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# ESTIMATING PARAMETERS OF COMPACTING FINE-GRAINED INTERBEDS WITHIN A CONFINED AQUIFER SYSTEM BY A ONE-DIMENSIONAL SIMULATION OF FIELD OBSERVATIONS

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Abstract

A one-dimensional mathematical model that calculates idealized aquifer-system compaction and expansion has been applied to observed water-level fluctuations and to the resulting observed transient compaction-and-expansion behavior of a total thickness interval at one site in the San Joaquin Valley and at several sites in the Santa Clara Valley. Values for total cumulative thickness $b'_{sum}$ of fine-grained interbeds within the confined aquifer system, the weighted-average thickness $b'_{equiv}$ of these interbeds, recoverable vertical compressibility $S'_{ske}$, and the initial distribution of preconsolidation pressure $p'_{max}(z, 0)$ must be approximated from field data independently of any subsequent simulation process. Only values for vertical components of hydraulic conductivity K' and nonrecoverable compressibility $S'_{skv}$ are adjusted by trial-and-error to fit calculated to observed compaction history. The estimated values for K' at the sites studied range from $1.7 \times 10^{-7}$ to $3.8 \times 10^{-6}$ m/day, those for $S'_{skv}$ range from $4.6 \times 10^{-4}$ to $4.3 \times 10^{-3}$ $m^{-1}$, and those for $S'_{ske}$ range from $7.2 \times 10^{-6}$ to $5.2 \times 10^{-5}$ $m^{-1}$. For established values of $b'_{sum}$, $b'_{equiv}$, and $p'_{max}(z,0)$, it is demonstrated that carefully evaluated values of K' and $S'_{skv}$ can be used to predict aquifer-system behavior with reasonable accuracy over periods of several decades.

A major problem facing hydrologists is how to predict land subsidence. Developing a reliable method for evaluating aquitard parameters for modeling purposes is a key to solving this problem. A field method of aquitard parameter evaluation has advantages over laboratory methods. For example, parameters estimated from direct macroscopic field measurements represent mechanical properties of large-scale behavior *in situ*. By contrast, after laboratory consolidation data have been analyzed the vertical distribution in the field of mechanical properties of soil-specimen behavior must be estimated statistically.

If pumping rates at the center of a well field are measured or projected, then for a computational model of skeletal (matrix) behavior to be considered reliable it must accurately account for both transient radial fluid flow (and the corresponding cumulative radial displacement of the skeletal matrix) and transient vertical fluid flow (and the corresponding cumulative vertical displacement of the skeletal matrix). Alternatively, if water-level fluctuations in the coarse-grained part of a confined aquifer system are directly measured or projected, the conceptual and computational problems are greatly simplified. For this second alternative a one-dimensional model of vertical deformation of the matrix is both sufficient and reliable.

A one-dimensional model of vertical deformation has been developed (Helm, 1975, 1976) that essentially uses Terzaghi's theory of consolidation. Using the concept of preconsolidation pressure developed in soil mechanics allows nonrecoverable compaction of fine-grained beds to be distinguished numerically from recoverable compaction and expansion. Computation of vertical deformation patterns is thereby simplified.

Extensometers in the field (Riley, 1969) measure the vertical behavior of a total thickness interval. According to standard convention, it is assumed that the nonrecoverable component of observed compaction occurs within fine-grained deposits. Hence the term $b'_{sum}$ will represent only the cumulative thickness of the fine-grained component of a total thickness interval. Because the depth intervals of interest are generally beneath the overlying semiconfining bed, the behavior of such a bed was not modeled. For simplicity of calculation, all deformation is assumed to occur within idealized doubly draining clay interbeds characterized by no-flow midplanes (aquitards).

For simulating field compaction the two parameters $S'_{skv}$ and $K'$ (nonrecoverable vertical specific storage and vertical hydraulic conductivity, respectively) are of critical importance. For macroscopic behavior $S'_{skv}$ is intimately related to total thickness of fine-grained compacting beds $b'_{sum}$ and to ultimate nonrecoverable compaction $\Delta b'_{ult}$ of the total thickness. In other words, $S'_{skv}$ equals $\gamma_w(\Delta b'_{ult}/b'_{sum})/\Delta p'_{max}$ where $\gamma_w$ is the unit weight of water. The use of $\Delta b'_{ult}$ implies that a specified increase in past maximum effective stress (preconsolidation pressure) $\Delta p'_{max}$ has progressed <u>in situ</u> throughout $b'_{sum}$ from one equilibrium distribution of $p'_{max}$ to another.

Unfortunately, because values for $K'$ of fine-grained interbeds are small and because water levels within adjacent coarse-grained material vary with time, $p'_{max}(z, t)$ seldom if ever reaches a new equilibrium distribution within thicker clay interbed members of a confined aquifer system. Correspondingly, observed compaction $\Delta b'_{sum}(t)$ is transient in the field and seldom reaches its theoretical ultimate value $\Delta b'_{ult}$ in response to boundary $\Delta p'_{max}$ observed in the coarse-grained material (aquifers).

