Calculation of instantaneous unit hydrographs in an urban area

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Abstract. This paper deals with a parameter identification problem for the rainfall–runoff relationship in an urban area. Unit hydrographs have been calculated for areas with different percentages of impervious cover with the aid of measured rainfall and runoff. Two case studies are reported here, one a sewer district in Enschede with low quality data and the other some specially equipped catchment areas with high quality data in Lelystad, a new town in one of the IJsselmeer polders. These catchment areas include a parking lot, a residential quarter and a shopping centre.

In the case study of a sewer district in Enschede one method was applied, based on Laguerre functions, to calculate unit hydrographs. Four methods were used in the second case study and the provisional results were compared. As a first method the Nash cascade was chosen as a model for the rainfall–runoff relationship. The parameters \( n \) and \( k \) were calculated by moments. Next some exercises were made with the Fourier transform method. The Nash cascade can be expanded with Laguerre functions. The last method concerns quadratic programming, which minimizes the differences between computed and recorded runoff in the sense of least squares. Besides the influence of the distribution of the losses on the unit hydrograph, the different unit hydrographs have been illustrated for the catchment areas. Finally, as the last stage of modelling, a start has been made validating the identification methods by the calculation of correlation coefficients.

LIST OF SPECIAL SYMBOLS

- \( ch(f), cp(j), cq(j) \): Laguerre coefficients of the IUH, \( P \) and \( Q \)
- \( h, h_N(t) \): instantaneous unit hydrograph (time \(^{-1}\)), IUH of the Nash model
- \( L_n(x) \): Laguerre polynomial of order \( n \) and argument \( x \)
- \( k \): reservoir coefficient (time)
- \( n \): number of reservoirs in a Nash cascade
- \( P(t) \): rainfall intensity (length/time)
- \( Q(t) \): runoff intensity (length/time)
- \( t, \tau \): time
- \( \omega \): frequency (l/time)
INTRODUCTION

The statement of the problem

During the last ten years progress towards optimal water resources management in urban areas has accentuated the lack of knowledge concerning the transformation of rainfall into sewer discharge in these areas (McPherson, 1976). Two aspects of this lack of knowledge can be distinguished; one concerning the way to measure the transformation and the other the physical processes involved. The practice of models begins with the formulation of the model, with attention to the suppositions, followed by the computation of parameters and the simulated runoff and terminated by the verification of the results by comparison with the measured runoff.

As a part of the research into rainfall—runoff transformation a number of case studies are carried out on areas provided with the best data.

The quality of the data

Most of the existing data concerning urban rainfall—runoff are gathered from existing structures. Usually these data are not of a very high quality and can only yield rough information about the processes under consideration. Only at very few places data are gathered in specially equipped areas. The automatic collection and digestion of these data often require more attention than expected. Only these high quality data should be used for such delicate studies as the comparison of models. Even with data from specially equipped catchment areas one has to be careful, because, especially during the first years after the construction is finished, changes of the infiltration capacity of both impervious and pervious surfaces, settling of the subsoil and changes in the vegetation may alter the rainfall—sewer discharge relation. Further, an accurate evaluation of the manner of recording is necessary to prevent systematic errors. The investigation of the propagation of data errors into the computations, for which results are presented here, is not yet performed. Data from these catchment areas should be completed with the results of experiments with artificial rainfall in these areas or in laboratories, and other experiments.

In this paper some results will be presented from two case studies about modelling. One study was carried out with data obtained with gaugings in an existing sewer in the town of Enschede. The other study concerns some specially equipped catchment areas which are situated in the new town of Lelystad in one of the IJsselmeer polders (Kraijenhoff van de Leur and Zuidema, 1969).

Schematization of the rainfall—runoff process

The transformation of rainfall into sewer discharge can be represented by the systems shown in the upper part of Fig.1. A more sophisticated scheme for system 2, used in the case study at Lelystad, is shown in the lower part of Fig.1.

