Water percolation through homogeneous snow

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ABSTRACT: The gravity-flow theory of water percolation through snow is generalized to include any power-law relationship between permeability to the water phase and effective-water saturation. Experimental observations of water percolation through homogeneous snow are described. It is found that the exponent in the power law is about 3 for homogeneous snow. The theory is used to construct diurnal meltwater waves and these compare favorably with the observed waves. The differences between the results found for natural snow and those found for repacked snow are discussed. The lower limit of applicability of the gravity-flow theory is uncertain.

RESUME: La théorie de l'écoulement par gravité à travers la neige est ici généralisée pour inclure toute loi en puissances exprimant la relation entre la perméabilité à l'eau et la saturation effective en eau. Les observations expérimentales de cet écoulement à travers la neige homogène sont décrites et la conclusion de ces expériences est que, pour la neige homogène, l'exposant est d'environ 3. La théorie est utilisée pour calculer des ondes de fonte diurnes qui sont en bon accord avec les ondes observées. La discussion porte ensuite sur les différences entre les résultats obtenus avec la neige naturelle et avec la neige retassée. On estime que la limite inférieure d'application de la théorie de l'écoulement par gravité est incertaine.

SYMBOLS

\( g \) acceleration due to gravity (m sec\(^{-2}\))
\( k \) total permeability (m\(^2\))
\( k_w \) permeability to the water phase (m\(^2\))
\( n \) exponent
\( S_w \) water saturation, water volume/pore volume
\( S^* \) effective water saturation, \((S_w - S_{wi})/(1 - S_{wi})\)
\( S_{wi} \) irreducible water saturation
\( t \) time (sec)
\( u_w \) volume flux of water (m sec\(^{-1}\))
\( U_{\text{max}} \) maximum value of volume flux at the surface
\( z \) depth below surface (m)
\( (dz/dt)_{S^*} \) speed of propagation of a wave of constant \( S^* \) (m sec\(^{-1}\))
\( (dz/dt)_{u_w} \) speed of propagation of a wave of constant \( u_w \) (m sec\(^{-1}\))
\( \alpha \) \( \rho g u_{w}^{-1}(5.47 \times 10^{6} \text{ m}^{-1} \text{ sec}^{-1}) \)
\( \rho \) density of water (Mg m\(^{-3}\))
\( \phi \) porosity, pore volume/total volume
\( \phi_e \) effective porosity, \( \phi(1 - S_{wi}) \)
\( \mu \) viscosity of water (Kg m\(^{-1}\)s\(^{-1}\))
INTRODUCTION

The importance of water flow through snow has been recognized for some time but little work of quantitative or predictive significance has been done. Colbeck [1] developed a theory to describe the vertical seepage of water through isothermal snow and data from the Upper Seward Glacier [2] were used to test the validity of that theory. The theory was found to be generally correct although Sharp's data was collected from natural snow and the effects of the inhomogeneous snow structures upon the distortion of diurnal-melt-water waves could not be isolated from the effects of the percolation processes.

In the development of the theory, it was necessary to postulate a relationship between the permeability to the water phase ($k_w$) and water saturation ($S_w$) because there was no experimentally derived relationship for snow-water systems (in fact the experiments might never be done because of the phase equilibrium problems). Based on Sharp's data, it was postulated that

$$k_w = kS^2,$$

where $k$ is the total permeability of the snow,

$$S^* = \frac{S_w - S_{w1}}{1 - S_{w1}},$$

and $S_{w1}$ is the "irreducible-water saturation" which is permanently retained by capillary forces. $S^*$ is called the "effective-water saturation" because it represents the part of the liquid water that is available for flow. When the available data were used to establish Equation (1), it was suggested that more carefully controlled experimental work would be necessary before the postulated relationship could be adopted. In the experiments described here, the unknown effects of the naturally occurring inhomogeneous features have been removed and a more definite basis established for the gravity-flow theory of water percolation through homogeneous snow.

