A STUDY ON MAXIMUM FLOOD DISCHARGE FORMULAS

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ABSTRACT

This paper describes a new formula for the calculation of approaching velocity of rain water, and a number of new formulas for the estimation of maximum flood discharge which have been developed by the author.

Many empirical formulas, which have limited application, exist. However, in devising his formulas, the author derived theoretically the form of the basic maximum discharge formula for the case of rivers with no tributaries, and determined stochastically the value of the coefficients in his basic formulas using the records of observed measurements. Then the author derived theoretically many different formulas for the case of rivers with tributaries to fit in the actual localities of the site under consideration, besides the basic formulas. So the author's formulas would be widely applicable for rivers or sewer nets, and also for any regions, countries, with different locality. The author could confirm these facts through the numerical examples. The author's formulas may be used not only for estimating the design flood, but also in flood routing. The author believes that his formulas would be very helpful in the planning of water resources development projects especially for those with inadequate data.

RESUME

L'auteur présente une nouvelle formule pour la vitesse de concentration d'un bassin et en suggère d'autres pour le débit de la crue maximale.

On trouve de nombreuses formules empiriques dans de nombreux manuels, mais ces formules sont d'une application limitée. L'auteur parvient cependant à asseoir la forme de sa formule sur des bases théoriques, lorsqu'il s'agit de cours d'eau sans affluents; il procède à l'évaluation des paramètres qu'elle contient par ajustement statistique aux données d'observation disponibles. Il généralise ensuite à différents cas de cours d'eau avec affluents. Les formules proposées devraient pouvoir être appliquées n'importe où, aussi bien pour les cours d'eau naturels que pour les réseaux d'assainissement; c'est ce que l'auteur peut confirmer par des applications numériques. Les formules peuvent servir non seulement au calcul des crues de projet, mais aussi à celui de la propagation des crues. L'auteur pense que ses formules devraient rendre de grands services dans la planification de l'aménagement des eaux, spécialement lorsque les données disponibles sont insuffisantes.

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I. INTRODUCTION

Charles F. Ruff* defined "maximum probable flood" as follows. "The maximum probable flood does not mean the largest flood possible but a flood so large that the chance of its being exceeded is no greater than the hazards normal to all of man's activities." The author will use here the term of "maximum flood discharge" with the same meaning of "maximum probable flood" as defined by Ruff.

It is very important to calculate maximum flood discharge correctly, and also it is a very difficult problem theoretically and practically. It may be impossible to establish a plan for flood control and water resources development or sewer nets projects without reckoning correctly the maximum flood discharge or the design flood.

There are many methods for calculation of maximum flood discharge, and we have to adopt the most suitable method in accordance with the completeness of the data. However, the method of calculation by the maximum flood discharge formulas, especially for the case of those with inadequate data, is easy and simple for practicing engineers. There are many empirical formulas devised by many authors such as Kuichling, Mead, Kresnik, Dickens, Metcalf and Eddy, Brix, Lauterburg, Possenti, Buerkli-Ziegler, Dr. Hisanaga, Kajiyama, and many others. These old formulas have been devised empirically and have limited application. It will be clear that one may be unable to apply them generally. Also it will not be strange to obtain results which may be 10 or 1/10 times of the correct values, according to selection of the coefficients in these formulas when these formulas are actually applied to practical problems.

Generally speaking, the flood discharge depends upon the shape of catchment, drainage area, amount of rainfall and the position attacked by the heavy rainfall, permeability, slope of the catchment, shape of the water course, status of the surface, geological status, etc. Strictly speaking, such status of catchment differs from others from season to season, for every flood, even in the same catchment as well as in different drainage basins. In other words, flood discharge depends also upon the intensity of rainfall which causes the flood, duration of the rainfall and the position of the center of the lows, or status of the ground in case of heavy rainfall, viz., dry ground or saturated ground, etc.

