ESTIMATION OF DESIGN FLOODS AND THE PROBLEM OF EQUATING THE PROBABILITY OF RAINFALL AND RUNOFF

by

M.A. Beran


ABSTRACT

Where data on river discharge are scarce it is a common engineering design practise to concoct a design flood with the aid of rainfall depth-duration-frequency information and a catchment response model. Two major weaknesses of this approach are (a) the problem of the sensitivity of the design to legitimate changes in the design assumptions and (b) the uncertainty of preserving the nominal rainfall return period in the design flood. A solution to these problems is proposed which makes use of a computer simulation investigating the sensitivity of flood magnitude to variations in return period, storm duration, temporal rainfall intensity pattern, infiltration loss rate, base flow and unit hydrograph shape. An extension to the sensitivity analysis allows an estimate to be made of any quantile of the distribution of flood magnitude based on sampling across all causative rainfall and antecedent conditions.

RESUME

El est courant, lorsque les données sur les débits sont insuffisantes, que l'ingénieur élabore la crue de projet à partir de l'information qu'il possède sur la distribution des pluies, en utilisant un modèle de transformation pluies-débits. Les deux inconvénients majeurs de ce procédé concernent (a) la sensibilité de l'aménagement à la variation des paramètres du projet, (b) la conservation de la période de retour (ou de la probabilité) lorsqu'on passe de la pluie de projet à la crue de projet. L'auteur propose une solution à ces problèmes, en utilisant une simulation pour rechercher la sensibilité de la grandeur de la crue aux variations de la période de retour, de la durée de l'averse, de la configuration du hyétogramme, de la capacité d'infiltration, du débit de base, de la forme de l'hydrogramme unitaire. Une extension de cette analyse de la sensibilité permet d'estimer n'importe quelle quantité de la distribution des crues, en se basant sur un échantillonnage des pluies et des conditions antécédentes.
1. INTRODUCTION.

Modern engineering practise requires the investigation of the likely behaviour of a proposed construction under extreme conditions. Because scarcity of data is the rule rather than the exception in the hydrological field it is commonly necessary to resort to indirect design procedures, for example the use of rainfall probability information with a catchment response model, to derive a design flood. One very common design procedure uses depth-duration-frequency information to construct a design storm of some nominal return period which is transformed to a design flood by means of a synthetic or observed unit hydrograph. Although conventionally the design flood is associated with the same return period as the storm which caused it it is clear that many sources of variation which might influence the return period, such as those due to storm duration, temporal and areal distribution of rainfall intensity, infiltration losses and base flow- are in fact ignored. The design engineer, in recognition of this fact must make stringent assumptions about these variables in order not to inadvertently reduce the return period of the flood.

It is relevant, therefore, to enquire into the effect of the ignored variation by posing such questions as (a) what is the expected flood following the T-year return-period rainfall?; (b) what is the sensitivity of the design flood to alterations to the assumed values of the variables?; (c) is there a single combination of values that can be assumed in order to approximate the expected flood?

An approach to the solution of such problems is offered in this paper. A simulation technique is described which samples the possible ways in which a rainstorm of T-year return-period can cause floods, and derives their probability distribution. (Section 2).

While this sensitivity aspect is relevant to current design practise, perhaps of greater interest is the possibility of using simulation technique to estimate the flood magnitude-frequency relationship*. The proposed method must therefore be extended to consider floods resulting from all maximum rainstorms and not merely from those of the nominal return-period (Section 3).

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* Nash (1) postulated a flood design procedure based on combining the incidence of flood producing factors. Chow and Ramaseshan (2), Evans (3) and Dyck and Kluge (4) present techniques conceptually not dissimilar to the one presented here. Recent work by Eagleson (5) and Leclerc and Schaake (6) approach the same problem in part analytically.
2. THE SAMPLING PROCEDURE.

The procedure follows closely the steps used to estimate the design flood.

(a) Determine a nominal return period

(b) Choose a storm duration and calculate the total depth of rainfall from the depth-duration-frequency relationship.