THEORY

The theory can be generalized by taking

$$k_w = kS^n$$

instead of postulating a specific value for $n$. Now applying the two-phase Darcian flow concepts to the air-water-snow system and simplifying the equation by neglecting capillarity [1],

$$n\alpha kS^{n-1} \frac{\partial S^*}{\partial z} + \phi (1 - S_{w1}) \frac{\partial S^*}{\partial t} = 0,$$

where $\alpha = \rho gu^{-1}$, $\rho$ is water density, $\mu$ is water viscosity, $g$ is acceleration due to gravity and $z$ is the vertical coordinate that starts at the surface and is positive downward. The porosity, $\phi$, is assumed to be constant with depth although a porosity gradient was incorporated into the original theory.

This equation can be solved for any surface input by the method
of characteristics. The general solution is

\[ \frac{dz}{dt} = a_n k \phi_e \left( 1 - S^* \right)^{-1} \]

where \( (dz/dt)_{S^*} \) is the speed of propagation of a value of constant \( S^* \) and \( \phi_e \), the "effective porosity", is given by

\[ \phi_e = \phi (1 - S_w) \]

Thus \( \phi_e \) is that fraction of the pore volume that is available for flow. In terms of volume flux, where \( u_w \) and \( S^* \) are related by Darcy's law,

\[ u_w = a k S^* \]

the solution is

\[ \left( \frac{dz}{dt} \right)_{u_w} = n (ak)^{1/n} \phi_e^{-1} u_w \left( \frac{n - 1}{n} \right) \]

where \( (dz/dt)_{u_w} \) is the speed of propagation of a value of constant \( u_w \). For any given input at the surface, this equation can be used to describe the propagation of the boundary values into the interior of the snow mass.

\[ \text{EXPERIMENTS} \]

These experiments were designed to test the validity of the theory in homogeneous snow where the unknown effects of ice layers were excluded. The experiments were done on the South Cascade Glacier, a small valley glacier in the Cascade Mountains, Washington, U.S.A. A site was chosen where a sufficient depth of the previous winter's snow cover was present throughout July and August. To achieve homogeneous snow masses, long columns of repacked snow were set in the natural snow cover on the glacier and exposed to ablation at the surface. The surrounding snow ensured an isothermal environment and, at the surface, the columns were indistinguishable from the surrounding surface. A large-diameter, lightweight coring auger was developed at CRREL with which to bore large holes (0.235 m). Initially five holes were bored to depths of 1 to 5 metres and a long 3-mil polyethylene tube with a funnel attached at the bottom was lowered into each hole. Then the tubes were packed with disaggregated snow. This procedure produced uniform columns of snow with densities of about 0.6 to 0.65 Mg m\(^{-3}\). The holes were placed in a line 1 m from the north-facing wall of a pit so that access to the funnels and attached flow lines was possible at the bottom of the holes. The discharges from the flow lines, which were entirely due to inputs at the surface, were collected either in automatic rain gauges with continuous readouts or in calibrated devices that were read to the nearest 0.1 cm\(^3\) (10\(^{-7}\) m\(^3\)) every hour. Data were collected only during long periods of fair weather when diurnal-meltwater waves resulting from day-time melt could be observed. In general, approximately 12 hr of surface melting and 12 hr of surface freezing were observed each day, a situation that
simplified the reconstruction of the surface fluxes during the subsequent analysis.

The original five tubes were designated A through E (placed at approximately 1 through 5 m respectively). This arrangement was chosen so that propagation of the diurnal-meltwater waves could be observed at five different depths in order to examine the changes experienced by each wave as it passed through 5 m of snow. When the shortest column melted out, the pit was deepened and another 5-m column was added. This procedure caused difficulties because the characteristics of the new column were different from those of the older columns and the character of the movement of the meltwater waves was somewhat different for each tube. Also, the original 5-m column (E) was replaced because its performance was unsatisfactory and the original 4-m column (D) behaved irregularly much of the time. Most of the differences in the columns can be satisfactorily handled using techniques discussed later but the results from the 4-m column are not considered as reliable as those from the other columns. In addition to the water-flow data, air temperature and radiation-balance records were obtained from the U.S. Geological Survey station located on a rock ridge near the experimental site. These data were used to help formulate the boundary conditions to be discussed later.