As the maximum flood discharge depends upon many factors, as stated above, it may be very difficult to express it in a formula. However, if we can consider theoretically correct value of approaching velocity of rain water and intensity of rainfall, we may deduce the maximum probable flood by getting the rainfall for a certain district. The principle of derivation of the author's formulas belongs to this process, and it may be said that this is an approach different from many scholars who had derived the old formulas.

* Ruff, Charles F.; "Maximum probable floods in Pennsylvania Streams", Transactions, American Society of Civil Engineers. Vol. 106, 1941, p. 1153
In the first step, the author thought out a method to ascertain correctly the approaching velocity of rain water. At the same time, the author found that the Rizha's (Germany) formula, the only complete one for this purpose, could not be applicable to solve practical problems as it gives too small values. In the second step, the author studied the rainfall intensity curve comprehensively, and found out theoretically when the maximum flood discharge may occur. In the third step, the author has theoretically derived the maximum discharge formulas for rivers with many tributaries by applying the general rules which he has determined by the first and second step. In the fourth step, the author determined the discharge coefficient in his formula from the actual records. The author was then able to calculate the value of the discharge coefficient, with a great degree of accuracy, of the rivers in Korea and Manchuria.

II. APPROACHING VELOCITY OF RAIN WATER

The approaching velocity of rain water (Ω) is defined as the mean velocity of rain water approaching from the farthest point F in a river basin to the point O where the maximum flood discharge is to be ascertained, in other words, the mean velocity of flow between F and O (Fig-1). There is only one formula to find such approaching velocity so far expressed in equation, given by Rizha, Germany, and a table given by Kraven, Germany.

1) RIZHA'S FORMULA

\[
\omega = 72 S^{0.6}
\]  (1)

where

\[
\omega = \text{Approaching velocity of rain water (km/hr)}
\]

\[
S = \frac{H}{L}
\]

\[
H = \text{Difference of elevation of height between O and F}
\]

\[
L = \text{Distance of OF (Length of water course)}
\]

2) KRAVEN'S TABLE

<table>
<thead>
<tr>
<th>S (km/hr)</th>
<th>above 0.01</th>
<th>0.01 - 0.005</th>
<th>below 0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega) (km/hr)</td>
<td>12.6</td>
<td>10.8</td>
<td>7.56</td>
</tr>
</tbody>
</table>

Kraven had expressed only about approximate limits of \(\omega\), the author tried to formulate his table, to pass through the medium points as follows.

\[
\omega = 135 S^{0.6/5}
\]  (2)

3) THE AUTHOR'S FORMULA

The author successfully devised a new method to determine the approaching velocity of rain water \(\omega\), theoretically which may be applicable for rivers where the hydrographic surveying was completed. Neglecting the principle and the process of derivation here, the result of the author's formula is illustrated as follows.

\[
\omega = \kappa S^{0.6}
\]  (km/hr)  (3)
The value of $\kappa$ for rivers in Manchuria calculated using eq(3) is shown in Table-I. As we see Table-I, the value of $\kappa$ lies between 133 and 177, and we may be able to recognize that the Rizha's formula will not be of practical use because of the reason that the value of $\kappa$ in his formula is too small, apparently, compared with the normal values. We can also see from Table-I that the value of $\kappa$ represents the bottom slope or the hydraulic slope of the river, and this fact coincides with practice. Also it can be seen that the value of $\kappa$ decreases gradually according as approaching the downstream of a river, and this fact shows that the bottom slope or the hydraulic slope of a river generally decreases gradually as we approach the downstream.

III. RAIFALL INTENSITY CURVE

We can express the rainfall intensity by the following equation.

