Performance of two deterministic hydrological models

G. W. Kite

Abstract. It was of interest to determine the extent to which results from a simple basin model with few parameters and an automatic optimization scheme would compare to the results of larger, more complex and more difficult-to-apply models. While the small model obviously does not have the versatility and range of application of the larger models, it can be easily and quickly applied to basins without a large amount of data. Comparisons were carried out for a basin of almost 2 000 km² area in northern Ontario using 12 months daily data for calibration of the models and 12 months data for verification.

Performances de deux modèles hydrologiques déterministes

Résumé. Il était intéressant de déterminer dans quelle mesure les résultats obtenus à partir d'un modèle de bassin simple de faibles dimensions comportant quelques paramètres et un programme d'optimisation automatique se rapprocheraient des résultats obtenus avec des modèles plus grands, plus complexes et plus difficiles. Le petit modèle ne possède évidemment pas la souplesse et l'étendue d'application des grands modèles, toutefois, il peut être utilisé rapidement et facilement pour des bassins pour lesquels on possède peu de données. Les comparaisons ont été effectuées sur un bassin couvrant presque 2 000 km² dans le nord de l'Ontario à partir de données quotidiennes enregistrées pendant 12 mois, pour l'ajustement des modèles et de données recueillies sur une période d'un an, pour la vérification.

INTRODUCTION

One of the functions of the Ottawa office of the Water Resources Branch, Canadian Department of Environment, is to act in an advisory capacity to regional offices located across Canada. In the case of the Network Planning and Forecasting Section, a specific function is to provide advice and assistance, when requested by a regional office, in the field of hydrological forecasting. In order to develop this potential for providing assistance, a number of operational hydrological models such as the SSARR (US Army Engineer Division, North Pacific, 1972) and NWS (US Department of Commerce, 1972) have been obtained, set up on the computer with data from a trial basin and are being studied.

Two problems quickly became apparent: (a) a considerable knowledge of the hydrology of the basin was needed to specify the detailed relationships required by the models, and (b) even with this knowledge a large number of trial runs were necessary before a reasonable reproduction of the recorded streamflow could be obtained. Since our objective was to supply models on a package basis so that they could be applied quickly and easily, it became evident that none of the existing models was wholly satisfactory for our purpose.

The ideal which we required was a simple but accurate model that used only readily available data and could be used on small computers by personnel without expert knowledge of either the hydrology of the specific basin or the inner workings of the specific model. Clearly this was not available. It was thought, however, that it might be possible to develop
reasonably quickly a simple watershed model with automatic parameter optimization. The operation of this simple model could then be compared with those of the more complex models. The SSARR model was chosen to represent these large complex models in this study.

PROCEDURE

(1) The basin
The basin selected for the model studies is that of the Magpie River in northern Ontario. The basin has an area of 1,900 km², and is situated in the Canadian Shield of Pre-Cambrian rock. The river has a length of just over 90 km and falls approximately 330 m before entering Lake Superior. The soil cover is a mixture of productive forest and nonproductive swamp forest. Further statistics on the basin are given by Kite and Egar (1974).

(2) The models
The two models whose results are described in this paper are the Corps of Engineer's SSARR model and the Water Resources Branch model. The SSARR model is so well-known and widely used that a description is almost superfluous. However, for purposes of completeness, a very brief discussion of the watershed portion of the model is given.

(a) SSARR model
The SSARR model has three possible modes of operation for snow-covered basins. Under the first option the basin is treated as a single watershed with rainfall and snowcover. The second option divides the basin into two separate watersheds, one snow-covered area and one snow-free area. The third option subdivides the basin into a number of bands or zones of equal elevation. Each snow-band is then treated as a separate watershed with its own characteristics. The first two options are not able to accumulate snowpack, they can only deplete an existing snowpack. In this case the more complicated snow-band option with its ability to accumulate and deplete snowpack on each band was used.

