Theory of radiation heat transfer between forest canopy and snowpacks

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ABSTRACT: The equations of radiative transfer have been applied to a forest canopy that transmits radiation to an underlying snowpack. Both long-wave and solar fluxes incident on the snowpack have been calculated for a model canopy represented by a homogeneous slab. Scattering of solar radiation has not been accounted for although the methods presented can be extended to the case of a canopy that scatters as well as absorbs solar radiation and receives reflected radiation from an underlying snowpack. The theory predicts that the total surface area of vegetation in the canopy is the relevant canopy parameter that determines the amount of radiation energy incident on a snowpack.

RESUME: Les équations concernant l'émission de radiation sont ici appliquées au problème de l'échange de radiation entre la forêt et le sol recouvert de neige. La forêt est ici remplacée par une couche uniforme sur laquelle la radiation solaire et l'infra-rouge est incidente. La dispersion de la radiation solaire n'est pas incluse ici, mais pourrait facilement être incorporée à la théorie. Ce modèle prédit que l'intensité de la radiation reçue par le sol ne dépend seulement que de la surface totale de végétation.

LIST OF SYMBOLS

Superscripts l and s refer to long-wave and solar radiation, respectively. In the following list, superscripts have been omitted from quantities that are defined for both long-wave and solar portions of the spectrum.

$I$ intensity of radiation (ly min$^{-1}$ steradian)
$I_-$ intensity for downward directions (-1 ≤ μ < 0)
$q_-$ downward flux (ly min$^{-1}$)
$q_+$ upward flux (ly min$^{-1}$)
$q$ net flux = $q_-$ - $q_+$
$S$ volumetric source (cal min$^{-1}$ cm$^{-3}$)
$\beta$ volumetric extinction coefficient (cm$^{-1}$)
$\kappa$ volumetric absorption coefficient (cm$^{-1}$)
$\gamma$ volumetric scattering coefficient (cm$^{-1}$)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{\hat{n}}$</td>
<td>unit vector specifying an arbitrary direction in space</td>
</tr>
<tr>
<td>$\theta$</td>
<td>colatitude angle; $\mu = \cos \theta$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>azimuthal angle</td>
</tr>
<tr>
<td>$z$</td>
<td>spatial coordinate (cm)</td>
</tr>
<tr>
<td>$d$</td>
<td>canopy depth (cm)</td>
</tr>
<tr>
<td>$A$</td>
<td>cross-sectional area of canopy (cm$^2$)</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>total surface area of canopy vegetation (cm$^2$)</td>
</tr>
<tr>
<td>$V_c$</td>
<td>canopy volume = $Ad$ (cm$^3$)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>vegetative area density = $1/4 , (\alpha_c/A)$</td>
</tr>
<tr>
<td>$x$</td>
<td>canopy closure</td>
</tr>
<tr>
<td>$r$</td>
<td>solar reflectance of canopy vegetation</td>
</tr>
<tr>
<td>$E_o^L$</td>
<td>downward long-wave flux incident on canopy (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$E_c^L$</td>
<td>canopy long-wave emission = $\sigma T_c^4$ (emissivity = 1) (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$T_c$</td>
<td>canopy temperature ($^\circ$K)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan's constant ($8.132 \times 10^{-11}$ ly -min$^{-1}$ K$^{-4}$ -min$^{-1}$)</td>
</tr>
<tr>
<td>$E_s^L$</td>
<td>snowpack long-wave emission (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$Q^L$</td>
<td>net long-wave radiant energy input to snowpack (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$E_o^S$</td>
<td>total solar flux incident on canopy (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$D_o^S$</td>
<td>diffuse solar flux incident on canopy (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$F_o^S$</td>
<td>solar direct beam flux in direction of beam (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$Q^S$</td>
<td>net solar radiant energy input to snowpack (ly -min$^{-1}$)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>snowpack solar albedo</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>cosine of angle between incident solar beam and outward normal to canopy</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>angle between solar beam and earth's surface; $\mu_o = -\sin \theta_h$</td>
</tr>
<tr>
<td>$E_n$</td>
<td>exponential integral of the nth kind</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Dirac delta function</td>
</tr>
</tbody>
</table>
INTRODUCTION

A forest canopy is a heterogeneous anisotropic system that absorbs, scatters, and emits radiant energy. The study of the equations of radiative transfer in homogeneous, isotropic media has been a subject of astrophysical research since the early part of this century [1]. In addition, these equations have been applied to problems in oceanography [2], radiant heat transfer [3, 4], and neutron transport theory [5]. In particular, the equations of neutron transport, which are formally identical to the equations of radiative transfer, have been used in the design of nuclear reactors [6]. Because of the successful application of the equations of transfer to a wide variety of problems, it is tempting to apply these same equations, with suitable modifications, to the problem of determining the net radiation input to a snowpack under forest canopy.