$$ I = \beta/(t + \alpha) \quad (4) $$

Table-I. The values of $\kappa$ and others for rivers in Manchuria

<table>
<thead>
<tr>
<th>Name of Rivers</th>
<th>$\kappa$</th>
<th>Range of S (°/oo)</th>
<th>$F$ (km$^2$)</th>
<th>$L$ (km)</th>
<th>$F/L$</th>
<th>$F/L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tumen R.</td>
<td>277</td>
<td>4.82 - 8.06</td>
<td>33,400</td>
<td>487.6</td>
<td>68.50</td>
<td>140</td>
</tr>
<tr>
<td>Whancheng R.</td>
<td>207</td>
<td>6.72 - 9.64</td>
<td>4,000</td>
<td>160.7</td>
<td>24.92</td>
<td>152</td>
</tr>
<tr>
<td>Roha R.</td>
<td>177</td>
<td>3.54 - 4.22</td>
<td>31,455</td>
<td>444.0</td>
<td>70.84</td>
<td>159</td>
</tr>
<tr>
<td>Searamorin R.</td>
<td>171</td>
<td>2.81 - 2.97</td>
<td>29,927</td>
<td>412.0</td>
<td>72.64</td>
<td>176</td>
</tr>
<tr>
<td>Sealeog R.</td>
<td>149</td>
<td>8.37 - 3.48</td>
<td>51,165</td>
<td>767.6</td>
<td>66.71</td>
<td>086</td>
</tr>
<tr>
<td>Tongleog R.</td>
<td>207</td>
<td>1.70 - 3.16</td>
<td>10,318</td>
<td>333.5</td>
<td>313.13</td>
<td>093</td>
</tr>
<tr>
<td>The upstream of the main Leog R.</td>
<td>150</td>
<td>1.83 - 2.30</td>
<td>178,699</td>
<td>1040.5</td>
<td>171.74</td>
<td>065</td>
</tr>
</tbody>
</table>

The middle of the Leog 158 1.62 - 1.77 187,250 1199.0 157.49 132
Cheng R. 159 3.90 - 5.51 4,958 163.0 30.42 186
Kwang R. 137 4.36 - 5.26 2,129 94.0 22.65 231
Van R. 149 7.07 - 18.66 1,072 102.5 10.46 102
Po R. 150 3.68 - 6.06 2,361 178.0 13.26 074
Senkai R. 134 1.43 - 1.96 515 51.0 10.10 198

$F$: catchment area
$L$: length of main water course

where

- $t$: Duration
- $I$: Average intensity of rainfall during duration $t$
- $\alpha$, $\beta$: Any constant

Eq(4) represents a kind of hyperbola, and the constants $\alpha$ and $\beta$ can be found by eq(5) by the principles of the method of the least square.

$$ \alpha = \frac{n(I^2 t) - (I)(It)}{(I)^2 - n(I^2)} \quad (5) $$

$$ \beta = \frac{(I)(I^2 t) - (It)(I)}{(I)^2 - n(I^2)} $$

$n$: number of observations

Next let $R$ be total amount of rainfall during the duration $t$,

$$ R = I t = \beta t/(t + \alpha) \quad (6) $$
Table-2 illustrates the values of the constants $\alpha$ and $\beta$ in eq(4) for various regions. In this table, those for the regions of Korea and Manchuria show the absolute maximum rainfall intensity curves during those periods. Those for the regions marked with the asterisk (*) were calculated by the author himself by the records of the recording gauges.

