Grain and bond growth in wet snow

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Abstract. Grain growth, bond growth and densification of wet snow are partly determined by the distribution of equilibrium temperature in the snow matrix. At high water saturations, the equilibrium temperature decreases with grain size causing small particles to melt and large particles to grow. Melting also occurs at intergrain contacts, causing low strength and rapid densification. At low saturations the capillary pressure controls temperatures and particle size has a smaller effect. Grain growth proceeds slowly although melting at the contacts does increase with overburden pressure. At low saturations the water ‘tension’ causes large attractive forces and large bonding strengths occur.

Résumé. La croissance du grain, la croissance du liant, et la densification de la neige humide sont déterminées par la distribution de la température stable dans la matrice de la neige. Lorsqu’il existe de fortes saturations en eau, la température stable diminue avec la dimension du grain provoquant la fusion des petites particules et la croissance des grandes particules. La fusion se produit aussi aux points de contact entre les grains provoquant une faible résistance et une densification rapide. Lorsque les saturations sont faibles, la pression capillaire contrôle les températures et la dimension des particules a moins d’effet. La croissance des grains se produit lentement bien que la fusion aux points de contact s’accroisse avec la pression surchargée. Lorsque les saturations sont faibles, la ‘tension’ de l’eau provoque de larges forces attirantes et des forces liantes considérables se produisent.

NOTATION

\(a, b\) \hspace{1cm} \text{constants}

\(A_b\) \hspace{1cm} \text{area of the grain-to-grain bond (mm}^2\text{)}

\(A_w\) \hspace{1cm} \text{area of the water phase (mm}^2\text{)}

\(k\) \hspace{1cm} \text{thermal conductivity of water (W m}^{-1}\text{K}^{-1}\text{)}

\(p_c\) \hspace{1cm} \text{capillary pressure, } p_g - p_l \text{ (N m}^{-2}\text{)}

\(p_g\) \hspace{1cm} \text{pressure of the gaseous phase (N m}^{-2}\text{)}

\(p_l\) \hspace{1cm} \text{pressure of the liquid phase (N m}^{-2}\text{)}

\(P\) \hspace{1cm} \text{particle stress, assumed to be homogeneous and an increasing function of overburden pressure (N m}^{-2}\text{)}

\(q\) \hspace{1cm} \text{heat flow (W)}

\(r_1\) \hspace{1cm} \text{radii of the smaller particle (mm)}

\(r_2\) \hspace{1cm} \text{radii of the larger particle (mm)}

\(r_{1,0}\) \hspace{1cm} \text{initial radius of the smaller particle (mm)}

\(r_{g1}\) \hspace{1cm} \text{radius of the gas/liquid interface (mm)}

\(r_{ij}\) \hspace{1cm} \text{radius of the } i/j \text{ interface (mm)}

\(r_{lg}\) \hspace{1cm} \text{radius of the liquid/gas interface (mm)}

\(r_{sg}\) \hspace{1cm} \text{radius of the solid/gas interface (mm)}

\(r_{sl}\) \hspace{1cm} \text{radius of the solid/liquid interface (mm)}

\(r_b\) \hspace{1cm} \text{radius of the bond (mm)}

\(r_p\) \hspace{1cm} \text{radius of the particle (mm)}

\(R_1, R_2\) \hspace{1cm} \text{radii of curvature of an interface (mm)}

\(S_i\) \hspace{1cm} \text{entropy per unit mass of the } i\text{th phase (J kg}^{-1}\text{K}^{-1}\text{)}
INTRODUCTION

More knowledge of the physical properties of wet snow is necessary for an increased understanding of such phenomena as wet snow avalanches and water runoff from seasonal snowpacks. Many observations of the 'ripening' of snow have been made and its effects have been known for some time (Gerdel, 1954). Very little is known about the actual processes occurring during ripening or the effects of subsequent variations of the liquid water content. The growth of grains and bonds determines the permeability and rheology which account for most of the significant material parameters of a snowpack. Clearly an improved understanding of the interaction between liquid water and snow grains is necessary.