(c) Distribute the total rainfall within the duration to form the gross rainfall hyetograph.

(d) Subtract from this an infiltration loss to form the net rainfall hyetograph.

(e) Convolute the net rainfall hyetograph with the unit hydrograph to form the design inflow hydrograph.

(f) Process the inflow hydrograph and extract the particular flood magnitude measure of interest.

In practical engineering application an arbitrary single choice is made at each step (a) to (f); in the procedure described in this paper, however, the choice is made from a selection of possible values, each one with a frequency proportional to its probability of occurrence. Figure 1 illustrates the procedure as a tree diagram on which the "single choice" method would be represented by a single path.

As implied in figure 1 the continuous distributions of variables such as rainfall duration are "discretized" so that each variable is made to assume only one of a finite number of possible values to each of which a probability weight is attached. Twelve values of duration, 36 temporal intensity patterns and 12 values of catchment wetness index (CWI — and index of antecedent conditions governing infiltration loss and base flow) are used. In a separate study to provide rainfall information (Appendix 1) no dependences were noted between the rainfall variables and this assumption was made throughout the simulation. This means that the weights associated with each sampled variable was itself invariable; for example the weights associated with each of the 12 CWI values is the same for 3 hour as for 48 hour duration storms. This particular consequence might represent some departure from actuality as, in the United Kingdom, both are seasonal variables.

However assuming independence and discretizing allowed considerable simplification in the programming and allowed the associated weights of each of the $12 \times 12 \times 36$ combinations to be calculated from the product of the weights of each of the contributing variables. This product weight is associated with the flood magnitude in calculating statistics or assembling data into histograms.

To summarize, let $p_i$ be the weight (or probability) of the $i$th duration, $D_i$; let $q_j$ be the weight (or probability) of the $j$th hyetograph distribution,
let \( r_k \) be the weight (or probability) of the \( k \)th CWI, \( C_k \); and let \( Q_{ijk} \) be the flood magnitude resulting from the combination of \( D_i \), \( H_j \) and \( C_k \). Then under the assumption of independence the weight or probability to be associated with \( Q_{ijk} \) is \( W_{ijk} = p_i q_j r_k \) and the expected flood magnitude is calculated from \( \sum_i \sum_j \sum_k p_i q_j r_k Q_{ijk} \), while the mean flood magnitude following all storms of say the fourth duration is calculated from \( \sum_k W_{ijk} \) (Figures 2A and 2B).

Table 1 shows the results of the simulation for the 10-year return-period at Burbage and Grendon. The contingent distributions show the effect of different assumed values on the peak discharge. One noticeable result is that changes to the rainfall variables have small effect on the average peak discharge showing that the design flood would be insensitive to variations in hyetograph pattern or storm duration. This is not to say that floods resulting from storms following particular combinations of duration and hyetograph pattern cannot be found that depart from the average, but as can be seen from the low standard deviations of peaks contingent on chosen CWI values centrally chosen rainfall variables will introduce little bias into the design flood. It has been found that this same effect is even more marked when the measure of flooding being investigated involves some element of storage.

On the other hand, small changes in the CWI have a marked effect on the resulting flood. It happens that a CWI value chosen to be near the median of the distribution of CWI would have yielded a peak discharge only 5% in excess of the expected flood.

Figure 3 shows some of the histograms of flood peaks following the 100-year storm. These are noticeably negatively skewed and the modal value is typically 20% to 30% in excess of the mean. The inference from this is that a single choice of each of the variables is likely to yield a flood that exceeds the average flood. The sharpness of the histograms contingent upon CWI and the discrete sampling is responsible for the spikey nature of the other histograms.

3. RAINFALL AND DISCHARGE DISTRIBUTIONS.

It had been noted in Section 2 and Figure 3 that the probability distribution of floods following rainfalls of fixed return period is negatively skewed. One might anticipate from this that T-year return-period storms tend on average to give rise to more floods with return period less than T-years than floods of return period greater than T-years.