### IV. THE AUTHOR'S MAXIMUM FLOOD DISCHARGE FORMULAS

1) **FUNDAMENTAL FORMULA FOR THE CASE OF A RIVER WITH NON-TRIBUTARY**

**Table-2. The values of $\alpha$ and $\beta$ in eq(4)**

<table>
<thead>
<tr>
<th>Region</th>
<th>$\alpha$ (min)</th>
<th>$\beta$ (hour)</th>
<th>Period taken</th>
<th>Range of t the records (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seoul, K</td>
<td>59</td>
<td>0.938</td>
<td>1905 - 1920</td>
<td>5-60 min</td>
</tr>
<tr>
<td>Inchon, K</td>
<td>37.5</td>
<td>0.625</td>
<td>1914 - 1920</td>
<td>ditto</td>
</tr>
<tr>
<td>Pyangyang, K</td>
<td>41</td>
<td>0.683</td>
<td>1914 - 1935</td>
<td>10min-24hr</td>
</tr>
<tr>
<td>Pusan, K</td>
<td>106.1</td>
<td>1.777</td>
<td>1914 - 1920</td>
<td>5-240 min</td>
</tr>
<tr>
<td>Wonsan, K</td>
<td>75</td>
<td>1.250</td>
<td>1914 - 1920</td>
<td>5-240 min</td>
</tr>
<tr>
<td>Taegu, K</td>
<td>40.2</td>
<td>0.67</td>
<td>1929 - 1953</td>
<td>10min-24hr</td>
</tr>
<tr>
<td>Chonju, K</td>
<td>81.1</td>
<td>1.35</td>
<td>1918 - 1954</td>
<td>ditto</td>
</tr>
<tr>
<td>Kwangju, K</td>
<td>90.4</td>
<td>1.51</td>
<td>1938 - 1954</td>
<td>ditto</td>
</tr>
<tr>
<td>Mokpo, K</td>
<td>101.8</td>
<td>1.70</td>
<td>1916 - 1953</td>
<td>ditto</td>
</tr>
<tr>
<td>Changoheng, M</td>
<td>40.5</td>
<td>0.675</td>
<td>1937 - 1943</td>
<td>10min-48hr</td>
</tr>
<tr>
<td>Sping, M</td>
<td>45.1</td>
<td>0.752</td>
<td>1934 - 1944</td>
<td>ditto</td>
</tr>
<tr>
<td>Tokyo, J</td>
<td>50</td>
<td>0.833</td>
<td>1891 - 1911</td>
<td>5-60min</td>
</tr>
</tbody>
</table>

where $a = \alpha/60$  $b = \beta/60$  $K=Korea$  $M=Manchuria$  $J=Japan$

Prior to describing the flood discharge formula, the definition of "retardation" must be understood. Now let O and F be the point under consideration and the farthest point of a catchment respectively, l be the length of water course between O and F, $\omega$ the approaching velocity of rain water flowing from F to O, $t_c$ the time of concentration, i.e., the time necessary for reaching O from F, $t_r$ the duration of rainfall, i.e., the period between the beginning and ending of a rainfall (see Fig-1), then,

$$t_c = 1/\omega$$  \hspace{1cm} (7)

$$T = t_r + t_c = t_r + 1/\omega$$  \hspace{1cm} (8)

where

$T$ = The time of the period between the beginning of a rainfall and the ending of the run-off due to the rainfall at the point O

It may be better to use the author's formula for determining $t_c$. When it becomes $t_r < t_c$, the run-off is subjected to retardation, and in this case the rain water falling the whole catchment would not reach the proposed point simultaneously. Because as the rain water
falling at F reached O, the rainfall would have ceased. In other words, the rainfall causing the maximum flood discharge is the rainfall which falls in a part of the catchment.

(b) Fundamental Formula for the case of non-Retardation

The fundamental principles of the author's maximum flood discharge formulas have already been described. The author adopted deductive and inductive theories for the derivation.

Since it is unable to derive the rational equation of the flood discharge hydrograph, the author expressed the peak of the flood discharge hydrograph for the case of non-tributary and non-retardation by the following equation.

\[ q_m - q_o = C \phi A R / T \]  

where

- \( q_m \) = Peak discharge in flood time
- \( q_o \) = Discharge of run-off in normal time
- \( T \) = Duration of flood = \( t_r + t_c \)
- \( t_r \) = Duration of rainfall
- \( t_c \) = Approaching time or time of concentration
- \( R \) = Total amount of rainfall during duration of \( t_r \)
- \( A \) = Catchment area
- \( \phi \) = Average run-off factor
- \( C \) = A coefficient depending upon the shape of the flood discharge hydrograph

Now let Fig.-? show a discharge hydrograph during a flood period. Then the peak discharge \( q_m - q_o \) may be represented by eq(9) and the product \( \phi AR \) of eq(9) shows the total run-off during the period of flood \( T \). As this also represents the area of DMED of Fig.-2 geometrically, we may affirm that the author's fundamental formula is reasonable analytically or graphically. Replacing the value of \( R \) of eq(6) into eq(9),

\[ q_m - q_o = C \phi AR / T = C \phi A b t_r / (t_r + t_c) (t_r + a) \]  

