Friction coefficients and speed of flowing avalanches

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Abstract. Observations were made of the speed of avalanches by timing the advance of their front over a section of track covered with deep snow. The track under observation had uniform incline and cross-section, and the avalanches could be assumed to have constant mass and a uniform flowing motion. The observed speeds \(u\) could best be expressed by

\[u^2 = 1420 R (\sin \alpha - f \cos \alpha)\]

with \(R\) the hydraulic radius and \(\alpha\) the inclination of the track. The coefficient of kinetic friction \(f\) was found to be dependent primarily on the speed. Tables are presented for estimating the depth and density of the flowing snow from observations of the avalanche snow after it comes to rest.

Résumé. On observe la vitesse des avalanches en chronométrant la progression de leur front dans un couloir couvert de neige profonde et dont l'inclinaison et le profil sont uniformes. On peut supposer que la masse des avalanches est constante et que leur mouvement d'écoulement est uniforme. On peut le mieux exprimer les vitesses observées \(u\) au moyen de

\[u^2 = 1420 R (\sin \alpha - f \cos \alpha)\]

\(R\) étant le rayon hydraulique et \(\alpha\) l'inclinaison du couloir. Le coefficient de frottement de glissement \(f\) dépend surtout de la vitesse. Des tableaux donnent l'évaluation de la profondeur et de la densité de la neige en écoulement sur la base d'observations de la neige lorsque l'avalanche a cessé.

CHARACTER OF A MOVING AVALANCHE

An avalanche starts when a snow slab breaks away or when snow grains lose their cohesion. Initially, the slab slides as a rigid body but it breaks into small fragments after a short time. During this first stage the movement may be considered to be one of laminar motion, similar to that of a fluid. The speed increases rapidly on steep terrain, and the motion becomes turbulent. It is difficult to determine exactly the critical speed for the transition from laminar to turbulent flow, but it must be low (Mellor, 1968). Mechanical properties of the snow, as well as the roughness of the sliding surface, have an influence on the critical speed.

The smallest particles of the disintegrated snow mix with the air at the surface and at the front of the avalanche. A typical mature avalanche consists of a core of dense snow flowing along the terrain, accompanied by a cloud of snow dust (Fig. 1). The dust cloud is well developed in avalanches of dry snow with little cohesion. It is less pronounced in moist snow, and is absent in wet snow avalanches. Although the snow dust is the spectacular part of the avalanche, the flowing part contains most of the mass. It has a density 20–50 times that of the snow dust and can be much more destructive. On steep terrain the snow dust usually moves at a speed equal to or slower than the flowing dense snow. On slopes with little incline, such as the outrun zone, the snow dust often overtakes the flowing part and travels a greater distance.

The flowing snow and the airborne snow dust are two different materials with different characteristics and behaviour and, for studies of their dynamics, they must be treated as separate entities. In this paper only the motion and speed of the dense flowing part of avalanches are discussed.

THEORETICAL MODEL

The acceleration and speed \(u\) of an avalanche on a slope with incline \(\alpha\) are determined
by the balance between the driving force due to its weight \((m \cdot g)\) and the forces of resistance \(F\). This balance is expressed by the momentum equation

\[
\frac{d}{dt} (m \cdot u) = m \cdot g \cdot \sin \alpha - \sum F
\]

The following resistance forces act on a moving avalanche:

1. Kinetic friction, proportional to the forces perpendicular to the sliding surface.
2. Resistance proportional to the speed \(u\), due to an apparent viscosity of the moving snow. The influence of this resistance, however, appears to be small and for practical purposes may be neglected (Salm, 1965).
3. Turbulent resistance similar to the resistance of turbulent fluids, proportional to \(u^2\).
4. Friction between the upper boundary and the air or snow dust. From studies of density currents Shen and Roper (1970) concluded that this friction is proportional to \(u^2\).
5. Air resistance at the front, proportional to \(u^2\).
6. Resistance of the snow cover at the front. The front would push against a layer of unstable snow, probably breaking it up and incorporating it into the avalanche.

A complete solution of equation (1) is not possible unless simplifications are made. Earlier investigators have treated the avalanche as a solid body sliding on an inclined surface with the kinetic friction acting as the only resistance force. This would be satisfactory for the slow motion in the initial stage of the avalanche and just before it stops in the outrun zone, but leads to unrealistic values for well developed avalanches. Recent authors in the U.S.S.R. have attempted a solution for turbulent motion and variable mass, but had to limit the number of resistance forces (Moskalev, 1966). Salm (1965) has made a complete analysis of the resistance forces, but assumes a constant mass.