To test this and to derive the flood distribution the simulation was generalised to sample the distribution of storm depths. Instead of sampling only storms of depth and duration such as lie on a line of equal return period the sampling is now conducted across all combinations of storm depth and duration. The depth-duration-frequency is again used in order to calculate the probability of occurrence of any combination (Figure 2C).

Figure 4 shows a comparison between the flood frequency relation as derived
from the two simulations and from recorded flood peaks. In the case of Grendon Underwood there is an apparent tendency for the simulated relation to underestimate the flood discharge based on the recorded peaks. although independent evidence from regional analyses has suggested that the distribution as estimated from the six only annual maxima would overestimate floods quite severely. However the agreement with Burbage Brook, a small upland catchment in the Derbyshire pennines with 43 years of data, is rather better. At small return periods the generalised simulation produced lower flood values than the expected flood following storms of that same return period.

4. CONCLUSIONS.

A technique has been described whereby the solution of several problems pertinent to hydrological design in regions of inadequate data may be approached. In particular, the sensitivity of the design flood to design assumptions can be assessed. Experience with the technique suggests that the size of the flood is determined more by the total depth of the rainfall than by its temporal distribution through the storm's duration. Correct choice of loss rate is in consequence most important.

It appears that median values of duration, temporal distribution and loss rate yield a design flood not far removed from the overall average flood following the T-year storm. Because of the skewed nature of the flood distribution a random choice of duration etc. would be more likely to yield a design flood rather larger than the overall average.

The ability of the technique to reproduce tolerably well the flood magnitude frequency relation could be of very great value at a site where flow data are scarce, whilst even at a well-endowed location the simulation result may be used with profit to augment the flow record.

While attention has been concentrated on peak discharge as the measure of flooding it should be emphasized that the technique is suited to more complex design criteria. The hydrograph may be treated as an inflow and routed through the scheme and so the actual design criteria of interest may be calculated. Examples are:— (a) volume between inflow and outflow hydrographs for reservoir freeboard design; (b) time to peak for a flood warning scheme; (c) volume over a threshold level for a levee design.

The technique may also be adopted to use an entirely different catchment response model such as that inherent in the rational formula, a multiple regression equation or conceptual model although it can be expected that the data requirements will be rather different from those of this investigation.

5. FUTURE RESEARCH.

The simulation appears promising as a tool for assessing the sensitivity of design floods to variations in their causative factors and in estimating the magnitude-frequency relationship for small return periods. However the technique has not succeeded in reproducing the observed rapid growth in flood discharge with increasing return period and it is here that further research is being
directed.

It is felt that the disparity between the definition of storms used to determine the distribution of depth and duration (Appendix A - Introduction) could be responsible for the "slow" growth and so long term autographic rainfall records are to be analysed to provide information on the distribution of the type of storm used for the duration statistics.

Further investigation into dependencies between the variables could produce results which would affect the discharge distribution. For example seasonal simulation would reduce the coincidence of winter storm types with low summer CWI's and vice versa.

The dependence of CWI on losses and base flow is essentially statistical and this source of variability could be preserved in the simulation by the addition of a random quantity to the values predicted from the best fit lines.

6. ACKNOWLEDGEMENTS.

Although few references have been cited the labours and opinions of others have played no small part in the development of the procedure. Colleagues and consultants of the United Kingdom Floods Studies Team Dr. J.V. Sutcliffe, Professor J.E. Nash, Mr. M.J. Lowing, Mr. C. Cunnane, Mr. R.T. Clarke and Mr. A.F. Jenkinson have all provided advice and encouragement. Mrs. J. Haworth was responsible for the FORTRAN computer program and the numerical experiments were run on the ICL 1906A of the Science Research Council's computing laboratory.