The rate of observed compaction $d(\Delta b'_{sum})/dt$ is significant in evaluating $K'$. Rate of compaction depends essentially on three things: (1) the number N and thickness $b'_i$ of individual fine-grained compacting beds, (2) the effective value $K_i$ for each i-<u>th</u> bed, and (3) the history of boundary stress values at all the 2N aquitard-aquifer interfaces. It is not possible in the field to measure $d(\Delta b'_i)/dt$ independently for each bed, nor thereby to estimate a distinct value of $K_i$ for each i-<u>th</u> bed. Because rate of compaction is observed in the field for a total aggregate of coarse- and fine-grained material, only a single $K'$ value (or function) for cumulative observed behavior at a site can be approximated.

Helm (1975) tested the computational appropriateness and efficiency of using a weighted-average clay-bed thickness $b'_{equiv}$ in simulating an observed rate of compaction of a series of 21 doubly draining clay interbeds within a single confined aquifer system near Pixley, California. Instead of calculating each $\Delta b'_i(t)$ and assuming an identical $K'$ and $S'_{skv}$ for each i-<u>th</u> bed and then summing over i, $\Delta b'_{equiv}(t)$ was calculated and multiplied by a constant of proportionality $N_{equiv}$; this change in procedure reduced computation time by more than an order of magnitude. In other words, considerable computer time was saved, and observed $\Delta b'_{sum}(t)$ was as closely approximated by calculating $N_{equiv}\,\Delta b'_{equiv}(t)$ as it would have been by calculating $\sum_{i=1}^{N} \Delta b'_i(t)$ where near Pixley $N_{equiv}$ equals 18 and N equals 21.

A third parameter of importance is a vertical recoverable specific storage parameter $S'_{ske}$. Its value can be estimated from field data only where vertical expansion is observed. Although $S'_{ske} \ll S'_{skv}$, Helm (1976) has shown that $S'_{ske}$ noticeably affects stress/strain behavior in the field, not only when current effective stress $p'(z, t)$ is less than the past maximum effective stress, as expected, but also when $p'(z, t)$ equals $p'_{max}(z,t)$. In other words, the influence of $S'_{ske}$ is not completely masked by $S'_{skv}$ even when stresses reach preconsolidation levels.

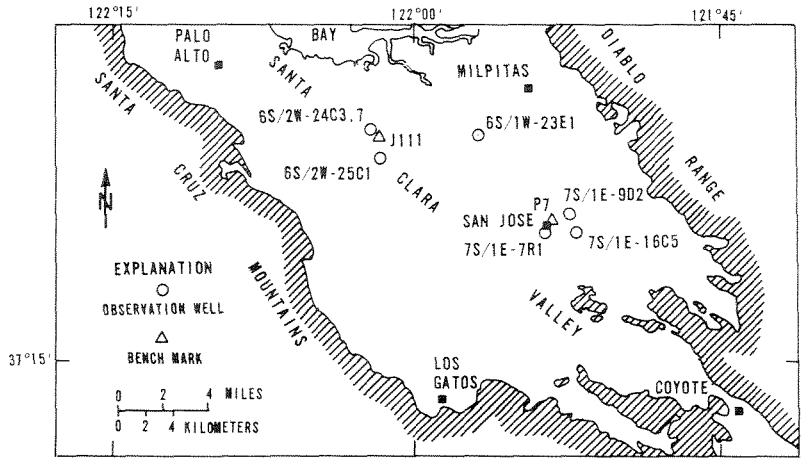


Figure 1.--Location of selected wells in Santa Clara Valley, California

Before the calculation process begins, it is necessary to estimate the initial value of $p'_{max}(z, t)$, at every nodal point within the model. The observed stress at which only net vertical expansion is actually measured in situ gives an indication of a preconsolidation pressure at the midplane of the thickest aquitard within the measured depth interval. This information, and measurement of stress history within coarse-grained material, allow one to estimate an initial distribution of preconsolidation pressure $p'_{max}(z, 0)$ within an idealized or model aquitard.

The procedure used by the author to evaluate system-behavior parameters from observed water-level fluctuations and compaction is to match calculated to observed compaction and expansion by trial-and-error adjustment of parameters. The model has been applied to field data from six sites in the Santa Clara Valley (fig. 1) and one site (Pixley) in the San Joaquin Valley, Calif. The location of the Pixley site is shown by Lofgren and Klausing (1969) and by Helm (1975). The model has also been applied to two additional sets of field data in the Santa Clara Valley where infrequently collected observations cover periods of 54 and 60 years.

Table 1 lists the values for $b'_{sum}$, $b'_{equiv}$, $K'$, $S'_{skv}$, and $S'_{ske}$ that were used to simulate observed compaction and expansion at selected sites (fig. 1). Table 1 also lists periods of record of field data at these sites. Wherever possible, the value for $S'_{ske}$ was estimated directly from rebound field data. After values for $b'_{sum}$, $b'_{equiv}$, $S'_{ske}$ and the initial distribution of $p'_{max}$ had been approximated for a site, they were not changed. Only values for $K'$ and $S'_{skv}$ were adjusted in order to simulate observed compaction history as closely as possible. The history of transient effective stress at the idealized aquitard-aquifer interface is estimated directly from observed water-level fluctuation within the confined aquifer by a method developed by Lofgren (Lofgren and Klausing, 1969, p. B64-B68). Using boundary stress fluctuations, a thickness value, and constant parameter values in a manner described by Helm (1975), compaction values can be calculated for each site.