System 1 is a fiction, to ease the computations.  
System 2a is formed roughly by the pervious and impervious surfaces.  
System 2b is formed roughly by the sewer system to outlet (so far there is no leakage in the sewer)

Each input is composed of a deterministic and a stochastic part. The models discussed here suppose linear and time invariant systems. Then the deterministic part is
The losses
The losses in system 1 are formed by evaporation, infiltration, wetting of surfaces and filling of depressions. For the critical storms of short duration and high intensities evaporation is negligible. Wetting of surfaces and filling of depressions can be considered as an initial loss, which amounts on the impervious areas usually only to a few millimetres (Reinhold, 1955; Zondervan and Dommerholt, 1976; Miller and Viessman, 1970).

Under Dutch circumstances, a flat country and West European ocean climate, sewer runoff will in most cases be generated only by roofs and pavements. The infiltration rate into pavements can however be considerable. For a pavement consisting of concrete paving-stones in sand, which is a common pavement in new Dutch towns, it was found that the infiltration rate started with dry initial conditions at a value of 25 mm/h and had after some hours of rainfall, dropped to 15 mm/h (Zondervan and Dommerholt, 1976). It can be expected that the infiltration rate will be much smaller for older pavements, due to silting up of the joints between the paving stones.

These observations about losses make clear that before proper mathematical modelling can be applied, a proper physical background concerning the losses is needed. The losses can be dealt with using one of the following concepts, eventually in combination with an initial loss.

In the case studies three very crude types can be distinguished:

1. a so called $\phi$-index that means the loss intensity is a constant during the event,
2. the losses accepted as a fixed percentage of the rainfall intensity,
3. an adjustment of the infiltration law of Horton, then the loss intensity follows a negative $e$-power in time.

These types are far from sophisticated, but the differences in the results of the models give an insight to the importance of the crude distribution of losses. For application of the formula of Horton one is referred to van den Berg et al. (1977).

MODELLING

Introduction
The models which are discussed are confined to linear time invariant black-box and con-
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Conceptual models. As will be known in linear black-box modelling no assumptions are made about the form of the unit hydrograph. For both types of model the system is considered to behave in a linear fashion and to be time invariant.

Black-box models don't make assumptions about the impulse response. Conceptual models are based on physical considerations which state in most cases the shape of the impulse response. In this situation the impulse response is normally determined by (the value of) a few parameters.

The simple models considered here are based on the properties of linearity and time invariance. For these a convolution integral

\[ Q(t) = \int_{0}^{t} h(t - \tau) P(\tau) d\tau \]  

(1)

can be postulated for the relation between net rainfall \( P(t) \) and runoff \( Q(t) \). The function \( h(t) \) is the impulse response or instantaneous unit hydrograph (IUH).

The black-box and conceptual models are applied to subsystem 2 as described in the introduction. The obvious nonlinearity of a rainfall–runoff system as a whole will be taken into account, at least partially, in formulating the structure of the first system.

Most models are essentially concerned with curve fitting. So the accuracy of the data is important for a successful application.

**Nash model**

Because runoff due to rainfall does not show any wave form in the hydrodynamic sense so that in the absence of resonance phenomena with a linear state-space description the conclusion of a cascade model can intuitively be justified. Giving all time lags of the cascade the same value, \( k \), one obtains the Nash model. This model belongs to the class of conceptual models. The IUH of the model can be written as

\[ h_{N}(t) = h_{N}(t; n, k) = \frac{t^{n-1}}{k^{n} \Gamma(n)} e^{-t/k} \quad (t > 0) \]

\[ = 0 \quad (t < 0) \]  

(2)

Substituting the rainfall and runoff data in the convolution integral (1) give a possibility for a fit of \( n \) and \( k \). This is normally effected by some transformations of (1). The simple method of moments has been applied here (Kraijenhoff van de Leur, 1966).

The accuracy of the results for \( n \) and \( k \) depends mainly on the accuracy of the data for \( P(t) \) and \( Q(t) \) at large values of \( t \) and also on the way in which the net precipitation is assumed. So one is forced to consider the calculated value of \( n \) and \( k \) as a first approximation in an optimization procedure which uses sensitivity functions. This procedure gives iteratively a better fit for \( n \) and \( k \).