The meltwater waves generated on 14 July, 26 July, and 8 August, 1971, are shown as examples (Figs. 1, 2, and 3). In general, all the data verify the characteristics of meltwater waves which the gravity-flow theory predicted:

1. The maximum flow rate decreases with depth;
2. The minimum flow rate increases with depth;
3. The leading edge of each wave distorts into a shape which can be approximated as a shock front;
4. The trailing edge of each wave becomes increasingly elongated with depth;
5. The shock front initially grows and then decays with depth until, at sufficient depth, steady-state flow occurs.

DRAINAGE

Although this qualitative verification of the theory shows that the wave distortion was predominantly the result of the percolation processes and not the natural inhomogeneous snow structures, much further work was necessary to test the postulates and assumptions made during the development of the theory. Therefore, at 2000 PST on 28 July, 1971, a drainage experiment was started by closing the tubes at the surface to prevent further influx. Then the columns were covered with tarps and several feet of snow to exclude solar radiation on subsequent days. Under these conditions of no further input of liquid water, drainage of the columns proceeded for several days, at which time the cover was removed and normal operation of the experiments was resumed.

The results of the drainage experiments are analyzed in such a way as to test the validity of Equation (4). First it is necessary to establish the surface-volume flux due to melting. A truncated-sine wave of surface flux as a function of time is assumed for lack of an accurate method of determining surface melt on an hourly basis. Although this choice is arbitrary, it is verified by some surface measurements discussed later, and in any case, only an approximation to the boundary condition is needed; as shown by Colbeck [1], melt-
water waves at depth are more sensitive to the effects of the percolation processes than to the exact shape of the boundary condition. Thus a truncated-sine wave is assumed where, for each column, the amplitude is calculated by using the total volume flux for the first 24 hr following the arrival of the wave generated on 28 July. The surface flux is assumed to peak at 1200 PST each day because both the air temperature and radiation balance peaked about then. Also, surface melting generally began about 0600 PST and surface freezing about 1800 PST. In Figures 4 and 5, $\Delta z/\Delta t$ is plotted against $u_w$ for each of the columns usable at that time ($\Delta z$ is the column length and $\Delta t$ is the transit time over $\Delta z$ of a particular value of $u_w$). If the theory is correct, the data points should form a straight line.

On Figure 4 the data for column B are shown. This relatively poor result can readily be attributed to effects which result from the short length of this column:

1. Penetration of solar radiation over a large portion of the column's length (before the column was covered),
2. Capillary-end effects propagate over a large portion of the column's length,
3. Surface subsidence caused a significant change in the length of the column during the last day's melt.

Figure 5 shows the best data, those for columns C, D, and E. By using Equation (4) to analyze the data,

$$\frac{d\Delta z}{dt} = n(ak)^{1/n} e^{-1} u_w (n-1)/n,$$

the slope of the line fitted to each set of data points is $(n-1)/n$ and the intercept of the line is $n(ak)^{1/n} e^{-1}$. Columns C and E are thought to be the most nearly representative of natural conditions (B was too short and D frequently gave inconsistent results). The average $n$ of these two columns is 3.05 whereas for all four columns it is 3.3 but an integer (3) is adopted for the sake of convenience (especially in calculating shock-front speed and concurrent work which is attempting to include capillarity into the theory). Also, this value receives fairly wide use in the general theory of flow through porous media (e.g., [3, 4, 5]). This relationship is of fundamental importance in the theory of water percolation in snow and, because it has not been determined in the laboratory, it is most important that the best form be deduced from these experiments. However because of the wide variation in the calculated values of $n$, no definite conclusion can be reached. A line corresponding to Equation (5),

$$k_w = kS^2,$$

is shown on Figure 5 and, based on the foregoing arguments, this relationship is adopted, although a slightly larger value was actually found from the drainage tests.