This paper is concerned with the solution of the equations of transfer for a model canopy which preserves many essential physical characteristics of an actual canopy and its environment.

MODEL CANOPY

We replace the actual heterogeneous canopy with a homogeneous slab of thickness d, the properties of which are assumed to be uniform and isotropic (Fig. 1). The horizontal extent of the slab is large compared with d.

All radiation is divided into two discrete groups—long-wave and solar. The canopy absorbs, transmits, and emits long-wave radiation and absorbs, transmits, and isotropically scatters solar radiation.

EQUATIONS OF TRANSFER

The intensity of radiation, I, is the amount of radiant energy that crosses a unit area with normal in the direction \( \hat{n} = \hat{n}(\theta, \phi) \) in unit time, and which is confined to a unit solid angle about \( \theta \). In general, I depends on three spatial coordinates and two direction coordinates. However, for the model chosen, I depends only on \( \theta \) and \( \mu \), where \( \mu = \cos \theta \). Therefore, the time independent equations of radiative transfer, which are applicable to our problem, can be written:

\[
\frac{\mu dI^L(\mu, \theta)}{d\theta} + \beta^L I^L(\theta, \mu) = \frac{S^L}{4\pi} \quad (1)
\]

\[
\frac{\mu dI^S(\mu, \theta)}{d\theta} + \beta^S I^S(\theta, \mu) = \gamma^S \int_{-1}^{1} I^S(\mu', \theta) d\mu' \quad (2)
\]

where the superscripts \( L \) and \( S \) refer to long-wave and solar, respectively. \( S^L \) is the amount of long-wave radiant energy that is isotropically emitted by a unit volume per unit time. The volumetric extinction coefficient, \( \beta^L \), is the sum of an absorption coefficient, \( \kappa^L \), and a scattering coefficient, \( \gamma^L \). As we assume that there is no scattering of long-wave radiation, \( \beta^L = \kappa^L \). Derivations of the equations of transfer may be found in references [1, 3, 7 and 8]. Reference [9] deals with a rigorous discussion of the mathematical foundations of radiative transfer.
The net radiant energy (long-wave or solar) which crosses a unit area at \( z \) (net flux) is \( q(z) = q_+(z) - q_-(z) \) where:

\[
q_+(z) = 2\pi \int_0^1 I(\mu', z) \mu' \, d\mu'
\]
\[
q_-(z) = 2\pi \int_0^{-1} I(\mu', z) \mu' \, d\mu'
\]

\( q_+ \) is the upward flux and \( q_- \) is the downward flux.

SCATTERING AND ABSORPTION COEFFICIENTS

Consider a uniform parallel beam of radiation, \( F \), which is incident on a homogeneous medium consisting of "black" (perfectly absorbing) spheres of radius \( R \). In travelling a distance \( d \), the beam is diminished by an amount \( \Delta F \), where \( \Delta F = -\beta F d \) and \( \beta \) is the extinction coefficient. If there are \( N/V \) spheres per unit volume of the medium, then the cross-sectional area of spheres per unit area of a slab of thickness \( d \) is \( N\pi R^2 d/V \). But this is the area which is effective in removing radiation from the incident beam. Therefore \( \Delta F = -F\pi R^2 \Delta s/V = -\beta F d \). The total surface area, \( a \), of the \( N \) spheres is \( N\pi R^2 \). Thus:

\[
\beta = \frac{a}{4V}
\]

If the spheres have a reflectance \( r \), then \( \gamma = r\beta \) and \( \kappa = (1 - r) \beta \). Therefore, for our model we assume that:

\[
\beta^L = \frac{a_c}{4V} = \kappa^L = \beta ; \quad \beta^S = \beta ; \quad \kappa^S = (1 - r) \beta
\]

where \( a_c \) is the total surface area of the canopy vegetation, \( V_c \) is the volume of the canopy, and \( r \) is the solar reflectance of the canopy vegetation. A method for estimating \( a_c \) for a ponderosa pine forest is presented in reference [10].