Within each of the watershed options, the snowmelt can be simulated in two manners. Firstly, the temperature index method can be used in which the snowpack is multiplied by a melt factor and by a degree-day temperature index. Secondly, the more accurate energy budget approach can be used in a generalized snowmelt equation. Because of a lack of the type of meteorological data needed in the energy budget approach, it was decided to use the simple temperature index method. Snowmelt and any rainfall precipitation are combined to give a total moisture input to the basin. This moisture input is then divided into (a) direct runoff, (b) soil moisture, (c) evaporation, (d) deep percolation. The proportion of the input going to runoff depends on a user-specified function of a soil moisture index. This soil moisture index is a daily updated measure of water input less evapotranspiration. The evapotranspiration was input, in this case, as a monthly index and was modified by a measure of rainfall intensity.

The runoff generated is then distributed by user-specified functions to surface flow, subsurface flow and baseflow. All three components are routed through variable volume and times of storage before being combined as
streamflow. In the particular configuration used for this study the SSARR model required estimates of 61 parameters.

(b) Water Resources Branch model
The Water Resources Branch (WRB) model was designed to be very simple and to require minimal input data. The only continuous input data required are daily precipitation and daily mean temperature. For calibration runs daily streamflow data are also needed. Initial estimates and upper and lower limits of 20 parameters are also required input but these do not need to be accurately specified since optimum values of all the parameters are determined in a calibration run.

Depending upon the mean daily temperature the precipitation input on a specific day is assigned either to snowfall or to rain. If it is assumed to be snowfall, then it is immediately added to the first storage reservoir, the snowpack. If the precipitation is rain then a check is made whether or not a snowpack exists. If there is a snowpack, then the rain is added to the pack. If there is no snowpack, then the rain goes directly to the next lower storage reservoir, surface storage.

Snowpack, surface storage and the two other reservoirs, upper groundwater and lower groundwater are all linear and are represented in the computer program by a standard subroutine. A call to the subroutine gives the values of input flow, current storage and potential evapotranspiration and returns from the subroutine with updated current storage, runoff, flow to the next lower storage level (percolation) and actual evapotranspiration. Potential evapotranspiration is computed as an annual potential divided into daily rates on the basis of the ratio of the mean daily temperature to the sum of the mean daily temperatures for the year. If the rainfall for the specific day exceeds a given value, the potential evapotranspiration is reduced to simulate cloudy conditions. The potential evapotranspiration for a particular day is divided parametrically into a potential to be satisfied from the snowpack or surface storage and a potential to be satisfied from the upper groundwater (to represent transpiration from the root zone).

The percolation from the snowpack and surface storage enters firstly the upper groundwater storage and secondly the lower groundwater storage. Parameters for these reservoirs control the volumes and rates of flow. The model follows the continuity law in that over any period the sum of the inputs equals the sum of the outputs; there is no water lost to deep groundwater.

During a calibration run optimum values of all parameters are determined by minimizing the sum of squares of differences between computed and observed flows.

(3) Method
The main interest in developing the simple parametric model was for flood forecasting. In most areas of Canada, the largest floods are caused by snowmelt and so a major emphasis was put on having a model that worked well over the snowmelt period from April to June. Since the model may also be required for other purposes, a secondary emphasis was put on ensuring a reasonable accuracy over the whole of the year. Accordingly, the two models were first calibrated over the 3-month period 1 April-30 June, 1972. Using the
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### Table 1. Results of model applications