We know through eq(10) that the peak discharge is a function of \( t_r \), and it will take a limiting value to make the peak discharge maximum. So differentiating eq(10) with respect to \( t_r \),

\[ t_r = \sqrt{a t_c} \quad \text{(unit in hours)} \]  

and

\[ T = t_r + t_c = \sqrt{a t_c} + t_c \]

Hence we know that the maximum flood discharge will occur when the duration of rainfall \( t_r \) satisfies eq(11). Up-to-date, we have taken \( t_r \) generally without definite reason as follows: 5 or 10 minutes for design of sewers, 3 or 4 hours for small rivers flowing the vicinity of a city, 24 hours or more for big rivers. However according to the author's theory, the value of \( t_r \) must satisfy eq(11) to cause the maximum flood discharge. Substituting eq(11) into eq(10),

\[ q_m - q_o = C \phi AR / T = C \phi b \sqrt{a t_c} A / (t_c + \sqrt{a t_c}) (a + \sqrt{a t_c}) \]
If we express in metric units, i.e., \( A(\text{km}^2), R(\text{mm}), t(\text{hr}), q(\text{cms}) \), (13) becomes,
\[
q_m - q_o = 0.2778C \phi b \sqrt{a t_c} \frac{A}{(t_c + \sqrt{a t_c})} (a + \sqrt{a t_c})
\]
where
\[
a, b = \text{any constants depending upon rainfall (see Table-2)}
\]
The value of \( t_c \) can be found from eq(3). (see Table-1)

(c) The value of the coefficient \( C \)

As stated above, the coefficient \( C \) in eq(9) depends upon the shape of the discharge hydrograph of the region. The relation between the kinds of the curve consisting the discharge hydrograph and the value of \( C \) is illustrated deductively as follows.

Table-3. The value of \( C \) found by deduction

<table>
<thead>
<tr>
<th>Kind of curve</th>
<th>Kind of curve</th>
<th>( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>parabola</td>
<td>cosine curve</td>
<td>2.0</td>
</tr>
<tr>
<td>triangle (straight line)</td>
<td>probability curve</td>
<td>2.394</td>
</tr>
</tbody>
</table>

Also, the value of \( C \) can be calculated inductively from a discharge hydrograph by using eq(9), which gives,
\[
C = \frac{(q_m - q_o)T}{\phi A R} = \frac{(q_m - q_o)T}{V}
\]
where
\[ V = \text{The volume of run-off represented by the area DMED of Fig-2} \]
The value of \( C \) for the rivers in Manchuria found by the author using eq(15) are given in Table-4.

Table-4. The value of \( C \) for Manchrian rivers found by induction

<table>
<thead>
<tr>
<th>Name of river</th>
<th>Site of measurement</th>
<th>Duration of flood taken from the records (hr, day-hr, day, month, yr)</th>
<th>Value of ( T )</th>
<th>Value of ( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tongcleog Ho</td>
<td>Tidathergtse</td>
<td>15, 3rd-21, 4th, Aug, 1940</td>
<td>30</td>
<td>1.664</td>
</tr>
<tr>
<td>*</td>
<td>Sankankeu</td>
<td>12, 10-5, 21, Sep, 1939</td>
<td>257</td>
<td>1.697</td>
</tr>
<tr>
<td>Whan Ho</td>
<td>Peidakeng</td>
<td>15, 24, -19, 27, Aug, 1940</td>
<td>76</td>
<td>1.543</td>
</tr>
<tr>
<td>Main stream of Leog Ho</td>
<td>Chengsenkong</td>
<td>19, 2nd-6, 6th, Sep, 1939</td>
<td>83</td>
<td>2.090</td>
</tr>
<tr>
<td>ditto</td>
<td>ditto</td>
<td>7, 7th-7, 10, Sep, 1939</td>
<td>72</td>
<td>1.966</td>
</tr>
<tr>
<td>Taitse Ho</td>
<td>Whelongbo</td>
<td>12, 31, Jul-3, 3rd, Aug, 1940</td>
<td>63</td>
<td>1.754</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>9, 4th-17, 6th, Aug, 1940</td>
<td>56</td>
<td>1.975</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>16, 6th-19, 8th, &quot;</td>
<td>51</td>
<td>1.887</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>17, 2nd-8, 5th, Sep, 1939</td>
<td>63</td>
<td>2.137</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>9, 5th-16, 9th, &quot;</td>
<td>103</td>
<td>1.806</td>
</tr>
<tr>
<td>&quot;</td>
<td>&quot;</td>
<td>5, 6th-13, 9th, Jul, &quot;</td>
<td>80</td>
<td>2.204*</td>
</tr>
</tbody>
</table>