On a uniform incline the avalanche reaches a maximum speed \(u_T\) after a relatively short time. Voellmy (1955), Salm (1965), Moskalev (1966), Mellor (1968) and Shen and Roper (1970) have developed equations giving \(u_T\). The equations determined by all the authors have the same basic form if a resistance proportional to \(u\) is neglected and the mass is assumed to be constant:

\[
u_T^2 = \frac{1}{R} (\sin \alpha - f \cdot \cos \alpha - a)\]
Friction coefficients and speed of flowing avalanches

where $\xi = \text{coefficient of turbulent friction (m s}^{-2}), f = \text{coefficient of kinetic friction,}$

$a = \text{coefficient for resistances that are independent of speed, }\alpha = \text{angle of incline of the avalanche track, } R = \text{hydraulic radius (m)}.

$R$ is equal to the flow depth $H'$ for avalanches that move over an open slope or in channels with a width greater than the flow depth by an order of magnitude.

OBSERVATION OF SPEEDS

Rogers Pass in British Columbia offers an opportunity for studying avalanche motion because the controlled release of avalanches in this area is an important part of the protection of the Trans Canada Highway. The terrain at Rogers Pass is rugged with steep, long slopes and is typical of the mountains in the Southern Interior of British Columbia, Canada. The avalanches start to slide at elevations between 1500 and 2300 m and run out between 900 and 1320 m.

Visual observations were made of the speed of released avalanches by timing the advance of their front over a known distance with a stopwatch. The sections of track under observation had lengths between 100 and 400 m, uniform inclines and uniform cross-sections. Features of terrain that could easily be identified on airphotos and maps, e.g. junction of gullies, major rock outcrops and clumps of trees, were chosen as upper and lower boundaries. The distance and difference of elevation over which the avalanches had moved were measured on aerial photographs and contour maps with scale 1 : 5000. The angle of slope of the track under observation ranged between 27 and 44°.

The sections were sufficiently far from the starting and the outrun zones that the avalanches could be assumed to be full size and to have a well developed, turbulent, uniform motion at a constant speed $u_f$. Unstable snow that was removed from the track made a negligible contribution, and for this reason the mass could be assumed constant.

A study of avalanche impact pressures was an additional source for speed observations. Several pressure sensors, mounted in the track of an avalanche, produced signals that were recorded and timed on an oscillograph as the avalanche front passed (Schaerer, 1973). The pressure sensors, located at various levels above ground, also permitted conclusions concerning the depth of the moving snow to be drawn.

After checking a large number of visual and pressure observations for accuracy and consistency with the assumption of a uniform flow and constant mass, a total of 47 observations from 21 different sites were retained for analysis. Snow dust avalanches were excluded. Avalanches that are controlled regularly by gunfire are generally small to medium sized, this being one of the objectives of the control with guns. The observations are, therefore, somewhat biased towards medium-sized avalanches of low speed, but fortunately some data for large avalanches with high speeds were obtained and are included.

Observation of flow depth of the moving snow is difficult. The most reliable information was obtained for occurrences at two pressure study sites. Other means of obtaining information concerning the depth were to observe flow marks in the snow on the side of gullies and the height to which snow was pressed against tree trunks in the avalanche path.

Most avalanche outrun zones at Rogers Pass are talus slopes or alluvial fans with surface slopes of 10—25°; the snow deposited by the avalanches is well spread in length and its depth varies little. The observed flow depths $H'$ in the tracks above the outrun zones were found to be related to the average depth $H$ of the avalanche snow after it came to rest (Table 1).

The bulk density of the flowing snow may be obtained from measurements of the density of the avalanche deposits and the relations given in Table 1. Table 2 gives
TABLE 1. Flow depths

<table>
<thead>
<tr>
<th>Type of avalanche</th>
<th>Dry snow (particles range from powder to diameter 100 mm)</th>
<th>Damp snow (particles mainly in the range 10–200 mm)</th>
<th>Wet snow (heavy, as it is usually observed in spring, particles &gt; 100 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H' = 4H$</td>
<td>$H' = 3H$</td>
<td>$H' = 1.5H$</td>
</tr>
</tbody>
</table>

TABLE 2. Average density of flowing snow (kg m$^{-3}$)

<table>
<thead>
<tr>
<th>Type of avalanche</th>
<th>Small size</th>
<th>Medium size</th>
<th>Large size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry snow</td>
<td>60</td>
<td>75</td>
<td>90</td>
</tr>
<tr>
<td>Damp snow</td>
<td>100</td>
<td>125</td>
<td>150</td>
</tr>
<tr>
<td>Wet snow</td>
<td>300</td>
<td>350</td>
<td>400</td>
</tr>
</tbody>
</table>

the calculated average densities of the flowing snow using observations made for seven years at Rogers Pass.