LONG-WAVE RADIATION

The long-wave radiation emitted per unit area of canopy surface is \( E_c^L = \sigma T_c^L \). Therefore, \( S^L/4\pi = E_c^L a_c/4\pi V = E_c^L \beta/\pi \). Because we are interested only in the downward flux from the canopy, we can write eq. (1) in the form:

\[
\frac{dI^L}{d\mu} + \beta I^L = \frac{\beta E_c^L}{\pi} \quad -1 \leq \mu < 0
\]

Multiplying both sides of eq (7) by the integrating factor \( \exp(\beta \pi/\mu) \) and integrating the resulting equation from \( \mu \) to \( d \) yields:

\[
I^L_-(\mu, z) = I^L_-(\mu, d) \frac{e^{\beta(d-z)/\mu}}{\mu} + E_c^L \left\{ 1 - e^{\beta(d-z)/\mu} \right\}/\pi
\]
As $V_c = Ad$, where $A$ is the cross-sectional area of the canopy, $\beta_d = \frac{1}{4} (\frac{2c/A}{1}) = \xi$, which we will call the vegetative area density. The boundary condition at the surface $z = d$ is

$$I_\infty^S (\mu, d) = \frac{E_\infty^S}{\pi}$$  (9)

where $E_\infty^S$ is the downward flux of isotropic long-wave radiation from the atmosphere. Substituting eq. (8) into eq. (4) for $z = 0$ and subtracting $E_\infty^S$, the long-wave emission from the snowpack, yields the final result for $Q^S$, the net long-wave radiant energy input to the snowpack:

$$Q^S = E_\infty^S 2E_3 (\xi) + E_\infty^3 \{ 1 - 2E_3 (\xi) \} - E_\infty^S$$  (10)

where:

$$E_\infty^3 (\xi) = \int_0^1 \mu n^{-2} e^{-\xi/\mu} d\mu$$  (11)

The exponential integrals, $E_\infty^3 (\xi)$, are discussed in reference [7] and tables of $E_3$ are presented in reference [11].

In Figure 2 we plot the quantity $(Q^S + E_\infty^S)/E_\infty^S$ versus $\xi$ for $E_\infty^S/E_\infty^3 = 0.70$, and compare it with the results of a theory which represents the canopy by a plane [12]; $x$ is the canopy closure. There is good agreement between the two theories although there is not a one-to-one correspondence between $x$ and $\xi$; therefore, quantitative comparison is difficult. As $\xi \to \infty$, $(Q^S + E_\infty^S)/E_\infty^S \to 1$. The physical interpretation of this is that an increase in the amount of vegetation in a canopy does not result in a corresponding increase in the long-wave input to an underlying snowpack. Additional vegetation absorbs as well as emits radiation and a saturated condition is reached for $\xi > 2$.

**SOLAR RADIATION**

The equation for the solar intensity (eq. (2)) is an integrodifferential equation that can be solved, at least approximately, by applying various quadrature formulae to the right-hand side and reducing the equation to a set of coupled ordinary differential equations [1]. Limited space prevents discussion of the general case in which the canopy scatters radiation and there is an underlying reflecting snowpack. The problem of multiple reflections between canopy and snowpack has been discussed elsewhere for a plane canopy [12]. If we assume that $\gamma_S = 0$, then the equation for the downward intensity reduces to:

$$\mu dI_\infty^S + \beta I_\infty^S = 0 \quad -1 \leq \mu < 0$$  (12)

which can be immediately solved:

$$I_\infty^S (\mu, z) = I_\infty^S (\mu, d) e^{\beta (d-z)/\mu}$$  (13)
The boundary condition at \( z = d \) is:

\[
I_z^S(\mu, d) = F_o^S \delta(\mu - \mu_o) + D_o^S \frac{1}{2\pi} \delta \frac{1}{\pi}
\]

where \( F_o^S \) is the direct beam solar flux, \( D_o^S \) is the diffuse solar flux (assumed isotropic), \( \mu_o \) is the cosine of the angle between the incoming direct beam solar flux and the outward normal to the canopy, and the Dirac delta function \( \delta \), is defined by