Figure 2 shows model results for a San Joaquin Valley site near Pixley, Calif., using parameters estimated by Riley (1969) (table 1). The solid line in figure 2A represents calculated compaction; the dotted line

TABLE 1.--VALUES OF PARAMETERS USED FOR SIMULATING OBSERVED COMPACTION AT SELECTED SITES IN SANTA CLARA AND SAN JOAQUIN VALLEYS, CALIFORNIA

| SITE OF WATER-LEVEL DATA | PERIOD OF RECORD (YEARS) | | ESTIMATED THICKNESS (METERS) | | ESTIMATED VALUES OF PARAMETERS | | |
|---|---|---|---|---|---|---|---|
| | WATER-LEVEL DATA | COMPACTION DATA | TOTAL $b'_{sum}$ | WEIGHTED AVERAGE $b'_{equiv}$ | $K'$ $\frac{METER}{DAY} \times 10^{-6}$ | $S'_{skv}$ $\frac{1}{METER} \times 10^{-4}$ | $S'_{ske}$ $\frac{1}{METER} \times 10^{-6}$ |
| SANTA CLARA VALLEY | | | | | | | |
| 6S/1W-23E1 | 1958-72 | 1958-72 | 57.6 | 4.6 | 0.36 | 4.6 | 24.6 |
| 6S/2W-24C3 | 1960-72 | 1960-72 | 77.7 | 5.5 | 0.17 | 43.0 | 13.0 |
| 6S/2W-24C7 | 1960-72 | 1960-72 | 128.6 | 16.2 | 3.8 | 11.0 | 8.5 |
| 6S/2W-25C1 | 1960-72 | 1960-72 | 77.7 | 5.5 | 0.23 | 10.7 | 52.0 |
| 6S/2W-25C1 | 1921-74 | 1932-74 | 77.7 | 5.5 | 0.22 | 10.8 | -- |
| 7S/1E-7R1 | 1915-74 | 1916-74 | 145.4 | 5.5 | 0.57 | 7.38 | -- |
| 7S/1E-9D2 | 1958-72 | 1960-63, 1965-72 | 78.6 | 5.5 | 0.25 | 20.0 | 26.0 |
| 7S/1E-16C5 | 1958-72 | 1960-72 | 145.4 | 5.5 | 1.0 | 7.55 | 7.2 |
| SAN JOAQUIN VALLEY | | | | | | | |
| 23S/25E-16N3 | 1952-71 | 1959-71 | 84.7 | 4.9 | 2.5 | 7.55 | 15.0 |

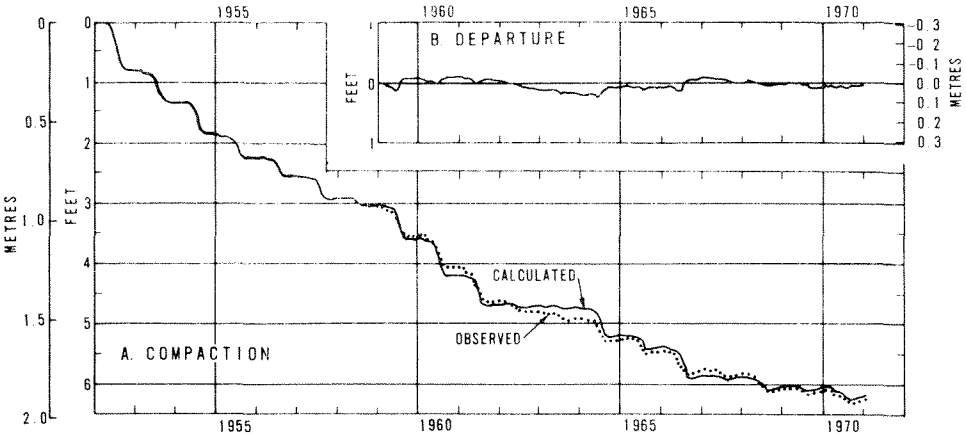


Figure 2.---Simulation of compaction based on water-level data for well 23S/25E-16N3 (1952-71) and on the difference in compaction observed in wells -16N3 and -16N1. (From Helm, 1975, Figure 8.)

represents observed compaction. Figure 2B shows the difference between calculated and observed compaction from 1959 to 1971. Transient boundary effective stress values were plotted and discussed by Riley (1969) and Helm (1975, 1976) and are not repeated here.