* shows the value calculated by estimation because of non-measurement at the vicinity of the peak discharge.

(d) The fundamental formula for the case of retardation of flow
The basic formula for the case of non-retardation, mentioned above, is applicable for the case of retardation of flow, too. But it is necessary to multiply the coefficient $\rho$ due to retardation, viz.,

$$q_m - q_o = \rho \cdot C \cdot b \cdot \sqrt{at_c} \cdot A \cdot \left( \frac{t_c}{\sqrt{at_c}} \right) \cdot (a + \sqrt{at_c}) \quad (16)$$

$$\rho = f\left(\frac{t_c}{t_r}\right) \quad (17)$$

It is clear that the value of the coefficient $\rho$ equals to 1 for the case of non-retardation, but it becomes less than 1 for the case of retardation. The value of $\rho$ varies inversely with that of $t_c / t_r$.

It is necessary to find out a general form of $f(t_c / t_r)$ for practical calculation. So the author tried to find out the general form of the function $f(t_c / t_r)$ stochastically using some data obtained for rivers in Korea by some other methods. The author would like to assume the general form of the function of $\rho$ as follows.

$$\rho = \frac{1 + k}{(t_c / t_r) + k}$$

where

$k = \text{Any constant}$

Finding the value of $k$ in above equation by the method of the least squares, we get $k = 4.802$. Accordingly,

$$\rho = \frac{5.802}{(t_c / t_r) + 4.802} \quad (18)$$

2) THE MAXIMUM FLOOD DISCHARGE FORMULAS FOR THE CASE OF RIVERS WITH TRIBUTARIES

(a) The maximum flood discharge at the confluence of a tributary

The author found that existence of tributaries affect greatly the peak discharge of flood flow at the proposed site of the main stream. So the author derived many different formulas of maximum discharge for the case of rivers with tributaries, besides the basic formula for the case of those with non-tributary. Therefore it would be said that this is a great approach different from many scholars who never considered the influence of tributaries in their traditional formulas.

Now assume one of the simplest case as Fig-3. The discharge hydrograph for this case may be illustrated as Fig-4. The value of $q_1$ in Fig-4 shows the peak discharge of the tributary(I), and the value of $q_2$ shows that of the main river (II) alone, excluding that of the tributary(I), also $Q_m$ shows that of the composed maximum discharge to be occurred at the proposed site. The rational equation of the curve, i.e., the true shape of the discharge hydrograph is unknown. But the author would like to discuss about the shape of the curve in the following. Let us consider two cases, one of them the simplest case, i.e., the case assumed that the discharge hydrograph consists of an isosceles triangle, and the other the case assumed that it consists of a parabolic curve, to seek the effect of the nature of the discharge hydrograph which influences on the peak discharge $Q_m$. 
(i) The case of an isosceles triangle

In this case, it is evident from Fig-5, 
\[ q_m = q_2 + q_1(2 - \frac{T_x}{T_y}) \]  
(19)