The thermodynamics of wet snow is used to describe the processes of grain growth, bond growth and densification. It is convenient to categorize the saturation regimes in wet snow as either pendular or funicular, i.e. low or high saturation respectively. At the low values of saturation which normally exist in wet snow, the water volume is greater than the capillary requirement (the irreducible water saturation) but less than that necessary to cause neighbouring rings to coalesce (see Fig. 1(a)). In this regime the water pressure is much less than atmospheric pressure and the air phase exists in more or less continuous paths throughout the snow matrix. In the funicular regime (see Fig. 1(b)), rings of water coalesce forming isolated bubbles of air trapped between the ice grains and the water phase exists in continuous paths completely surrounding the
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The permeability to liquid water is greatly increased at larger saturations and the capillary pressure, or ‘tension’, of the liquid water is reduced.

The water saturation during gravity drainage of homogeneous snow generally lies within the pendular regime. The upper limit of the pendular regime is estimated as 14 per cent of the pore volume for a packing of uniform spheres (Smith, 1933; see also Stallman, 1964) but could be somewhat larger for a random pack of ice grains. The funicular regime occurs over impermeable boundaries, at interfaces, and possibly in the highly permeable drains where large flow rates occur. Physically these regimes differ in that the ice grains are surrounded by water at high saturations but are exposed to air at low saturations. The effects of this difference are described in the following analysis.

FUNICULAR REGIME

The rapid metamorphism of snow at high water saturations has been observed many times. In particular, three phenomena require explanation — rapid grain growth, rapid densification and loss of mechanical strength. These rapid changes are caused by large temperature gradients among the ice particles.

When three phases of water occur in equilibrium the Gibbs—Duhem equation describes each phase (i):

\[-S_i \, d \, T + \nu_i \, d \rho_i = du\]  

(1)

where for the gaseous phases, the equation is applied only to the water vapour component of the air (Defay and Prigogine, 1951, p. 219). The pressure difference \((p_i - p_j)\) across an interface is related to the principal radius of curvature \(r_{ij}\) by the Kelvin equations:

\[p_i - p_j = \frac{2 \sigma_{ij}}{r_{ij}}\]  

(2)

Usually these interfaces have two radii of curvature \((R_1\) and \(R_2\)) which are related to the principal radius by

\[\frac{2}{r_{ij}} = \frac{1}{R_1} + \frac{1}{R_2}\]  

(3)

Equations (1) and (2) are combined to give (see Colbeck, 1973 for more details)

\[\ln \left( \frac{T}{T_0} \right) = \frac{2}{L} \left( \frac{1}{\rho_i} - \frac{1}{\rho_s} \right) \sigma_{lg} \frac{r_{lg}}{r_{lg} - 2 \sigma_{sl}} \]  

(4)

where \(T_0\) is the equilibrium temperature over a flat surface. For small differences between \(T\) and \(T_0\), the equilibrium temperature on the Celsius scale \(T_m\) is approximately

\[T_m = 1.11 \times 10^{-5} r_{gl}^{-1} - 5.84 \times 10^{-5} r_{sl}^{-1}\]  

(5)

where \(r_{gl}\) is the radius of the air bubbles and \(r_{sl}\) is the radius of the ice particles in millimetres. In Fig. 2, \(T_m\) is shown as a function of \(r_{sl}\) for various values of \(r_{gl}\).

Grain growth

In the funicular regime there are two opposing effects — the melting temperature increases as the size of the air bubbles decreases but decreases as the size of the ice grains decreases. The melting temperature could be quite high but the size of the air bubbles is limited by the fact that smaller bubbles tend to escape by buoyancy and only the larger bubbles remain trapped in the snow matrix. For any given bubble size
the temperature distribution at larger saturations is determined by the size of the individual particles. The smaller particles equalize at a lower temperature and therefore melt in the presence of larger particles. This is a powerful mechanism driving grain capture because the controlling temperature gradient increases as grain capture progresses and the difference in size between the particles increases.