The deposit width of most avalanches observed was not significantly different from the track width, and the flow depth was determined according to Table 1. When the track was a narrow channel and the avalanche spread out after leaving it, the flow depth was increased to produce a cross-section area corresponding to that of the deposit.

COEFFICIENTS OF FRICTION

The observations of avalanche speeds were fitted to equation (2) by stepwise linear regression. The coefficient, $a$, which would account for the resistance at the front of the avalanche due to the snow cover and perhaps other sources, proved to be insignificant. This can be explained by visual observations that tend to confirm that the front of dry and damp avalanches overrides the snow surface. Furthermore, the

FIGURE 2. Observed avalanche speeds.
density of the moving snow (Table 2) tends to be slightly lower than that of the undisturbed snow in the track, and it would be reasonable to assume that dry avalanches would float on the new snow. Only wet snow avalanches would penetrate the light snow at the surface. The new snow on the top of a snow cover is probably removed by shear along the bottom of the avalanche, and the force necessary would be part of the kinetic friction term in equation (2).

The kinetic friction term proved to have a significant influence on the speed, but the coefficient \( f \) does not appear to be a constant. Other investigators have found a speed dependence for the friction between snow and snow, as well as between snow and other materials (Mellor, 1964). Good agreement with the observations was obtained in the present study by using:

\[
\frac{5}{u} = f
\]

with \( u = \) avalanche speed in m s\(^{-1}\).

Equation (2) may be written as

\[
u R = \xi \left( \frac{5}{u} \right) \sin \alpha - \frac{5}{u} \cos \alpha\]  

(4)

High speed avalanches appear to have a kinetic friction coefficient even lower than that given by equation (3), but the number of observations was insufficient to permit definite conclusions. For practical purposes, the kinetic friction may be neglected when the speed is greater than 50 m s\(^{-1}\).

Slow moving avalanches at Rogers Pass were frequently observed to stop on inclines less than 27° but to continue their motion on steeper slopes. A slope angle of 27° corresponds to a sliding friction coefficient of about 0.5. Roch (1965) gives a kinetic friction coefficient of 0.5 for the slow motion of fine-grained dry snow, which would be the type found in dry avalanches. Sommerhalder (1972), from observations of friction coefficients on snow sheds, found an average value of 0.32 and a maximum of about 0.5. It would be appropriate, therefore, to assume 0.5 as an upper limit for the kinetic friction coefficient.

The coefficient of turbulent friction includes all influences on friction forces that are dependent on \( u^2 \). The best fit for equation (4) was obtained for a value \( \xi = 1420 \) m s\(^{-2}\). When the friction coefficient, \( f \), is small and negligible, equation (2) becomes the velocity equation developed by Chézy for the motion of fluids in pipes and open channels:

\[
u = C \cdot \sqrt{(R \cdot S)}
\]

where \( C = \sqrt{\xi} \) is the Chézy discharge coefficient, and \( S = \sin \alpha \). Voellmy (1955) has suggested a value for \( \xi \) of between 400 and 600 m s\(^{-2}\). Shen and Roper (1970), using results of experiments with density currents, recommended 750 m s\(^{-2}\). These values are in the range of the Chézy discharge coefficients for the flow of mountain rivers in a bed of coarse boulders, and they would be adequate for avalanches that run over rough ground. Observations reported in this present paper, however, were made of avalanches on deep snow, usually dense old avalanche deposits with few rocks and no trees. The value obtained for \( \xi \) was between 1000 and 1800 s\(^{-2}\), the range of Chézy coefficients \( C^2 \) recommended for rivers flowing in channels of earth with few rocks and poor alignment.

The observations did not permit a full analysis of the factors that might influence the friction coefficients, because the variations of speed produced by them are of the
same magnitude as variations caused by observational errors, e.g. distance under observation, slope angles, flow depth, and mass.

