

(b) The gradually varied flow equation:
$$\frac{\partial y}{\partial x} = \frac{\partial Z}{\partial x} - \frac{n^2 Q^2}{B^2 y^{10/3}} - \frac{\alpha Q^2}{gB^2 y^{2}}$$
where

\( n \) is the Manning coefficient;
\( Q \) is the water discharge;
\( \alpha \) is the Saint-Venant coefficient.

(c) The bed-load equation — as mentioned above the following three formulae will be considered:

**Meyer-Peter formula**

\[
q_s = \left( q^{2/3} I - \frac{A_m d}{B_m} \right)^{2/3}
\]

where

\( A_m, B_m \) are the constants for the particular sediment;
\( q \) is the water discharge per unit time and unit width;
\( d = d_{35} \).

**Meyer-Peter–Müller formula**

\[
q_s = 8 \left( \frac{g}{\gamma} \right)^{1/2} \left( \frac{\gamma_s}{\gamma_s - \gamma} \right) (\tau - \tau_0)^{3/2}
\]

where

\( \gamma, \gamma_s \) are the specific weights for water and sediment respectively;
\( \tau = \gamma y \gamma_s \);
\( \tau_0 \) — according to Shields.

**Gontcharoff formula**

\[
q_s = 5.3 (1 + \phi) v_0 d \left( \frac{v^3}{v_0^3} - 1 \right) \left( \frac{v}{v_0} - 1 \right)
\]

where

\( v_0 = \log \frac{8.8 y}{d_{35}} \sqrt[3]{ \frac{2g}{3.5} (\gamma_s - \gamma) } \)

\( v \) is the mean velocity;
\( d = d_{35} \)
\( \phi \) is the turbulence parameter.

The resistance is expressed by the Manning formula, \( n - d \) relation either by Strickler formula or by an empirical relation based on measurements, \( n = f(d) \); in the investigation the initial \( n - d \) relation does not differ significantly from the final one and is of the following form:

\( n = 0.2 d^{1/6} \)

After differentiating equations (3), (4), (5) and by some rearranging the following equations were obtained:

from equation (3):

\[
\frac{\partial z}{\partial t} = -\frac{3}{\gamma_s} q^{2/3} I \left( q^{2/3} I - \frac{A_m d}{B_m} \right)^{1/2} \left[ \frac{1}{B} \frac{\partial B}{\partial x} - \frac{5}{3} \frac{1}{y} \frac{\partial y}{\partial x} + \left( \frac{1}{6d} - \frac{A_m}{2q^{2/3} B} \right) \frac{\partial d}{\partial x} \right]
\]
from equation (4):
\[
\frac{\partial z}{\partial t} = - \frac{1}{\gamma_s} \phi (\tau - \tau_0)^{1/2} \left[ \frac{1}{d} \left( \frac{1}{6} - \frac{\tau_0}{\tau} \right) \frac{\partial d}{\partial x} - \frac{10}{3} \frac{1}{y} \frac{\partial y}{\partial x} - \frac{1}{B} \frac{\partial B}{\partial x} \right]
\]  
(7)

where
\[
\phi = 12 \left( \frac{g}{\gamma} \right)^{1/2} \left( \gamma \frac{\gamma_s}{\gamma_s - \gamma} \right)
\]

from equation (5):
\[
\frac{\partial z}{\partial t} = - \frac{1}{\gamma_s} \left[ \left( \frac{4v^3}{v_0^3} - 1 \right) \frac{\partial v}{\partial x} \left( 1 - \frac{3v^2}{v_0^2} \right) \frac{\partial v_0}{\partial x} \right]
\]

where
\[
\frac{\partial v_0}{\partial x} = \frac{N}{d_{eq}} \sqrt{Kd} + \log \left( \frac{N}{d_{eq}} \right) \frac{K}{\sqrt{Kd}} \frac{\partial d}{\partial x}
\]

where \(N, K, k\) are cumulated constants.

Equations (6) to (8) are derived without neglecting any term.

The solution of \(\partial z/\partial t\) equations

The differential equations for the bed erosion below a dam cannot be solved analytically, firstly because there are more unknowns than equations. The \(B(x)\) relation can be found from the river cross sections, in most cases, however, not in analytical form.

The \(d(x, t)\) relation is more difficult to establish. Relation (5) might possibly be based on the standard deviation of the particle size distribution of river-bed sediments

\[
\sigma_\phi = \sqrt{\frac{d_{eq}}{d_0}}
\]

(9)

In our investigation \(\sigma_\phi = 4 \div 5\), and the formula for the final \(d\) distribution is

\[
d_f = 16d_0 \exp(-0.12x)
\]

(10)

where \(d_0\) is the initial diameter.