The most significant effect of air bubbles in wet snow is to decrease the area available for heat flow between particles. A description of grain growth at high water saturations is formulated for a typical case. Air bubbles trapped in the snow matrix are set equal to the smallest bubble which could be trapped in a cubic packing of equal spheres, or

$$r_{gl} = 0.414r_{sl}$$

These air bubbles correspond to an air saturation of about 8 per cent of the pore volume, a close approximation for wet snow saturated by imbibition. The metamorphism of snow at lower funicular saturations is not examined here but clearly the rate of heat flow between the grains would be decreased and the rate of grain growth would be proportionately less. Since air bubble trapping prevents 100 per cent water saturation under normal circumstances, it is not necessary to consider large saturations.

Combining equations (5) and (6), the melting temperature is given by

$$T_m = -3.16 \times 10^{-5} \frac{T}{R}$$

which is shown in Fig. 2. The average heat flow between two adjoining spheres is approximated as

$$q = k \frac{\text{area/length}}{\Delta T}$$

where $\Delta T$ is the difference between the melting temperatures of the two spheres, $(\text{area/length})$ is approximated as the cross-sectional area of the small particle divided by one-fourth of the distance between centres, and $k$ is the conductivity of the water. In unsaturated porous media, only the heat flow through the liquid phase need be
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The heat flow between two neighbouring particles is

\[ q = 22.2 \times 10^{-8} \frac{r_1}{r_2} \frac{r_2 - r_1}{r_2 + r_1} \]  

(9)

where \( r_1 \) and \( r_2 \) are the radii of the smaller and larger particles respectively. Two competing factors affect heat flow between the particles — the temperature difference increases as the smaller particle melts but the cross-sectional area available for heat flow also decreases. For this reason the heat flow reaches a maximum when \( r_2 \) is about twice as large as \( r_1 \) rather than increasing monotonically as the smaller particle disappears. In Fig. 3 the heat flow is shown as a function of the radius of the larger particle for various sizes of smaller particles.

\[ q(w) \]

![Graph](image)

**FIGURE 3.** Heat flow as a function of the radii of the larger particles for various sizes of smaller particles.

The conservation of mass and energy between two particles (see Colbeck, 1973) is used to derive

\[ \frac{dr_1}{dt} = 5.8 \times 10^{-8} \frac{1}{r_1 r_2} \frac{r_2 - r_1}{r_2 + r_1} \]  

(10)

Upon integrating equation (10):

\[ r_2(r_1^2 - r_{1,0}^2) + 4r_2^2(r_1 - r_{1,0}) + 4r_2^2 \ln \frac{r_2 - r_1}{r_2 - r_{1,0}} = 11.6 \times 10^{-8} t \]  

(11)

This analysis shows that the rate of decrease of \( r_1 \) increases monotonically in spite of the fact that heat flow continuously decreases, i.e. \( q \) is approximately proportional to \( r_1 \) whereas \( dr_1/dt \) increases approximately with \( 1/r_1 \). This analysis also shows that small particles are very effective at capturing their smaller neighbours. Therefore, the metamorphism of a snowpack does not depend on the presence of a few large particles. Metamorphism proceeds rapidly for any distribution of particle sizes which includes smaller particles. The rate of metamorphism progressively decreases as the average size of the particles increases. Figure 4 shows the decay of smaller particles in the presence of a larger particle of 1.0 mm radius. The life expectancy \( (t_D) \) of a smaller particle increases approximately as the square of its radius:

\[ t_D = (a + b r_2) r_{1,0}^2 \]  

(12)

Wakahama's (1968) results show that when snow is immersed in water, grains smaller than 0.1 mm in radius disappear within 3 days but grains smaller than 0.5 mm take
FIGURE 4. Radius of curvature of the smaller particle as a function of time for various initial values of its radius. The larger particle is taken as 1.0 mm.