**Fourier analysis**

This method, based on the theory of Fourier analysis, has the advantage that the convolution integral can be solved algebraically in the frequency domain. We define the Fourier transform of a time function \( f(t) \) by

\[ F\{f; \omega\} = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \]  

(3)
With this definition one can conclude from (1) that

\[ F\{Q; \omega\} = F\{h; \omega\} F\{P; \omega\} \]

so that

\[ F\{h; \omega\} = \frac{F\{Q; \omega\}}{F\{P; \omega\}} \quad (4) \]

In a numerical calculation of \( F\{Q; \omega\} \) and \( F\{P; \omega\} \) from the original data, some complications can arise due to the finite length of the time interval between the measurements. Using the inverse transform one has for black-box models the possibility of calculating the IUH by

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F\{h; \omega\} e^{i\omega t} d\omega \quad (5) \]

For conceptual models other procedures may be possible. The Nash model gives theoretically

\[ F\{h_N; \omega\} = \frac{1}{(1 + i\omega k)^n} \]

By regression analysis one now can calculate \( n \) and \( k \) on the basis of the formula

\[ (1 + i\omega k)^n = \frac{F\{P; \omega\}}{F\{Q; \omega\}} \quad (6) \]

**Laguerre functions**

The Nash model can be expanded into a more general model by means of Laguerre functions. We define Laguerre polynomials of order \( n \) and argument \( x \) by

\[ L_n(x) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} \frac{x^j}{j!} \quad (x > 0) \quad (7) \]

and the Laguerre functions by

\[ \Phi_n(x) = e^{-(1/2)x} L_n(x) \quad (8) \]

A sketch of the first three Laguerre functions is given in Fig.2. One immediately sees that for \( x = 2t/k \) the exponential is of the same type as in the Nash model. Furthermore one has for example

\[ \Phi_0 \left( \frac{2t}{k} \right) = e^{-t/k} L_0 \left( \frac{2t}{k} \right) = e^{-t/k} = k h_N(t; 1, k) \]
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The Nash-3 unit hydrograph has been drawn with a dotted line.

\[
\phi_1 \left( \frac{2t}{k} \right) = \left( 1 - \frac{2t}{k} \right) e^{-t/k} = k h_N(t; 1, k) - 2k h_N(t; 2, k)
\]

\[
\phi_2 \left( \frac{2t}{k} \right) = \left( 1 - \frac{4t}{k} + \frac{2t^2}{k^2} \right) e^{-t/k} = k h_N(t; 1, k) - 4k h_N(t; 2, k) + 4k h_N(t; 3, k)
\]

Now it follows that

\[
h_N(t; 3, k) = \frac{1}{4k} \phi_0 \left( \frac{2t}{k} \right) - \frac{1}{2k} \phi_1 \left( \frac{2t}{k} \right) + \frac{1}{4k} \phi_2 \left( \frac{2t}{k} \right) \tag{9}
\]

This result can be generalized as

\[
h_N(t; n, k) = \sum_{j=0}^{n} \alpha_j \phi_j \left( \frac{2t}{k} \right)
\]

in which the coefficients \(\alpha_j\) are appropriately chosen.
A further generalization can be made by stating that

\[ h_L(t; k) = \frac{2}{k} \sum_{j=0}^{\infty} c_{h_j} \Phi_j \left( \frac{2t}{k} \right) \]  

represents a Laguerre IUH.

By integrating (1) over \((0, \infty)\) and using the water-balance one obtains for every IUH the restriction

\[ \int_{0}^{\infty} h(t) dt = 1 \]  

This must also hold for the Laguerre IUH. Substituting (10) in (11) shows that the constraint

\[ \sum_{j=0}^{\infty} (-1)^j c_{h_j} = \frac{1}{2} \]  

must hold.

Besides the similarity between Nash and Laguerre models, there is also a similarity between Laguerre functions and Fourier analysis. Our main interest in this last similarity is that the Laguerre functions form an orthonormal set, so that

\[ \int \Phi_n(x) \Phi_m(x) dx = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases} \]  

For this Laguerre model two methods are possible for calculation of the coefficients \(c_{h_j}\). The first method also expands the rainfall and runoff as a Laguerre series

\[ P(t) = \sum_{j=0}^{\infty} c_{p_j} \Phi_j \left( \frac{2t}{k} \right), \quad Q(t) = \sum_{j=0}^{\infty} c_{q_j} \Phi_j \left( \frac{2t}{k} \right) \]  

where, due to orthonormality

\[ c_{p_j} = \frac{2}{k} \int_{0}^{\infty} P(t) \Phi_j \left( \frac{2t}{k} \right) dt, \quad c_{q_j} = \frac{2}{k} \int_{0}^{\infty} Q(t) \Phi_j \left( \frac{2t}{k} \right) dt \]  