There was little difference between the average friction coefficients of wet snow and dry snow avalanches. It is known that wet snow avalanches are slower than dry ones, but this is due to a smaller flow depth (Table 1) rather than higher friction coefficient.

The presence or absence of new snow in the track had little influence on the speed. For six avalanches that moved over old avalanche snow not covered with new snow, the mean coefficient of turbulent friction was \( 1470 \text{ m s}^{-2} \). This value is not significantly different from \( 1420 \text{ m s}^{-2} \) determined for avalanches moving over a layer of new snow.

Voellmy (1955) has suggested a relation between the avalanche speed and the depth of unstable new snow. No such direct dependence could be established from our observations. There is a relationship between the depth of the sliding snow layer and the volume, and hence the depth of the flowing snow, but it also depends on the size and character of the area where snow starts to slide as well as on the character of the avalanche track.

CONCLUSIONS

The speed of the dense flowing component of fully developed dry and wet snow avalanches was analysed. Characteristics of this component determine the maximum impact pressure and are significant, therefore, for the design of structures in the avalanche path. Observations have confirmed that the equation first suggested by Voellmy (1955),

\[
\tau_1^2 = \xi \cdot R (\sin \alpha - f \cdot \cos \alpha)
\]

is adequate for determining the speed in the range 10–50 m s\(^{-1}\) for avalanches that have flow depths between 0.5 and 5.0 m and move over terrain steeper than 25°.

The value selected for the friction coefficient \( \xi \) must take into consideration the condition of the avalanche track. Values between 1 000 and 1 800 m s\(^{-2}\) with 1 400 m s\(^{-2}\) as an average, should be used for avalanches that move over deep, dense snow, e.g. old avalanche deposits. Values of 400–800 m s\(^{-2}\) would be applicable for avalanches that flow over rough terrain covered with boulders and trees.

The friction coefficient, \( f \), varies inversely with speed. Speed refers to the apparent speed of the front of the avalanche, which might be smaller than the speed of individual particles behind the front. It would probably be safe to assume the frontal speed to be close to the mean speed of the moving snow for fully developed turbulent avalanches that have little resistance at the front.

The avalanche speeds were related to the depth of flow, which in turn was related to the depth of snow after the avalanche came to rest. This relationship was satisfactory for obtaining values for friction coefficients and for making an assessment of the factors that influence them. For practical purposes it would not usually be possible to rely on the depth of the deposited snow. The flow depth would have to be estimated from the amount of snow that is expected to be set in motion in the starting zone, and from the character of the track.

The confidence in the analysis of speeds was strengthened by a study of the observations obtained with pressure sensors. They provided the most reliable and accurate measurements of speed and deposited snow, and yielded direct information concerning the flow depth. The sample of seven observations at the pressure sites had a correlation coefficient of 0.87 with respect to equation (4) and \( \xi = 1420 \), whereas the correlation coefficient for all observations was 0.81.
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REFERENCES


DISCUSSION

B. Salm:
The measurements are showing that obviously $f$ is not constant. It would be desirable to establish a theory with this assumption, $f = f(u)$, so that we get $u$ only as a function of known quantities.

Peter A. Schaerer:
It is agreed that further studies should be carried out in this direction.

W. H. Graf:
What is the physical significance of the $\xi$ and $f$ values in equation (2)? I cannot help feeling that a Chézy-type equation — as given in equation (5) — is just as good! That is to say ‘all effects’ should be lumped in the Chézy factor for avalanches. (The Chézy relation is used with reasonable practical success in channel hydraulics.)

Peter A. Schaerer:
The observations indicated that the Chézy equation is valid only for avalanches with high speeds, but leads to unrealistic speeds on terrain with low incline and avalanches with small flow depth. Fitting the observations to equation (4) produce a correlation coefficient of 0.81, and a fit to equation (5) of 0.68. A slightly better correlation was found with the Manning equation.

K. Lied:
Were observations made on the pressure waves before the arrival of the avalanche?
Peter A. Schaerer:
No pressures due to wind blast were recorded with the load cells.

R. Sommerfeld:
In our films we see pressure waves moving the trees before the arrival of the avalanche.

W. Fritzsche:
In addition to Mr Schaerer’s method of electronic means for speed observation, I will mention the Doppler radar can become useful. For primary tests we used little battery operated Doppler radar units (frequency 10 GHz). Even lumps of 0.5 m diameter could be measured. The sensitivity depends on the distance.