7. REFERENCES.


### TABLE 1

**FLOOD DISCHARGE FOLLOWING 10-YEAR RETURN PERIOD STORMS.**

<table>
<thead>
<tr>
<th></th>
<th>Grendon Underwood</th>
<th>Burbage Brook</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td><strong>STANDARD</strong></td>
<td><strong>MAXIMUM</strong></td>
</tr>
<tr>
<td>m³/s</td>
<td>m³/s</td>
<td>m³/s</td>
</tr>
<tr>
<td><strong>DISCHARGE</strong></td>
<td><strong>DISCHARGE</strong></td>
<td><strong>DISCHARGE</strong></td>
</tr>
<tr>
<td>Overall</td>
<td>5.9</td>
<td>2.0</td>
</tr>
<tr>
<td>1 hour</td>
<td>4.5</td>
<td>1.6</td>
</tr>
<tr>
<td>3 hour</td>
<td>5.7</td>
<td>2.0</td>
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<td>6 hour</td>
<td>6.2</td>
<td>2.1</td>
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<tr>
<td>9 hour</td>
<td>6.2</td>
<td>2.0</td>
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<td>12 hour</td>
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<td>2.0</td>
</tr>
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<td>15 hour</td>
<td>6.0</td>
<td>1.9</td>
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<td>18 hour</td>
<td>5.9</td>
<td>1.8</td>
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<td>1.7</td>
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<td>24 hour</td>
<td>5.0</td>
<td>1.5</td>
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<td>30 hour</td>
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<td>36 hour</td>
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<td>4.1</td>
<td>1.2</td>
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<td><strong>Constant Quartile Type</strong></td>
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<td></td>
</tr>
<tr>
<td>I</td>
<td>5.8</td>
<td>2.0</td>
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<tr>
<td>II</td>
<td>6.0</td>
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<td>5.8</td>
<td>2.0</td>
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<tr>
<td><strong>Constant CWI</strong></td>
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<tr>
<td>15 50*</td>
<td>1.6</td>
<td>0.2</td>
</tr>
<tr>
<td>35 60</td>
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<td>3.9</td>
<td>0.4</td>
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<td>70 80</td>
<td>4.7</td>
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<td>80 90</td>
<td>5.2</td>
<td>0.6</td>
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<tr>
<td>90 100</td>
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<td>110 120</td>
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<td>130 140</td>
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<tr>
<td>140 150</td>
<td>8.3</td>
<td>0.9</td>
</tr>
<tr>
<td>150 165</td>
<td>9.4</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Recorded data**

- Graphical fit: 12.0
- Max. Likelihood: 13.1

*First figure refers to assumed CWI at Grendon, second to Burbage Brook.*
INTRODUCTION.

Statistical distributions were required for the three modes of rainfall variability: depth, duration and temporal variability for each catchment investigated. In order not to predetermine any of the variability modes it was necessary to define a storm in a manner unlike that of the customary rainfall depth-duration-frequency diagram. The definition was expressed in terms of the conditions for starting and ending a storm: a storm was considered to begin at the onset of rain and to end when in the preceding Y hours not more than X mms of rain occurred. X and Y were chosen to represent the conditions under which a flood hydrograph would return to near base flow and allowed short spells of zero rainfall to occur within a storm event.

Hourly analysis of catchment average rainfall was available from three catchments; Grendon Underwood, Coalburn and Plynlimon (Wye). Sufficient records were available to permit an investigation into statistical distribution of storm durations and temporal patterns but not to conduct an investigation into storm depth. For this element of the simulation, results of a depth-duration-frequency analysis of the entire country were available from A.F. Jenkinson (Ref. A1).

The catchment response model is one currently under investigation by the Floods Study Team. A relation between catchment wetness index (CWI) and total storm losses, and CWI and base flow was used. A unit hydrograph based on recorded unit hydrographs from the catchments were convoluted with the gross rainfall less losses.

DETAILS OF THE SIMULATION DATA.

(a) Rainfall depth: The basic equation used to relate the T-year return period rainfall of any duration (MT) to that of the five-year return period rainfall (M5) is

\[ MT/M5 = (T/5)^c \]

where c is the "growth factor" and is related uniquely to M5 which is mapped for the entire United Kingdom. Other necessary information required by the simulation and provided in Ref. A1 concerns areal reduction factors to convert point to areal rainfall.