Figures 3 through 10 show simulation results for sites in Santa Clara Valley (fig. 1) using parameter values listed in table 1. As mentioned earlier, measured and continuously changing depth-to-water values are used at each site in order to estimate transient boundary effective stress values

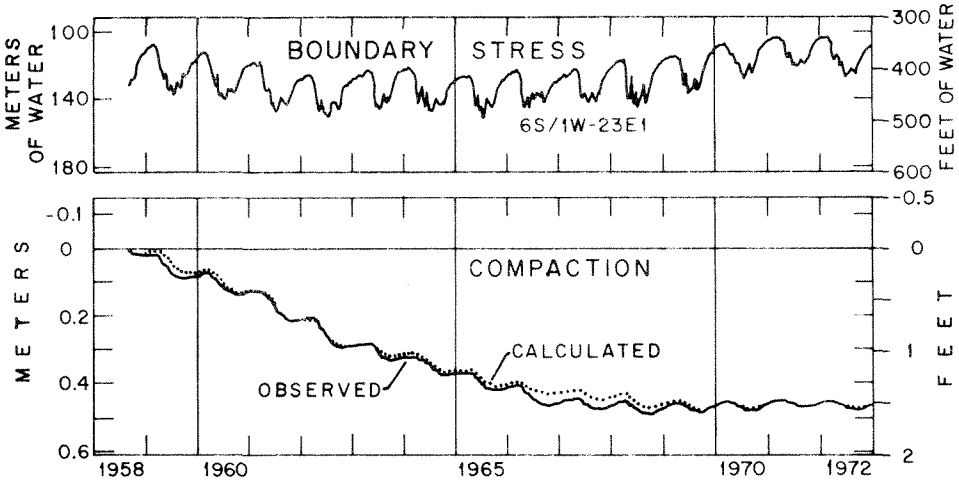


Figure 3.--Simulation of compaction based on water-level and compaction data for well 6S/1W-23E1 (1958-72).

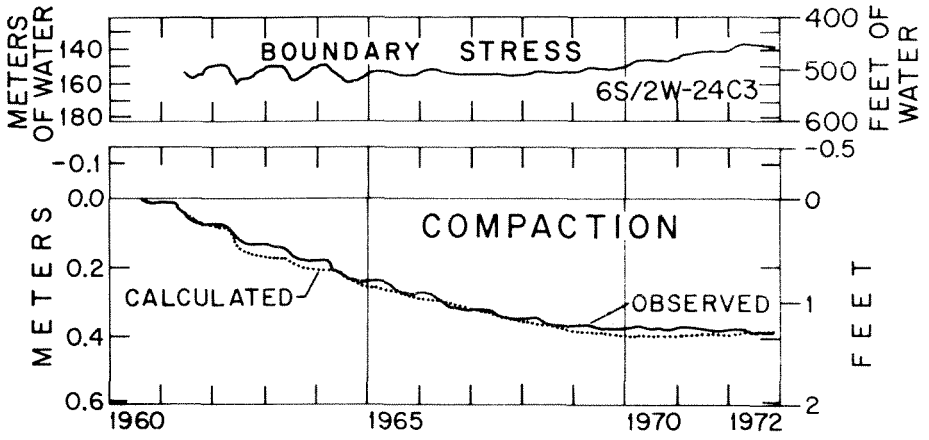


Figure 4.--Simulation of compaction based on water-level and compaction data for well 6S/2W-24C3 (1960-72).

at the top of figures 3 through 10 that are needed for calculating compaction. Observed compaction in figures 2 and 5 represent compaction of a lower depth interval at two sites. Hence a difference in observed compaction in two wells at each site is used in figures 2 and 5. Calculated compaction does not differ greatly from observed compaction at any site (figs. 3-10). Estimated values for K' in the Santa Clara Valley range from $1.7 \times 10^{-7}$ to $3.8 \times 10^{-6}$ m/day; estimated values for $S'_{skv}$ range from $4.6 \times 10^{-4}$ to $4.3 \times 10^{-3}$ $m^{-1}$; and estimated values for $S'_{ske}$ range from $7.2 \times 10^{-6}$ to $5.2 \times 10^{-5}$ $m^{-1}$.

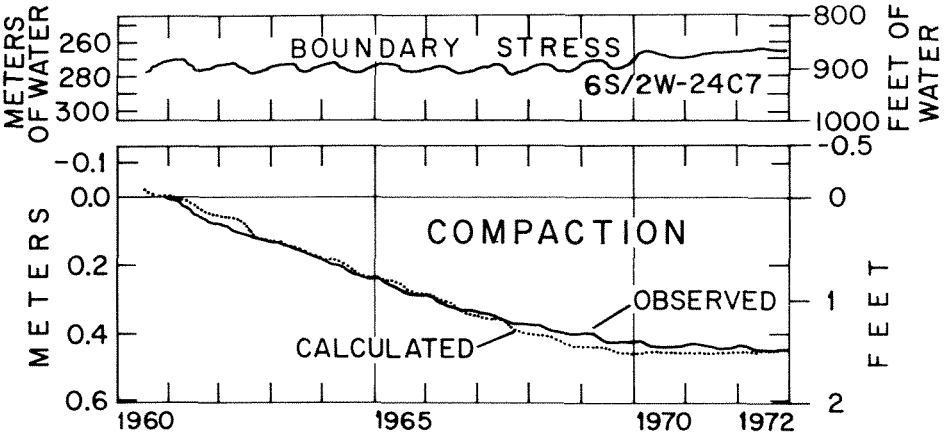


Figure 5.--Simulation of compaction based on water-level data for well 6S/2W-24C7 (1960-72) and on the difference in compaction observed in wells -24C7 and -24C3.