Colbeck [1] previously postulated $n = 2$, for snow with naturally occurring inhomogeneities. It seems unlikely that the occurrence of ice layers (which should cause further distortions of the waves) could cause an apparent decrease of the value of $n$. While drainage-tubes (see [6] for a discussion of these features) may have caused the observed wave distortions in natural snow, no
Table 1

Results from the drainage tests

<table>
<thead>
<tr>
<th>Column</th>
<th>Length</th>
<th>n</th>
<th>$k^{1/n} \phi^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.6 m</td>
<td>3.1</td>
<td>0.014 m²/n</td>
</tr>
<tr>
<td>C</td>
<td>1.8</td>
<td>2.8</td>
<td>0.00095</td>
</tr>
<tr>
<td>D</td>
<td>2.8</td>
<td>4.0</td>
<td>0.0144</td>
</tr>
<tr>
<td>E</td>
<td>3.95</td>
<td>3.3</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

definite conclusions about the effects of these inhomogeneous structures can be given at this time. Obviously further work will be required to test the applicability of this theory in natural-snow masses.

Where values of snow density ($\rho_s$) are available (the remaining columns were excavated upon completion of the experiments), the porosities and permeabilities are calculated (using an estimated value of water saturation). These values of permeability (see Table 2) are close to those ($k = 1.4 \cdot 10^{-10} m^2$) found by Kuroiwa [7] for snow with a similar porosity. Although these values are not necessary for the analysis of the results, it is encouraging that reasonable values can be calculated from this theory. Values of water saturation at the surface on 14 July are calculated for the two columns for which values of permeability are available. Equation (3), where $n = 3$, and the measured value of $S_w$ indicates these boundary conditions shown on Figure 6, along with the values of water saturation measured with a di-electric meter on the snow surface. Although neither of these methods gives definite values for water saturation, a reasonable qualitative agreement exists between them at least, and thus, the assumed shape of the boundary condition is verified.

COMPARISON

Based on the information currently available, Equation (5) is adopted for the relationship between $k_w$ and $S_*$. This postulate is now tested by constructing theoretical fluxes and comparing them with the measured fluxes. For each column, the boundary condition is constructed (using the procedure described above) and then values of $k^{1/3} \phi^{-1}$ are calculated. This is accomplished by choosing a value of $u_w$, calculating $\Delta z/\Delta t$ (for each column) for that particular value of $u_w$, and solving Equation (4) for $k^{1/3} \phi^{-1}$. This procedure is equivalent to matching the calculated and measured waves at one point. While this argument is cyclic, there is no other sufficiently accurate method of calculating values for $k^{1/3} \phi^{-1}$, even when the snow density is accurately known. Much further effort will have to be devoted to methods of measuring permeability, porosity, and saturation of water-bearing snow before this parameter can be accurately calculated for snow where no lysimeter data is available.

For the present, it is sufficient to compare the character of the computed and measured waves to decide whether the theory is capable of predicting the propagation of diurnal-meltwater waves through snow when the necessary parameters are already known. The data from 26 July, 1971 are used for this comparison. The calculated values of $k^{1/3} \phi^{-1}$ are shown in Table 2 and the curves of volume flux as a function of time are shown on Figures 7 through 11.
The method of characteristics was used to construct the solutions since the method of solution described by Colbeck [1] is only applicable when a vertical porosity gradient occurs. From the measurements of snow density made at the termination of these experiments, it was concluded that no significant decrease of porosity existed in the columns.