(b) Rainfall duration: This distribution is dependent upon the storm definition and for the values X = 2 mms, Y = 5 hours used for both Grendon Underwood and Burbage Brook simulation is given below

<table>
<thead>
<tr>
<th>STORM DURATION (HOURS)</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
<th>30</th>
<th>38</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>RELATIVE FREQUENCY (PER CENT)</td>
<td>5</td>
<td>12</td>
<td>26</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
It was found that the distribution was very similar for both upland and lowland rainfall stations and varied slowly with changes to X and Y, longer storms becoming commoner as the conditions for ending a storm were relaxed.

(c) Temporal distribution of storm rainfall: Several alternative schemes for describing the hydrograph shape were investigated. The one chosen was due to F. Huff (Ref. A2) in which four quartile types are recognised depending upon in which of the four quarters of the storm duration the largest rainfall fell. The fine detail of the hyetograph is sampled by plotting all curves of the same quartile type on a graph showing accumulating fraction of storm depth against fraction of total storm duration. Composite storms can then be constructed by connecting points which are exceeded by 10%, 20%, 30% etc. of all storms. Sampling from these composite storms is analogous to sampling at regular intervals from a distribution function in order to sample a variable in proportion to its frequency of occurrence and were used by the simulation. The shapes of the composite storms were found to be insensitive to changes to the storm definition and were nearly indistinguishable between upland and lowland catchments. The percentage frequency of the four quartile types were 12% type I, 32% type II, 35% type III, 21% type IV.

(d) CWI distribution: CWI is calculated in mms. from the soil moisture deficit (SMD) as computed by the Meteorological Office (Ref. A3) and a five day antecedent precipitation index (API5) using a daily decay constant of 0.5. The formula used was \(\text{CWI} = 125 - \text{SMD} + \text{API5}\). It had been observed in a recent study (Ref. A4) that wet day rainfall and SMD were statistically independent and so the end of month values were adopted as representative of all CWI values. Oxford data was used to provide the distribution for Grendon Underwood and Buxton for Burbage Brook. In the simulation a linear relation with CWI was used to calculate total storm losses and the reciprocal of the temporal variation of CWI as the storm progresses (assuming no evaporation to increase SMD) determined the loss rate curve. An exponential relationship with CWI determined the base flow.

REFERENCES.

Consider case where two variables only affect discharge $Q$, for example storm duration and CWI (Figure 2A).

For each combination of duration and CWI a value of $Q$ and a probability of occurrence can be calculated. For example combining the duration in the fourth interval, $D_4$, with the CWI in the second interval, $C_2$, a discharge $q(H)$ and a probability $p(H) = p(D_4)p(C_2)$ are found.

Summing all the probabilities in each discharge interval a discharge distribution (Figure 2B) may be constructed.

This concept can be generalised to sample from further variables.

NOTES

a) Depth and duration are plotted on the base plane (Figure 2C).
b) Each combination is associated with a probability of occurrence as given by the depth-duration-frequency diagram.
c) Contingent on each depth-duration combination a distribution of discharges like Figure 2B can be visualized on the vertical discharge axis.
d) Integrating such densities above all points on the base plane on a locus of equal return period yields the results of Section 2.
e) Integrating over the entire base plane yields the distribution of discharge of Section 3.
Burbage Brook—Floods following 100-year Rainfalls
Distribution of all floods
Floods from storms of given duration
Floods from storms of given CWI

FIGURE 3
Most likely peak following storms of given return period

Mean annual flood

Recorded flood peaks

Simulated flood peaks

Expected peak following storms of given return period

Peaks have been standardised by the arithmetic mean of the recorded annual maxima 5.39 m³/s.
Plotting position corresponds to expected value of order statistic.
Graphical fit to plotted points so 'recorded' line misses (1,1).

FIGURE 4