6 days to disappear. The decreasing rate of metamorphism is consistent with equation (12), which predicts that the life expectancy of medium-size grains will be much larger than that of the smaller grains.

Although the dominant mechanism of grain capture has been described, much more work remains to be done on the unsteady, nonequilibrium processes of grain capture. A complete description of grain capture would involve the interaction at multigrain contacts with complicated patterns of heat and mass flux among grains. Further work requires consideration of the distribution of grain sizes, which is difficult for a random packing of grains with a wide range of sizes.

Grain contacts
The rapid growth of the contact area between grains (Wakahama, 1968) is caused by the distribution of surface temperature around individual grains. The equilibrium temperature at a contact \( T_b \) is determined by the normal stress at the contact (capillary pressure is negligible in the funicular regime):

\[
T_b = -2.36 \times 10^{-8} P r_p^2 r_b^{-2}
\]

where the particle is assumed to have a homogeneous stress \( P \). When the temperature of the ice—water interface is given by equation (7), the temperature difference across a grain is (see Fig. 5):

\[
T_m - T_b = 2.36 \times 10^{-8} P r_p^2 r_b^{-2} - 3.16 \times 10^{-5} r_p^{-1}
\]

This equation shows that the contact area increases by melting especially when the overburden pressure is large or the bonds are small. Wakahama's (1968) results show a similar increase of contact area at high saturations. The rate of growth of the contact area decreases with increasing size of the area until thermal equilibrium is established for a given overburden pressure and particle size. Large overburden pressures greatly increase the rate of melting and, at greater depths in a snowpack, the quasi-stable size of the bonds is larger. When a variety of grain and bond sizes exist, the smaller grains and bonds compete for the heat flow from the larger particles. The relative rate of grain decay or contact area growth is determined by the size of the smaller particle,
bond size, and overburden pressures. Clearly a small contact area between two particles can delay the decay of a small particle while the bond area increases by melting. While melting occurs, the strength between particles should be quite low. Low 'hardness' values were observed by Wakahama et al. (1968) in water channels with high flow rates and large grains. The loss of grain-to-grain strength at water impoundments probably explains the deterioration of wet snow leading to slides (Mellor, 1968, p. 152). Tusima (1971) reported changes in hardness with water content but was not specific. The bond strength may increase as ice-to-ice adhesion develops but, in general, the strength of snow has low initial values at large water saturations.

**Densification**

The growth of the contact area between grains causes densification since the grains move closer together and assume a more efficient shape for close packing. The rate of densification increases with overburden pressure (all other factors being equal) as observed in Wakahama's (1968) experiments. The firnification process in a temperate glacier would occur much more rapidly if saturated conditions prevailed. Even with small overburdens the temperature differences developed across snow particles are as large as 0.02°C and metamorphism occurs by melting and refreezing rather than mechanical deformation.

**PENDULAR REGIME**

A smaller volume of water occurs in the pendular regime so a smaller area is available for heat flow. The most significant difference is the introduction of another variable, the solid—gas radius of curvature \( r_{sg} \). Equation (2) give directly the geometrical requirement for mechanical equilibrium among the three interfaces:

\[
\frac{\sigma_{sg}}{r_{sg}} + \frac{\sigma_{gl}}{r_{gl}} = \frac{\sigma_{sl}}{r_{sl}}
\]  

(15)
where $r_{sg}$ is the particle size and $r_{gl}$ is determined by the capillary pressure. The solid—liquid radius ($r_{sl}$) is specified by equation (15) for any given particle size and liquid saturation. Because $r_{sl}$ determines the bond geometry it is necessary to develop a complete understanding of the nature of capillary pressure in wet snow. Direct measurements of capillary pressure as a function of water saturation were made on wet snow samples in an adiabatic environment (the procedure is described by Scheidegger, 1957, p. 49). The results are shown on Fig. 6 and are typical of highly permeable aggregates of hygroscopic materials. The capillary pressure rises rapidly through the pendular regime and approaches a very high value near the irreducible liquid saturation. Capillary pressures in wet snow undergoing free drainage are greater than $700 \text{ N m}^{-2}$ and values greater than $9000 \text{ N m}^{-2}$ have been measured in situ in snowpacks (Colbeck, unpublished).