The prediction of the \(d\) relation at every cross section would be more correct.

In the \(d(x, t)\) relation the armouring coefficient (Egiazaroff, 1965; Komura and Simmons, 1967) is included; the simultaneous use of \(d(x, t)\) and an armouring coefficient as done by Komura and Simmons (1967), does not seem to be justified.

For a solution of equations (6), (7) and (8) at a transient state the trial-and-error computational procedure presented by Tinney (1962) can be adopted.

The first rate of degradation is computed for the known bed profile by equations (6), (7) or (8), and the new position of the bed at the end of time interval, \(\Delta t\), can be obtained from

\[
z = z + \left( \frac{\partial z}{\partial t} \right)_0 \Delta t
\]

(11)

For \(z\) so obtained the water surface is computed from equation (2) by the standard step method. The process is repeated until the equilibrium state is achieved. This
method allows the possibility of avoiding the assumptions in the second group, by the division of the time intervals and an individual treatment of every cross section, in the following way:

Assumptions (a) and (b) can be avoided by separate treatment of different discharge durations, as the use of a mixed model of unsteady and steady flow for the respective time periods is possible.

(c) At a section \( x_0 \) tributaries can be considered by a change in the continuity equation

\[
\frac{\partial q}{\partial x} + \gamma_s \frac{\partial x}{\partial t} + \mu \delta(x - x_0) = 0
\]

where

\( \mu \) is a constant connected with the amount of sediment;

\( \delta(x - x_0) \) is the Dirac \( \delta \) function.

(d) The change in the bed material in a certain section and after a certain time needs only the change of \( d, n = f(d) \) in equations (6), (7) and (8).

(e) The changes in \( n \) with change of Froude number for every time interval can be introduced in equations (6), (7) and (8). (In our investigation there was no significant change in the Froude number during the erosion time.)

The trial-and-error method is time-consuming and tedious, but can be easily adapted for computer use.

The final equilibrium river-bed profile

If the time duration of bed degradation is not the point of interest, the final equilibrium profile can be obtained in the following way (Komura and Simmons, 1967). At a final equilibrium state

\[
\frac{\partial x}{\partial t} = 0
\]

and the river slope can be expressed from equation (2)

\[
\frac{i_f}{a_2} = -\frac{\partial x}{\partial x} = \left(1 - \frac{y_{cr}^3}{y_f^3}\right) \frac{\partial y}{\partial x} - \frac{y_{cr}}{By_f} \frac{\partial B}{\partial x} + \frac{\tau_0}{\gamma y_f}
\]

where \( y_{cr} \) is the critical depth; \( y_f \) denotes a final state.

From either (6) or (7), \( \partial y/\partial x \) can be obtained. And from the known relations (15)

\[
y = \left(\frac{n_f^2 q^2}{a_c(\frac{\rho_s}{\rho} - 1) d_f}\right)^{3/7}
\]

From (14), (15) and \( \partial y/\partial x \) the final river-bed elevation is given by

\[
z_f = z_0 - \int_0^x i_f \, dx
\]

Usually direct integration is not possible and the solution is obtained by numerical integration

\[
z_f = z_0 + \sum_{i=0}^{i=n} \Delta Z
\]
This result with equation (7) yields

\[ z_f = z_0 + \sum_{i=0}^{n} \left[ \frac{A_{df}}{\gamma y_f} \Delta x + \frac{1}{4} \frac{y_f}{df} \left( 1 - \frac{y_f}{y_f^3} \right) \Delta d \Delta x \right] \]  

(18)

For constant \( B \) and equation (10) an analytical solution is possible in the form

\[ z_f = z_0 + \frac{4 A_{d0}}{5} \left[ \exp\left(\frac{(5/4)x}{y} - 1\right) + a y_0 \left( 1 - \exp\left(-b/4\right) \right) \right] \]

\[-\frac{a}{2} \frac{y_f^3}{y_0^2} \left[ \exp\left(\frac{(b/2)x}{y} - 1\right) \right] \]

(19)

where \( a = 16 \) and \( b = 0.12 \) from (10) and \( A = a_c (\rho_r - \rho) g \).

Then equation (6) yields

\[ z_f = z_0 + \sum_{i=0}^{n} \left[ \left( 1 - \frac{y_f^3}{y_f^3} \right) \left( \frac{1}{6d} - \frac{A_m}{0.08 q^{1/3}} \frac{y_f^{10/3}}{B_m d_f} \right) \Delta d + \frac{A_m d_f}{\gamma (y_f)} \right] \Delta x \]  

(20)

In this case no analytical solution is available and this is so for most bed-load formulae. The analytical solution has, however, only very limited use because it gives indicative values only.