(ii) The case of a parabola

Since it is evident as the nature of the parabola, at Fig-6, 
\[ q = 4q_o t/T - 4q_o (t/T)^2 \]  
(a)

we can get the following equation for Fig-7,
\[ Q = q_{ii} + q_{i2} = 4q_i t/T - 4q_i (t/T)^2 + 4q_2 t/T_2 - 4q_2 (t/T_2)^2 \]  
(b)
and by \( \frac{dQ}{dt} = 0 \)
\[ t_o = \frac{q_i/T_i + q_2/T_2}{2(q_i/T_i^2 + q_2/T_2^2)} \]  
(20)

Accordingly, substituting eq(20) into eq(b), we get
\[ Q_m = \left\{ q_2 + q_i(T_x/T_i) \right\}^2 /\{ q_2 + q_i(T_x/T_i)^2 \} \]  
(21)

NUMERICAL EXAMPLE

An illustration is given here to compare the degree of accuracy of the two cases mentioned above.
Given \( T_2 = 26 \) hr, \( T_1 = 20 \) hr, \( q_2 = 5000 \) cms, \( q_1 = 3000 \) cms. Then since \( T_2/T_1 = 26/20 = 1.3 \), from eq(21), the case of assuming as parabolic curve, \( q_m = (5000 + 3000x1.3)/(5000 + 3000x1.3x1.3) = 7865 \) cms

Next from eq(19), the straight line formula,
\[ Q_m = 5000 + 3000(2 - 1.3) = 7100 \]  

Hence, we know that there is not any remarkable difference on the results of calculation of the maximum discharge whether we assume the discharge hydrograph as straight lines or a parabolic curve through this numerical example. Also we can imagine that we shall obtain the similar results with this numerical example even in the cases we adopt some other curves else than parabola for the discharge hydrograph, e.g., cosine or probability curve. But adopting the case assumed as a parabolic curve is safer, easier to handle, and reasonable. So the author would like to suggest those of the parabolic curve as the general formula in this paper.

(b) The maximum flood discharge formula at the confluence for the case of a river where \( n-1 \) tributaries flow into the confluence (Fig-8)

If we assume the discharge hydrograph consists of a parabolic curve, by the same principle with that in the previous paragraph, we get
\[ t_o = \left( \sum_{i=1}^{n} q_i/T_i \right) / \left( 2 \sum_{i=1}^{n} q_i^2/T_i^2 \right) \]  
(22)
and
\[ Q_m = \left( \sum_{i=1}^{n} q_i/T_i \right)^2 / \left( \sum_{i=1}^{n} q_i^2/T_i^2 \right) \]  
(Fig-9)
(23)

(c) The maximum flood discharge at the proposed site which is located the downstream of a tributary
Now let 0 is the proposed site, O' the confluence of the tributary in Fig-10, and $t_1$ is the necessary time for reaching of rainwater from O' to 0. If we assume the discharge hydrograph consists of a parabolic curve, Fig-11, then

$$Q = q_i + q_z = 4q_i(t - t_1)/T_i + 4q_z(t - t_1)^2/T_z^2 - 4q_zt^2/T_z^2$$  

(c)

from $dQ/dt = 0$

$$t_0 = \frac{(q_i/T_i) + (q_z/T_z)}{2}$$  

(24)

and

$$Q_m = \frac{(q_i/T_i) \left\{ 1 + (2t_1/T_i) \right\} + (q_z/T_z)^2}{(q_i/T_i^2) + (q_z/T_z^2)} - 4q_i t_1/T_i \left\{ 1 + (t_1/T_i) \right\}$$  

(25)

(d) The maximum flood discharge at a proposed site where n-1 tributaries join to the main river at its upstream side. (Fig-12)

If we assume the discharge hydrograph consists of a parabolic curve, by the same principle with that in the previous paragraph, we get

$$t_0 = \left( \sum_{i=1}^{n} \frac{n}{x-i} q_i/T_i + 2 \sum_{z=i}^{n} q_z t_i/T_i^2 \right) / 2\left( \sum_{z=i}^{n} q_i/T_i^2 \right)$$  