![FIGURE 6. Capillary pressure as a function of water saturation. The various saturation regimes are shown.](image)

The equilibrium temperature in the pendular regime is

$$T_m = -0.814 \times 10^{-6} r - 194 \times 10^{-6} r^{-1}$$  \hspace{1cm} (16)

Normally the effect of capillary pressure (and therefore water saturation) on the equilibrium temperature dominates the effect of particle size. Wakahama (1968) observed grain growth at two saturations:

(1) Less than the capillary requirement of liquid was present in a snow sample at its equilibrium temperature; the capillary pressure was very high and only slight grain growth was observed.

(2) The addition of a small amount of liquid reduced the capillary pressure and increased the rate of grain growth. These observations closely agree with the analysis given here. The metamorphism and densification of wet snow occurs very slowly at low liquid saturations, mostly due to the large value of capillary pressure rather than the reduced area available for heat flow.
Much of the grain-to-grain strength in the pendular regime is caused by the water ‘tension’ drawing particles together. The mechanical equilibrium of two neighbouring particles is given by

\[ \sigma_b A_b = \pi r_p^2 P + A_w P_c \]  

(17)

where \( A_b \) is the cross-sectional area of the bond and \( A_w \) is the cross-sectional area through which the water tension acts. The temperature difference between the particle surface and the bond are derived from equations (13), (16) and (17):

\[ T_m - T_b = p_c (7.4 A_w/A_b - 81.4) \times 10^{-8} + 7.4 \frac{\pi r_p^2}{A_b} P \times 10^{-8} - 19400 r_p^{-1} \times 10^{-8} \]  

(18)

For large values of capillary pressure, the ratio \( A_w/A_b \) determines the sign of \( (T_m - T_b) \). If \( A_w \) is much greater than \( A_b \), further bond growth occurs by melting at the contact area and, if \( A_b \) is much greater than \( A_w \), freezing occurs at the inter-grain contacts preventing further enlargement and higher bond strengths. At large values of overburden pressure the contact area grows by melting but, in the pendular regime, the effect of the overburden is moderated by capillary pressure. At thermal equilibrium \( A_w \) approaches \( 11 A_b \) and the effect of the ice-to-ice adhesion on the interparticle strength is small compared to the effect of the liquid tension.

SYNOPSIS

The metamorphism, strength and densification of wet snow are determined by the occurrence of small temperature gradients around the particles. In order to describe these processes, it is convenient to categorize the saturation regimes of wet snow as either funicular or pendular. In the funicular regime saturations are greater than 14 per cent of the pore volume and the air occurs in discrete bubbles trapped between the ice particles. The equilibrium temperature of the snow matrix is controlled by the size of the air bubbles and the size of the ice particles and, for any given air content, the particle sizes dictate the distribution of temperature locally within the mixture of ice particles. The smaller particles exist at a lower equilibrium temperature, causing heat flow from the larger particles and rapid melting of the smaller particles. The result is the disappearance of the smaller particles and the subsequent growth of the intermediate and larger particles. The average particle size thereby increases without a significant change of density in the snow matrix. The time rate of change of the radius of the smaller particle increases as the particle disappears, or the life expectancy of a particle increases as the square of its radius. Therefore, the smaller particles are not observed in the processes of disappearing because of the speed with which they vanish. The rate of metamorphism decreases with time after the smaller particles disappear and the size difference between adjoining particles decreases.