Table 2

<table>
<thead>
<tr>
<th>Column</th>
<th>Length (m)</th>
<th>$U_{\text{max}}$ (m sec$^{-1}$)</th>
<th>$k_{1/3}\phi_{e^{-1}}$ (m$^{2/3}$)</th>
<th>$\rho_s$ (Mg m$^{-3}$)</th>
<th>$\phi$</th>
<th>$k$ (m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.25</td>
<td>1.81\times10^{-5}</td>
<td>0.000724</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>1.06</td>
<td>1.64\times10^{-6}</td>
<td>0.00162</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>C</td>
<td>2.05</td>
<td>1.59\times10^{-6}</td>
<td>0.00178</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>3.15</td>
<td>1.25\times10^{-6}</td>
<td>0.00159</td>
<td>0.653</td>
<td>0.32</td>
<td>1.2\times10^{-10}</td>
</tr>
<tr>
<td>E</td>
<td>4.30</td>
<td>1.36\times10^{-6}</td>
<td>0.00196</td>
<td>0.623</td>
<td>0.36</td>
<td>3.2\times10^{-10}</td>
</tr>
</tbody>
</table>

Columns A and B (Figs. 7 and 8) are short and suffer from the effects discussed above. For A especially, with a length of only 0.25 m, capillary-end effects and radiation effects propagate over the entire length of the column thus reducing the effect of gravity drainage. Their influence is also seen in the low value of $k_{1/3}\phi_{e^{-1}}$ calculated for A. For B the comparison is reasonably good but the best comparison occurs for C. D and E are also favorable comparisons although D, as usual, is not as good as desired. In general, these comparisons are reasonably good and show that the theory, with $n = 3$, is essentially correct and is capable of making accurate predictions.

The most important discrepancy between the measured and calculated meltwater waves is the minimum flow rate. The measured values are consistently larger than the calculated values, a fact that cannot be easily explained. Also, this behaviour is inconsistent with the results from the drainage experiments (see Figs. 4 and 5), which suggest that the theory of gravity drainage is valid at very low values of flow rates. These high values of the minimum flow rate are not limited to the data for 26 July; no flow rates of less than $0.2\times10^{-6}$ m sec$^{-1}$ were observed (see Figs. 1, 2, and 3) except during the drainage tests. Several possible explanations, including capillary-end effects at the bottom of the columns and other boundary conditions, have been considered and rejected. Increasing the value of $n$ to 3.3 improves the comparison for A, B, and C only slightly and higher values of $n$ would be inconsistent with the results of the drainage experiments.

DISCUSSION

The gravity-flow theory is capable of making reasonably accurate predictions of the movement of water through homogeneous snow where the appropriate parameters of the snow and the input at the surface are known. In one case of natural snow, the theory is approximately correct when the effects of ice layers can be included by reducing the apparent value of permeability [1]. Much further work should be done to improve our understanding of the effects of inhomogeneous snow structures on the percolation processes.

The application of the theory at flow rates below $0.2\times10^{-6}$ m sec$^{-1}$
is uncertain. The analysis of the data from the upper Seward Glacier [1] and the drainage experiments on the South Cascade Glacier suggest that the lower limit of applicability of the theory was less than the observed flow rates (less than $10^{-7} \text{m sec}^{-1}$). However, the analysis of a diurnal meltwater wave on the South Cascade Glacier suggests that the theory is only roughly correct at flow rates below $0.2 \times 10^{-5} \text{m sec}^{-1}$. Because much of the percolation occurs at these low flow rates, further efforts should be devoted to improving our understanding of the flow processes at low flow rates.

Unfortunately the relationship between $k_w$ and $S^*$ cannot be definitely established from these experiments. A wide range of scatter exists in the calculated values of $n$ (see Table 1), but apparently $n$ lies between 3.0 and 3.3 and the value of 3.2 is suggested as being most nearly correct. For analytical purposes, the value of 3.0 is adopted because of the convenience of using an integer, especially for calculating shock front speed and for inclusion of the effects of capillarity into the theory (Colbeck, to be published). This value was tested and accurate predictions were made which, for any practical purpose, were just as good as predictions based on the value of 3.3.