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<tbody>
<tr>
<td>QC</td>
<td>39.28 (SSARR) 50.13 (WRB)</td>
<td>62.98 (SSARR) 68.80 (WRB)</td>
<td>20.54 (SSARR) 26.09 (WRB)</td>
<td>18.56 (SSARR) 23.54 (WRB)</td>
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<td>QO</td>
<td>48.75 (SSARR) 48.75 (WRB)</td>
<td>67.44 (SSARR) 67.44 (WRB)</td>
<td>28.70 (SSARR) 28.70 (WRB)</td>
<td>32.49 (SSARR) 32.49 (WRB)</td>
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<tr>
<td>R</td>
<td>-0.184 (SSARR) 0.028 (WRB)</td>
<td>-0.049 (SSARR) 0.020 (WRB)</td>
<td>-0.259 (SSARR) 0.090 (WRB)</td>
<td>-0.410 (SSARR) -0.275 (WRB)</td>
</tr>
<tr>
<td>A</td>
<td>0.213 (SSARR) 0.084 (WRB)</td>
<td>0.273 (SSARR) 0.217 (WRB)</td>
<td>0.700 (SSARR) 0.240 (WRB)</td>
<td>0.686 (SSARR) 0.351 (WRB)</td>
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<tr>
<td>SE</td>
<td>13.79 (SSARR) 5.294 (WRB)</td>
<td>28.36 (SSARR) 21.17 (WRB)</td>
<td>28.20 (SSARR) 8.228 (WRB)</td>
<td>33.75 (SSARR) 20.64 (WRB)</td>
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<tr>
<td>Y</td>
<td>0.287 (SSARR) 0.109 (WRB)</td>
<td>0.428 (SSARR) 0.314 (WRB)</td>
<td>1.017 (SSARR) 0.287 (WRB)</td>
<td>1.071 (SSARR) 0.636 (WRB)</td>
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<tr>
<td>NSR2</td>
<td>0.832 (SSARR) 0.974 (WRB)</td>
<td>0.611 (SSARR) 0.770 (WRB)</td>
<td>-0.480 (SSARR) 0.866 (WRB)</td>
<td>-0.133 (SSARR) 0.555 (WRB)</td>
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<td>PHASE</td>
<td>1/2/3</td>
<td>2/3</td>
<td>3/5</td>
<td>3/5</td>
</tr>
<tr>
<td>STORAGE</td>
<td>71.70 (SSARR) 87.50 (WRB)</td>
<td>71.70 (SSARR) 34.40 (WRB)</td>
<td>71.70 (SSARR) 35.70 (WRB)</td>
<td>71.70 (SSARR) 34.40 (WRB)</td>
</tr>
<tr>
<td>TIME</td>
<td>278.2 (SSARR) 71.51 (WRB)</td>
<td>9.522 (SSARR) 7.229 (WRB)</td>
<td>250.0 (SSARR) 43.88 (WRB)</td>
<td>21.03 (SSARR) 53.06 (WRB)</td>
</tr>
</tbody>
</table>

parameters derived from these calibration data, the models were used in a verification mode on data for the period 1 April-30 June, 1973. Secondly, the models were re-calibrated using data for the hydrological year from 1 October 1971 to 30 September 1972 and verified over the period 1 October 1972-30 September 1973.

### RESULTS AND DISCUSSION

The results of the applications of the two models to the two calibration data sets and the two verification data sets are summarized in Table 1. The statistics used to compare the model performances are as follows:

- **QC**: the mean of the computed or simulated discharges over the $n$ days (m$^3$/s);
- **QO**: the mean of the observed or recorded discharges (m$^3$/s);
- **R**: the ratio of the relative error (computed-observed) to the mean of the observed discharges;
- **A**: the ratio of the absolute error to the mean of the observed discharges;
- **SE**: the standard error (m$^3$/s);
- **Y**: the coefficient of variation of the residuals;
- **NSR2**: the measure of model efficiency proposed by Nash and Sutcliffe (1970);
- **PHASE**: the ratio of the number of simulated peak flows incorrectly timed to the total number of peak flows simulated;
- **STORAGE**: the central core storage in thousands of bits required by the model on a CDC CYBER 74 system;
- **TIME**: the time in seconds for which the model is using the central processor. The time for verification is measured in the same way for both models; a single computer run using previously determined parameters. The time of calibration in the case of the WRB model is also for a single computer run. In the case of the SSARR model, however, where no automatic optimization is...
available, the time of calibration is the sum of the times of a series of computer runs comprising a manual optimization procedure. In the latter case, no time has been included for the time of the hydrologist in reviewing each computer run and determining the necessary changes in parameter values. The times shown are central processing times on the CYBER 74 only and do not include compilation time. This is because the WRB model was compiled separately for each of the four runs while the SSARR model was compiled initially and then kept in an object deck for all subsequent runs.