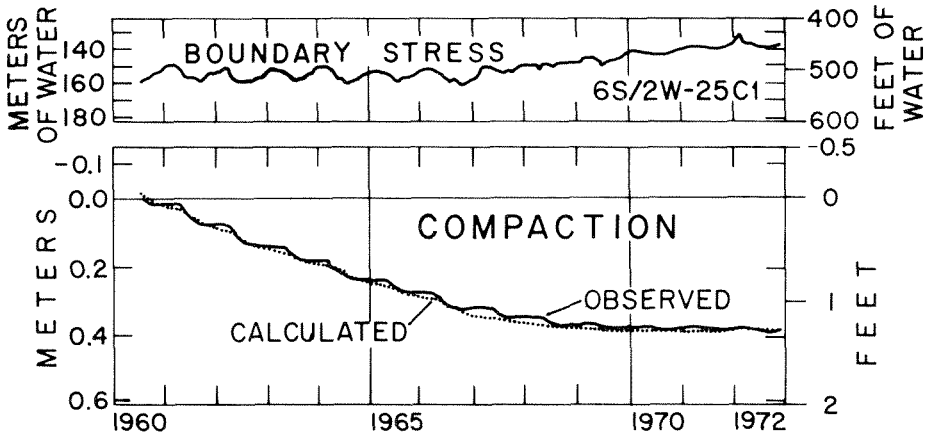


Figure 6.--Simulation of compaction based on water-level data for well 6S/2W-25C1 (1960-72) and compaction data observed in well -24C3.

It is instructive to discuss nonuniqueness of estimated parameter values and thereby the degree of confidence in selecting a single set of parameter values (or functions) as a predictive tool. If at any site (fig. 1) the value for either total compacting aquitard thickness $b'_{sum}$ or equivalent aquitard thickness $b'_{equiv}$ were changed, from the value listed in table 1, different values of $S'_{skv}$ and K' would also have to be found in order for calculated compaction to fit observed compaction accurately. This interdependence is evident because for known compaction history and predetermined bed-thicknesses, $S'_{skv}$ is related inversely to $b'_{sum}$ and K' is

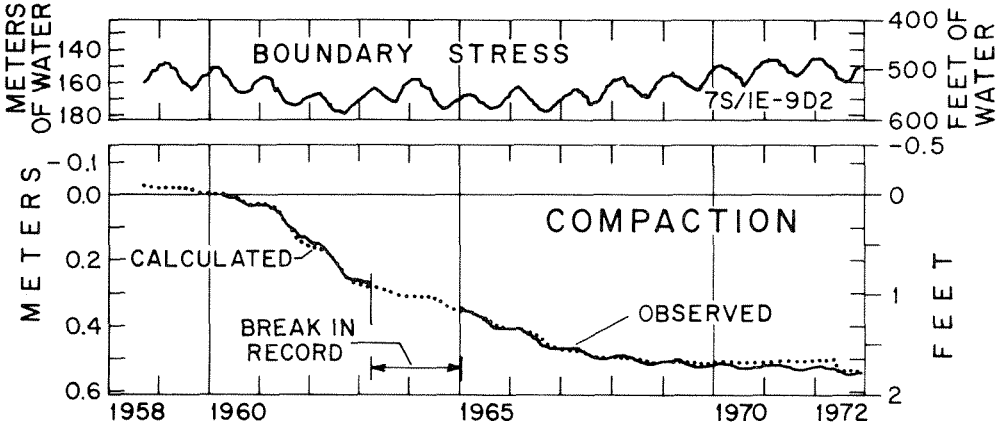


Figure 7.--Simulation of compaction based on water-level data (1958-72) and compaction data (1960-72) for well 7S/1E-9D2.