In the funicular regime the equilibrium temperature at a contact between grains is decreased by the compressive stress between the grains. The temperature depression is increased by overburden pressure causing melting of the intergrain contacts and removing any bond-to-bond strength. This explains the loss of strength that causes wet snow slides over impermeable layers where water saturations are high. As melting occurs at the contact areas, the grain centres move closer together and closer packings are possible. Thus, density increases as the grains assume a shape more suitable for close packing and, in highly saturated snow, the density of pore close-off is rapidly achieved.

The thermodynamics of the pendular regime is significantly different because of the lesser cross-sectional area of water available for heat flow and the existence of another interface, the gas—solid surface. The Kelvin equation can be solved to relate the three radii of curvature for the matrix, hence, given a water saturation and particle
size, the radii of curvature and shape of the bonds are determined. The equilibrium temperature of the matrix is a decreasing function of both capillary pressure and particle size. The capillary pressure, which is related experimentally to the water saturation, increases rapidly as the water saturation approaches the capillary regime. Therefore, as water saturation drops, the equilibrium temperature takes a large negative value which dominates the effect of the radius of curvature. At small water contents, the temperature differences between particles and the area of heat flow are both reduced and much lower rates of grain growth are observed. The large 'tensional' forces developed in the water phase give strong intergranular attractions and the bonds assume a finite size which is determined by the relative effects of capillary pressure and particle size. The strength of snow at low water saturations should be high. In spite of the large stresses induced by the attractive forces, no melting occurs at the grain contacts because the large values of capillary pressure reduce the temperature of the entire snow matrix. Without intergranular melting, metamorphism is controlled by the relatively slow processes of mechanical deformation and sintering. Therefore the observed rates of grain growth and densification are much lower at low water saturations.

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REFERENCES

DISCUSSION
G. Kovacs:
The importance of distinction between the two conditions of saturation (i.e. pendular and funicular) has been emphasized. On the basis of similarity between soil physics and snow dynamics, perhaps the dynamic explanation of the two different regimes achieved in soil physics could be useful when investigating the saturation of snow as well, although in the case of soil there is no transformation between solid and liquid phases.

In the zone where the particles are covered by thin water films, the main force balancing gravity is adhesion, which creates tension on the surfaces of the water film.
In this zone the relationship between elevation above the water table and the saturation can be approximated by a hyperbola of sixth order.

Capillary is another molecular force, basically different from adhesion. Its existence can be explained by the attraction of the water molecule and its final result is the capillary height, which is inversely proportional to the diameter of the pores. After characterizing the pore size distribution by a probability distribution curve, the latter can be transformed for describing the relationship between capillary weight and saturation.

As a summary it can be stated, that from the two curves mentioned, the vertical moisture distribution belonging to the dynamic equilibrium can be constructed, even numerically calculated. The basic requirement of this is the clear distinction of the adhesion and capillary zone, considering that the acting forces are different there.

P. M. B. Föhn:
Referring to Fig. 6 showing capillary pressure versus water saturation: How did you measure the saturation or better said the liquid water content of the snow samples?

S. C. Colbeck:
The capillary pressure experiments are performed by saturating a known weight of dry snow. Then (considering the temperature increase upon saturation) the ice and water volumes are known at the start and, during drainage, the volume of discharge is measured. Therefore the water saturation can be determined directly without resorting to one of the measuring devices.

The experiment progresses as the water pressure is decreased by increments. At each step, some water drains from the sample. When water drainage stops the pressure is decreased again, etc. The experiments were not done as slowly as desired because of the requirement of maintaining an adiabatic environment.

The pressure was measured through a porous, ceramic plate using a simple manometer.

P. M. B. Föhn:
But how do you know then, if the water has been distributed uniformly in your snow samples?

S. C. Colbeck:
We did assume that the distribution was uniform, using a semipermeable plate and taking care on this problem.

J. F. Nye:
As the temperature differences being considered are very small, do you think the effect of impurities on the melting point would be significant?

S. C. Colbeck:
The major effects which I have described are sufficiently large that impurities have only a second order effect. This will soon be tested when we measure actual temperatures in wet snow. Some revision of our ideas is likely to occur at that time.