**THE CALCULATION BY THE STEP METHOD OF THE FINAL PROFILE FOR THE EROSION BELOW THE DAM IN PRZEWÓZ ON THE VISTULA RIVER**

The Przewóz data consist of the bed profile and a few bed material samples taken before the construction of the dam (1951) and bed profiles from 1957 to 1960. In 1967 and 1968 intensive field studies were made by the Institute of Hydraulic Engineering (Witkowska et al., 1971). The river-bed profile was measured over 20 km, the cross sections at every kilometre, the velocity measurements, and the local velocity slopes and samples of the bed material were taken every 5 m in a cross section. The sediment diameters used for the calculations represent median values from

\[ d_m = \frac{\sum_{i=1}^{n} d v}{\sum_{i=1}^{n} v} \]  

(21)

where \( d \) is the diameter in the measured vertical and \( v \) is the mean velocity in vertical.

The hydrographs for km 9.0 were constructed and then all possible information concerning the geological characteristics, hydrological data, the exploitation of the bed material work of the structure and dredgers were collected (Witkowska et al., 1971).

The analysis of the hydrographs shows that at most cross sections the bank full discharge of the main channel is from 200 to 250 m³/s, and has no unsteady flow characteristics. The analysis of the final equilibrium depth \( y_f \) (Table 1) shows that the equilibrium state was achieved at a discharge of 60 m³/s to km 18, at 200 m³/s to km 15, and at 250 m³/s to km 13. From the last results, and from the analysis of the bed profile, it can be assumed that degradation occurs only over the first 13 km and can
TABLE 1. The final equilibrium depth

<table>
<thead>
<tr>
<th>No.</th>
<th>km</th>
<th>( y_f ) (m)</th>
<th>( y ) from hydrograph (m)</th>
<th>( y_f ) (m)</th>
<th>( y ) (m)</th>
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<td>( \dot{Q} = 60 \text{ m}^3/\text{s} )</td>
<td>( \dot{Q} = 200 \text{ m}^3/\text{s} )</td>
<td>( \dot{Q} = 250 \text{ m}^3/\text{s} )</td>
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continue slightly for the next 7 or 8 km. The calculations were made for the 20 km from the dam-site. They were performed for discharges 100 m$^3$/s, 150 m$^3$/s and 250 m$^3$/s; the mean values — according to the duration of flows — were the same as calculated for 200 m$^3$/s (up to 5 per cent). The last ones are given in Table 2. All three equations were checked at a few cross sections. The results obtained from the Gontcharoff formula were very far from reality, Meyer-Peter gave closer values, but

TABLE 2. The final degradation computation

<table>
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<tr>
<th>No.</th>
<th>( x ) (km)</th>
<th>( \Delta x ) (m)</th>
<th>( d ) (mm)</th>
<th>( \Delta d ) (mm)</th>
<th>( q ) ( (\text{m}^3/\text{s}) ) ( \text{m}^{-1} )</th>
<th>( y_{cr} ) (m)</th>
<th>( \Delta z ) (m)</th>
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Mathematical model of the river-bed erosion below a dam

the best were obtained with the Meyer-Peter–Müller formula. The results of the computation are given in Table 2. Column 8 gives the computed differences in bed elevation $\Delta z$, and column 9 gives the observed ones (from the measurements in 1967 and 1968). For the cross sections close to the dam-site 0–10 km, the computed results are very close to the observed ones. It has to be mentioned that the negative slopes $–6$ km (computed) correspond very closely to the measured ones. In previous works (Tinney, 1962; Komura and Simmons, 1967) the changing and negative slopes were not considered. The great differences in km 9.35–12.27 are caused by the exploitation of the bed material in this area between 1953 and 1959. The last kilometres 15.5–19 are not yet fully degraded; the results shown in Table 1, therefore, cannot be conclusive.

CONCLUSIONS

(1) The choice of mathematical model is determined by the geological, hydrological and sedimentation characteristics. It cannot be general and for every practical use has to be carefully chosen.

(2) The bed-load formulae are of the greatest importance. Their choice is crucial to the results.

(3) Three types of differential equations were derived and their solution by the Tinney trial-and-error method was shown. This method allows some basic assumptions to be avoided and allows the inclusion of additional parameters.

(4) The final equilibrium equations were derived by the step method.

(5) The calculations based on the field data show that the chosen model gives very good results. It allows then a nonregular variation of the $x$-coordinate of the sediment diameter and varied slope (even with negative values) to be taken into account.

(6) The armouring coefficient $a_c$ is not necessary if the final $d$ is properly expressed as a function of $x$.

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