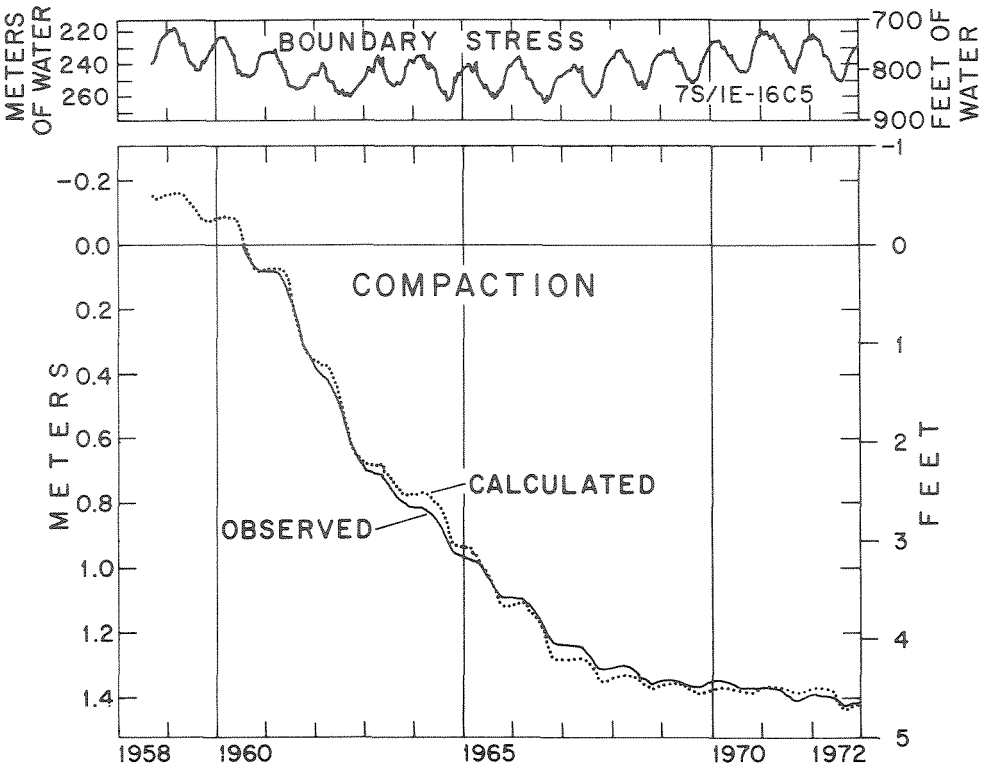


Figure 8.--Simulation of compaction based on water-level data for well 7S/1E-16C5 (1958-72) and on compaction data observed in wells -16C6 and -16C11.

related approximately to $b'_{equiv}$. Therefore, values of $b'_{sum}$ and $b'_{equiv}$ must be estimated unambiguously by geophysical or other field measurement and statistical methods. In the Santa Clara Valley such estimates have been made (J. F. Poland, written commun., 1973-76) for depth intervals of interest from the best available geophysical data.

A second problem of nonuniqueness is the initial distribution of $p'_{max}(z, t)$. This distribution in effect implies an assumed average degree of consolidation U of fine-grained material in the field at the beginning of simulation. This initial preconsolidation stress condition affects all subsequent computations of compaction and hence affects the estimated values of K' and $S'_{skv}$ that are needed to duplicate observed compaction. Changing $p'_{max}(z, 0)$ for any subsequent computer run will require searching anew for appropriate model values of K' and $S'_{skv}$.

A third more subtle problem of nonuniqueness will be discussed here in some detail (fig. 11). Using any set of values for K' and $S'_{skv}$ on the dashed line in figure 11 (or on extensions of the dashed line) within the computational model will give the correct net compaction at 6S/1W-23E1 (fig. 3) between the two dates March 1959 and December 1972. However, the calculated rate of compaction between these two dates also varies according

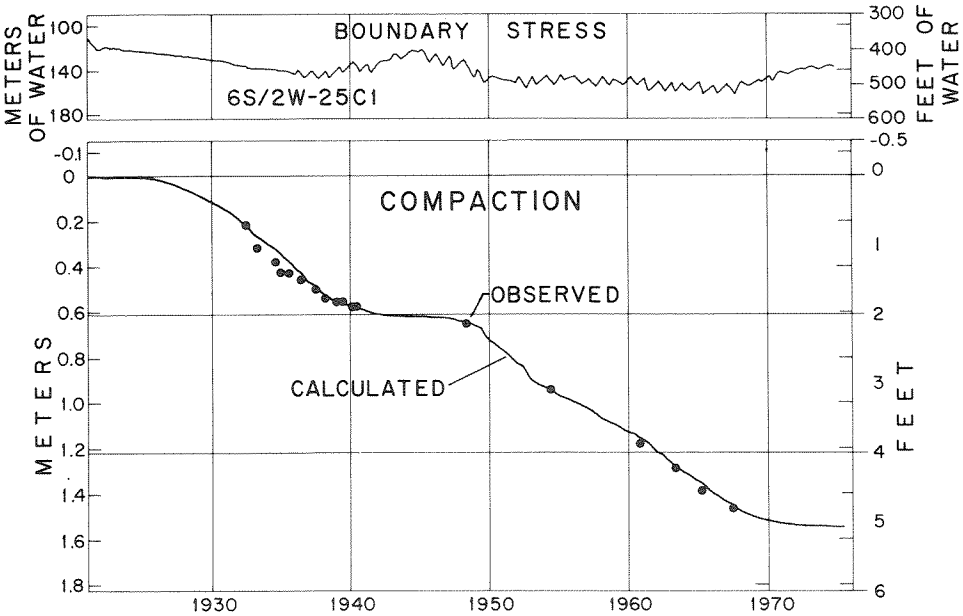


Figure 9.--Simulation of compaction based on water-level data for well 6S/2W-25C1 (1921-74) and on compaction estimated as a portion of subsidence measured at bench mark J111.

to choice of parameter values. In contrast, using any set of K' and $S'_{skv}$ values on the solid line in figure 11 gives the correct net compaction between the two dates March 1959 and December 1963. Similarly, using parameter values on the dotted line will duplicate net observed compaction between the two dates March 1959 and December 1966. To simulate observed compaction closely throughout the entire 1959-72 period, however, the choice of appropriate parameter values becomes more limited, namely near the area where these three zero error lines (dashed, solid, and dotted) tend to converge or cross. Solid circles in figure 11 represent some of the parameter values for which computer runs have been made. Each set of three values in parentheses is associated with parameter values at each solid circle. For each of the three indicated time periods a corresponding value in parentheses gives the departure of calculated compaction from observed compaction in percent of net observed compaction (namely, in percent of 0.45 m compaction observed between March 1959 and December 1972). For example: using parameter values for 6S/1W-23E1 (table 1), figure 11 indicates that the calculated compaction (fig. 3) between March 1959 and December 1963 is 2 percent less than the above-mentioned observed compaction, is 6 percent less between March 1959 and December 1966, and is 0.07 percent less between March 1959 and December 1972. In order to estimate parameter values from field data with sufficient confidence to make the method a predictive tool it is essential to know, to calculate, and to reproduce accurately the rate of compaction as well as the net observed compaction between two specified dates.