\[
\int f(\mu) \delta(\mu - \mu_o) \, d\mu = f(\mu_o)
\]

where \( f \) is any function and the range of integration includes the value \( \mu_o \). We can also write \( \mu_o = \sin \theta_h \), where \( \theta_h \) is the angle between the solar beam and the earth's surface. The final result for the net solar input to the snowpack is:

\[
Q^S = D_o^S (1 - \rho_s) 2E_3(\xi) + F_o^S \sin \theta_h (1 - \rho_s) e^{-\xi/\sin \theta_h}
\]

where \( \rho_s \) is the snowpack albedo. The total downward flux incident on the canopy is:

\[
F_o^S = \frac{F_o^S}{\sin \theta_h} + D_o^S
\]

Utilizing eq. (17), we can write eq. (16) in the form:

\[
\frac{Q^S}{E_o^S(1 - \rho_s)} = \left( \frac{D_o^S}{E_o^S} \right) 2E_3(\xi) + \left( 1 - \frac{D_o^S}{E_o^S} \right) e^{-\xi/\sin \theta_h}
\]

In Figure 3 we have plotted the ratio of net solar input with the canopy to net solar input without the canopy, \( Q^S/E_o^S(1 - \rho_s) \), versus \( \xi \) for \( \theta_h = 90^\circ, 45^\circ, 15^\circ \), with \( D_o^S/E_o^S = 0.10 \) and compared it with the results of a plane canopy theory in which the canopy albedo is zero [12]. We have not included the fact that the ratio of diffuse to global solar radiation, \( D_o^S/E_o^S \), is also dependent on \( \theta_h \). There is generally poor agreement between the slab canopy model and the plane canopy model for solar radiation.

CONCLUDING REMARKS

Other attempts have been made to solve the problem of radiation transmission through a vegetative canopy for various canopy models [14, 15], but without, apparently, explicitly solving the equations of radiative transfer. However, the equations of transfer, especially the discrete space form [9], are the governing equations for radiative transfer processes, and any rational attempt to solve problems should include these equations, at least as an underlying theoretical framework. The equations of transfer can be modified to include a variety of boundary conditions and model canopies which can be progressively refined until satisfactory agreement with experiments is obtained.
An obvious extension of the methods presented is to include the effect of canopy scattering and reflection from an underlying snowpack. It will then be possible to calculate the canopy albedo as a function of vegetative area density, solar reflectivity, and solar angle, both with and without an underlying snowpack. In addition, the inhomogeneity of the canopy can be approximately accounted for by suitably modifying the absorption and scattering coefficients.

If a two- or three-dimensional representation of the canopy is necessary to predict the effect of various cutting practices (i.e., strips, blocks, etc.) on radiant energy input, the transport equations in their most general form become unwieldy and it is necessary to resort to the diffusion approximation [6, 8]. Comparison of one-dimensional solutions to the diffusion and transport equations will enable boundary conditions and extinction coefficients to be selected, which will give the best agreement between the two approaches. It should be possible, then, to solve two- and three-dimensional problems, at least approximately, and to estimate the error introduced by the diffusion approximation.

ACKNOWLEDGMENTS

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REFERENCES


![Diagram](Fig. 1. Model canopy)
Fig. 2. Long-wave radiation transmitted by canopy; $\frac{E_o}{E_C} = 0.70$
Fig. 3. Solar radiation transmitted by canopy; no canopy scattering; $D_o^S/E_o^S = 0.10$
DISCUSSION

L. Gold (Canada) - Have you had a chance to compare your predictions with measurements?

C.F. Bohren (U.S.A.) - No, not with all the necessary measurements of sun angle, vegetation density, etc.

J.L. Smith (U.S.A.) - One can easily visualize an enormous number of combinations of solar angle effects (depending on date and time of day), ventilation effects, density of timber, and other factors that would have to be considered in a real-life situation. How do you plan to include these factors into your revised model?

C.F. Bohren (U.S.A.) - What I have presented is a first-step model. I am confident that the model can be modified to take account of inhomogeneities.

L. Gold (Canada) - With your theory you can determine the albedo of a forest cover and its dependence upon the density of the forest. Would it be possible to verify your model using, for example, the forest albedo data collected by aircraft for the University of Wisconsin?

C.F. Bohren (U.S.A.) - Yes, this should be possible.