Looking first of all at Fig. 1 comparing the recorded hydrograph for the period April-June 1972 with the calibration hydrographs from the SSARR and WRB models it is apparent that both models provide a reasonable fit although they do not reproduce recorded conditions exactly. The SSARR hydrograph shown is the best fitting of a series of 29 computer runs and while it could probably have been improved by making further runs there soon comes a point at which the rate of improvement becomes very slow. It is interesting to note that both the SSARR and WRB models fall below the recorded curve at the start of the flood (15-20 April) and then both rise above the recorded curve (22-26 April) before falling back in line. This may indicate an inconsistency in the meteorological data for this period.

Turning to the comparison of test statistics for the 3-month calibration period it is apparent that the WRB model provides a slightly better fit than the SSARR model. The preferred test statistic is the Nash and Sutcliffe

![Graph showing recorded and model hydrographs for April-June 1972](image-url)
efficiency (NSR2) since this is a direct measure of the proportion to the variance of the recorded flows explained by the model. Over the 3-month calibration period the WRB model accounted for 97 per cent of the original variance while the SSARR model accounted for 83 per cent. The computer time taken for the 29 runs of the SSARR model was 278 s compared to 72 s for the one optimization run of the WRB model. Perhaps more important than the computer operation time is the real office time spent on the calibration. Using a remote terminal to the CDC CYBER 74 with the SSARR model stored on magnetic disc in object form it was possible to run the model only 2 or 3 times per day. Including time taken to review results and make parameter adjustments it took nearly a month to make the 29 calibration runs. This month does not include the time necessary to get the program compiled correctly, to develop the data in its specialized form and to make a successful first run.

Figure 2 shows the recorded streamflow data for the verification period April-June, 1973, and the hydrographs computed by the two models. It is apparent that the recorded hydrograph is very complex, having three distinct peaks within a 1-month period. The first peak (17 April) is due to a sharp rainstorm while the second (23 April) and third (17 May) peaks are due to snowmelt. It is a very unusual hydrograph since the rainfall apparently went straight to runoff without causing immediate snowmelt. A study of temperature records indicates that these were higher for the second peak than for the third and yet the third peak is by far the highest. This sequence is not one which would normally be chosen to verify a model but since that is what happened in nature the verification was continued. Both models missed the

![FIGURE 2. Verification hydrographs: Magpie basin, April-June, 1972.](image-url)
first (rainfall) peak and the WRB model mis-timed and under-estimated both the second and third peaks. In terms of statistics, however, Table 1 shows that the WRB model performed better than SSARR both in standard error and in Nash and Sutcliffe efficiency. The single run times for the two models are reasonably close.

Columns 6-9 of Table 1 show the results of the model calibrations over the 12 month period from October 1971 to September 1972 and the verifications over the period from October 1972 to September 1973. Twelve runs of the SSARR model were made with various parameter changes before a satisfactory result was obtained. At an average CP time of just under 21 s this amounted to 250 s in total. For contrast the WRB model took 43 s for its one calibration run. During the verification runs, however, the SSARR model took less than half of the CP time of the WRB model.

CONCLUSIONS

A simple linear basin model has been developed which gives results which are statistically at least equal to those of a more complex model. At the same time this model can be applied with only one computer run compared to the many runs required with non-optimizing models. This saves computer time and money and more importantly, saves the time of the office engineer; relieving him from the tedious, frustrating job of manually adjusting parameters in a curve-fitting procedure. Care must always be taken, however, in any automatic optimizing procedure and the results must be carefully checked for hydrological realism.

It is suggested that the most appropriate use of the two different types of model is as follows. The small optimizing model is suitable for use in those cases in which results are required in a very short time. The results from this model can then be used as an indication of those components of the hydrological cycle important in the study basin. This information can then be used to obtain faster results from models such as SSARR by cutting down on the number of trial-and-error computer runs.

Development of the WRB model will continue in an attempt to improve its performance.

REFERENCES