Regarding the problem of model calibration, if for example one uses measurements of March 1959 and December 1963 alone to estimate parameter values, the solid line in figure 11 (and its extension) can be considered a zero-error calibration line. The choice of ($3.4 \times 10^{-7}$ m/day, $5.6 \times 10^{-4} m^{-1}$) or ($6 \times 10^{-7}$ m/day, $3.3\ m^{-1}$) for (K', $S'_{skv}$) would be as appropriate for such a calibration as using values given in table 1 for 6S/1W-23E1 and

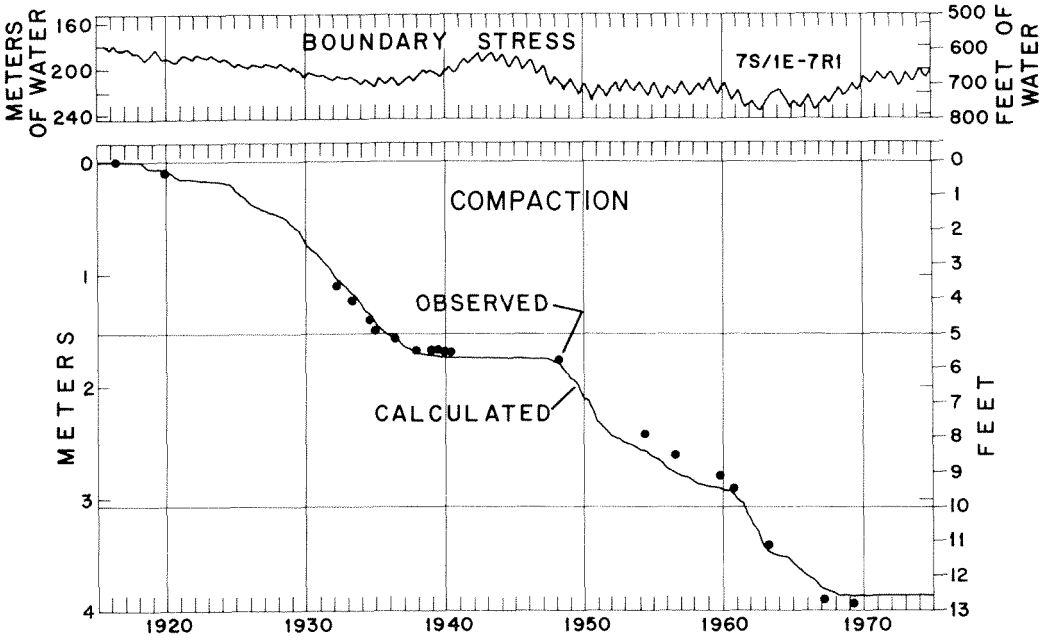


Figure 10.--Simulation of compaction based on water-level data for well 7S/1E-7R1 (1915-74) and on subsidence measured at bench mark P7.

discussed in the previous paragraph. However, according to figure 11, predicting observed behavior at this site through 1972 would be in error (5 percent too much in the one case and 13 percent too little in the other case). Simulation error for the 1959-72 period is even greater for parameter values lying on extensions of the solid line in figure 11.

The final question to be addressed in the present paper is whether constant values of K' and $S'_{skv}$ can be used to predict compaction at a site over an extended period of time. Attention is called to figure 6 where compaction at site 6S/2W-25C1 is computed (dotted line) using daily values of water-level data (transformed into boundary stress in figure 6) from 1960 through 1972, and also to figure 9 where compaction is computed (solid line) using periodic water-level measurements at the same site (collected into the boundary stress line in figure 9) from 1921 through 1974. Each solid circle in figure 9 represents a compaction value on a particular day which has been estimated from first-order leveling from a stable datum to nearby bench mark J111 at land surface. The solid line (and its extension in either direction) in figure 12A shows what values of K' and $S'_{skv}$ are needed to compute net observed compaction correctly between two specific dates, spring 1932 and spring 1939. The dashed line in figure 12A similarly shows appropriate parameter values for accurately simulating net observed compaction between the two dates spring 1932 and spring 1967. The upper left solid circle in figure 12A indicates parameter values (table 1) which were used for calculating compaction (solid line) in figure 9. The numbers in parentheses in figure 12A indicate that by using $2.2 \times 10^{-7}$ m/day for K' and $10.8 \times 10^{-4}$ $m^{-1}$ for $S'_{skv}$ the net calculated compaction between Spring 1932 and spring 1939 is 2 percent larger and between spring 1932 and spring 1967 is 0.8 percent larger than net observed compaction of 1.2 m between spring 1932 and spring 1967. The numbers in parentheses near the lower right solid circle indicate