(26)

and

$$Q_m = \frac{\left( \sum_{i=1}^{n} \frac{n}{x-i} q_i/T_i + 2 \sum_{z=i}^{n-1} q_z t_i/T_i^2 \right)^2}{\sum_{z=i}^{n} q_i/T_i^2} - 4\left( \sum_{z=i}^{n-1} q_i t_i/T_i + \sum_{z=i}^{n} q_i t_i^2 \right)$$  

(27)

(e) The maximum flood discharge at a proposed site where m tributaries flow into this site and n-1 tributaries join to the main river at its upstream side. (Fig-13)

This is the most general case. If we assume the discharge hydrograph consists of a parabolic curve, by the same principles, we get

$$t_c = \left( \sum_{i=1}^{m+n} \frac{n}{x-i} q_i/T_i + 2 \sum_{z=m+1}^{m+n} q_z t_i/T_i^2 \right) / 2\sum_{z=m+1}^{m+n} q_i t_i/T_i^2$$  

(28)

and

$$Q_c = \frac{\left( \sum_{i=1}^{m+n} \frac{n}{x-i} q_i/T_i + 2 \sum_{z=m+1}^{m+n} q_z t_i/T_i^2 \right)^2}{\sum_{z=m+1}^{m+n} q_i/T_i^2}$$  

(29)

V. CONCLUSION

The maximum flood discharge generally increase toward downstream, as the result of increment of the drainage area. But as the approaching time also increases approaching downstream, in other words, as the nearer approaching downstream, the greater effect of retardation. Accordingly the rate of increment of the peak discharge decreases generally approaching downstream; and sometimes, i.e., in such cases where the approaching time remarkably increases compared with the increment of the drainage area, not only the rate but also the actual absolute value of the peak discharge decreases at the downstream than those of the upstream. These facts are experienced sometimes in practice. In such cases, it was impossible to express this fact by the old formulas. However by the author's formulas, it is...
easy and theoretically sound to express this fact. Because as we see
the author's basic formulas—eq(9)–(14)—which represent the drainage
area A in the numerator and the factor of the approaching time tc in
the denominator. So it may also be said that the author's formulas
are very theoretical from the point of view of this fact.

As mentioned above, the author derived theoretically, i.e., ration-
ally or stochastically many formulas of maximum flood discharge—
the basic formulas for the case of a river with non-tributary and many
other different formulas for the case of rivers with tributaries. Be-
cause the author found that the existence of tributaries affect great-
ly not only the peak discharge but also the entire shape of the dis-
charge hydrograph at the proposed site of the downstream. Consequent-
ly it may be possible, by applying the author's formulas, to find the
real shape of the discharge hydrograph at the point under consider-
at ion to be occurred in some flood time.

Some scholars advocate that the actual shape of the flood dis-
charge hydrograph resembles to Fig-16. On the other hand, some other
scholars insist that it should be resembled to Fig-17. But the author
should say that these theories both advocated by the traditional
scholars are those have not been touched to the core of the true theo-
ries. The real shape of the discharge hydrograph depends upon the lo-
cality of the point under consideration, in other words, it depends on
the relative position of the proposed point and those of the conflu-
ences of the tributaries on the mainstream under consideration. Conse-
quently it is resembled to Fig-16 in some cases, and also it takes a
shape resembled to Fig-17 in some cases, in accordance with the locali-
ty of the point under consideration. As stated above, it would be able
to show the real shape of the discharge hydrograph just fitted in the
locality of the proposed site by applying the author's formulas.

The author's formulas also would be applicable not only for the
purpose of reckoning of the design flood, but also for that of esti-
mation of the flood routing for some floods. In the case of flood
routing, it would be possible to obtain more correct results by taking
the real value for tr instead of that calculated from eq(11) in some
cases, i.e., the real value of tr is greatly different from that calcu-
lated from eq(11).

The author's formulas would be widely applicable for rivers or
sewer nets, and also for any regions, countries with different locality,
and it would be possible to obtain correct and accurate results by
selecting or assuming the values of the coefficients in his formulas
appropriately. Accordingly the author should like to suggest that the
author's formulas shall be applied in practice in many regions and
also for many purposes as far as possible.