The theory appears to be essentially correct, although major difficulties exist with the methods of measuring permeability, porosity, and irreducible-water saturation for water-bearing snow. Also, the accuracy of the prediction is limited by the accuracy with which measurements of surface melt can be made. Until more refined methods of characterizing the snow and surface fluxes exist, the present theory is sufficient for predictive purposes. Once more refined techniques have been developed, a re-examination of the theory may be in order.

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Special thanks go to Dr. Mark Meier, Mr. Wendell Tangborn, and the staff of the U.S. Geological Survey who generously furnished the use of the facilities on the South Cascade Glacier, much of the necessary equipment, and part of the financial support. Their encouragement and cooperation was essential to the successful completion of the field program.

REFERENCES


Fig. 1. Meltwater waves generated on 14 July 1971.

Fig. 2. Meltwater waves generated on 26 July 1971.
Fig. 3. Meltwater waves generated on 8 August 1971.

Fig. 4. Values of $\Delta z/\Delta t$ and $u_w$ for column B.
Fig. 5. Values of $\Delta z/\Delta t$ and $u_w$ for columns C, D and E and a straight line corresponding to $n = 3$.

Fig. 6. Measured and calculated boundary conditions.
Fig. 7. Measured and computed meltwater waves for column A.
Fig. 8. Measured and computed meltwater waves for column B.

Fig. 9. Measured and computed meltwater waves for column C.
Fig. 10. Measured and computed meltwater waves for column D.

Fig. 11. Measured and computed meltwater waves for column E.
DISCUSSION

C. Obled (France) - Je voudrais d'abord dire que nous nous sommes penchés sur les précédentes publications de S.C. Colbeck et que nous pensons que les différences entre l'onde observée et calculée pourraient être dues à la compression de la lame d'air présente dans la colonne. (cf. Vachand, Morel-Seytoux)

D'autre part, que pensez-vous du schéma très simple qui est parfois proposé, à savoir:
(1) Un écoulement par fil et ruissellement dans le manteau?
(2) Puis un ressuayage du type Darcy à la base du manteau?

S.C. Colbeck (U.S.A.) - I think that Dr. de Quervain's paper clearly showed that film flow is restricted to very low flow rates in snow. When you have very, very small flow rates which are much slower than those that I am considering, film flow is a reasonable approximation. At those flow rates that I am considering, I do not think you can simplify the problem to the extent of talking about film flow in capillary tubes but you have to be concerned with both the movement of water through films and the seepage of water drops through the snow. I did not "break-down" Darcy's Law to discriminate between these two processes because I did not think it would be terribly useful at this point in my work.

As for the movement of air in snow, I have not neglected it but incorporated it into the derivation of the form of Darcy's Law applicable to the water phase given here by assuming counter flow where the percolating down must be balanced by a displacement of the air upwards. I think this is a very good approximation in snow because of the lower boundary effects caused by capillarity. Although I did include the movement of air, I believe I am safe in saying this about the range of saturations which we see in snow; air pressures do not significantly change because air is relatively free to move through the pores of the snow and is not trapped in pockets where high pressures can build up. If one were concerned with much larger flow rates than I am looking at, then air pressure would have to be considered.

D.M. Rockwood (U.S.A.) - Have you checked your theory with the observations of runoff from the bottom of the snowpack, as made at the Central Sierra Snow Laboratory Snow Lysimeter? Those observations are documented for certain cases of clear weather snowmelt in Chapter 5 of "Snow Hydrology". Those observations were made in the mid-1950's.

S.C. Colbeck (U.S.A.) - Yes, I have used some of these data but the only test that comes immediately to mind is the one where rain was put on the snowpack in the form of a step function. I have modeled this condition and found very good correlation between theory and experiment.