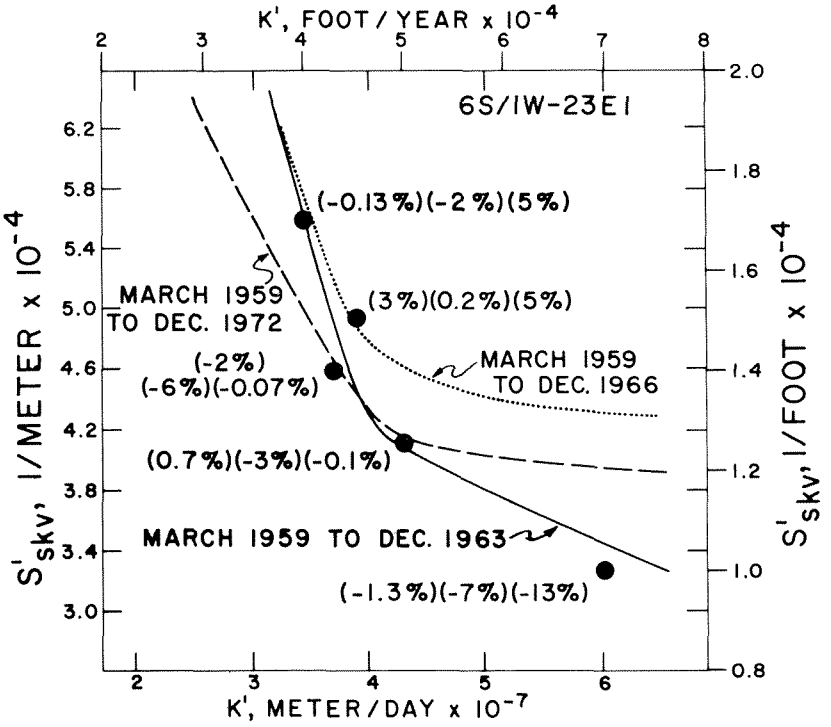


Figure 11.--Zero-error curves for simulation at well 6S/1W-23E1.

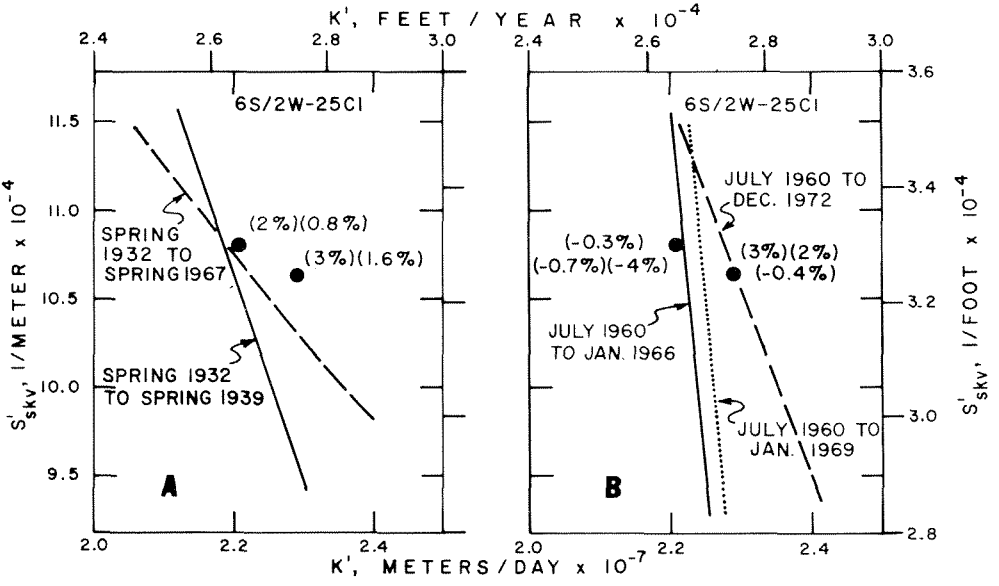


Figure 12.--Zero-error curves for simulation at well 6S/2W-25C1.

that by using $2.3 \times 10^{-7}$ m/day for K' and $10.7 \times 10^{-4}$ $m^{-1}$ for $S'_{skv}$ the net calculated compaction between spring 1932 and spring 1939 is 3 percent larger and between spring 1932 and spring 1967 is 1.6 percent larger than the aforementioned net observed compaction. These latter parameter values were used (table 1) for computing compaction at 6S/2W-25C1 from 1960 to 1972 (fig. 6). Hence the use of the 1960-to-1972-based parameters to duplicate compaction for several decades leads to very small error.

Figure 12B is also a plot of zero-error curves for site 6S/2W-25C1 between specified dates. Figure 12B is related to figure 6 as figures 11 and 12A are related to figures 3 and 9. Considering the difficulties of model calibration discussed earlier and the sparseness of stress and compaction data at 6S/2W-25C1 prior to 1960, examination of figures 12A and 12B suggests that carefully evaluated values of K' and $S'_{skv}$ can be used to predict both rate of compaction and total compaction with reasonable accuracy over periods of several decades.

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