By (15) we can obtain \(c_{p_j}\) and \(c_{q_j}\) as functions of the time lag \(k\) when measurements for \(P(t)\) and \(Q(t)\) are available.
Next the substitution of (10) and (14) into (1) gives

\[ ch_0 = \frac{cq_0}{c_p 0}, \quad ch_j = \frac{cq_j}{c_p 0} - \frac{1}{c_p 0} \sum_{m=0}^{j-1} (cp_{m+1} - cp_m)ch_{j-m-1} \]

\[(j = 1, 2, \ldots) \] (16)

Thus \( ch_j \) is known as a function of \( k \).

Substituting all \( ch_j \) into (12) now gives an equation for the unknown \( k \). The structure of the Laguerre functions is such that they have relative small values for large \( t \), due to the influence of the negative exponential. This implies that the data of \( P(t) \) and \( Q(t) \) with relative large errors are multiplied by a small number in (15). By this we can expect accurate numbers \( cp_j \) and \( cq_j \) and thus also for \( ch_j \). This is the main difference with Fourier analysis where the orthonormal function \( e^{-i\omega t} \) does not extinguish with increasing \( t \). Further we mention that for successful application it is necessary to restrict the series expansion (10) to a small finite number of Laguerre functions. The second method does not expand \( P(t) \) and \( Q(t) \) in Laguerre series, so that we lose our gain in accuracy. The advantage lies now in the possibility of generalizing the method by means of Volterra series for nonlinear rainfall—runoff relationships.

The first two Volterra series illustrate the character of this expansion

\[
Q(t) = \int_0^t h_1(t - \tau)P(\tau)d\tau + \int_0^t \int_0^t h_2(t - \tau_1, t - \tau_2)P(\tau_1)P(\tau_2)d\tau_1d\tau_2
\]

Introducing this second method we substitute (10) in (1). This gives

\[
Q(t) = \frac{2}{k} \sum_{j=0}^{\infty} ch_j \int_0^t \phi_j \left( \frac{2t - 2\tau}{k} \right) P(\tau)d\tau = \sum_{j=0}^{\infty} ch_j a_j(t; k)
\] (17)

where

\[
a_j(t; k) = \frac{2}{k} \int_0^t \phi_j \left( \frac{2t - 2\tau}{k} \right) P(\tau)d\tau
\] (18)

Thus \( a_j(t, k) \) can be calculated with the aid of rainfall data. The calculation of the coefficients \( ch_j \) is performed by minimization of the expression

\[
w = \int_0^{\infty} \left[ Q(t) - \sum_{j=0}^{\infty} ch_j a_j(t; k) \right]^2 dt
\] (19)

Setting \( \partial w / \partial ch_m = 0 \) yields
By (12) and (20) the unknown $k, ch_0, ch_1, \ldots$ can be solved. In practical applications with discrete data (18) is replaced by

$$a_j, L(k) = \frac{2}{k} \sum_{i=0}^{L} \phi_j \left( \frac{2L - 2i}{k} \Delta t \right) P(i \Delta t) \Delta t$$

in which $\Delta t$ is the interval between two successive measurements. The set of equations (20) is then rewritten as

$$\begin{bmatrix}
\sum_{l=0}^{L} a_{0,0,l} & \sum_{l=0}^{L} a_{0,1,l} & \cdots & \sum_{l=0}^{L} a_{0,N,l} \\
\vdots & \ddots & \ddots & \vdots \\
\sum_{l=0}^{L} a_{N,0,l} & \sum_{l=0}^{L} a_{N,1,l} & \cdots & \sum_{l=0}^{L} a_{N,N,l}
\end{bmatrix} \begin{bmatrix}
ch_0 \\
\vdots \\
ch_N
\end{bmatrix} = \begin{bmatrix}
\sum_{l=0}^{L} Q_l a_{0,l} \\
\vdots \\
\sum_{l=0}^{L} Q_l a_{N,l}
\end{bmatrix}$$

in which $Q_l = Q(l \Delta t)$ and $N$ the highest order Laguerre to be used.

**Quadratic programming**

Since neither the first nor the second method of calculating the IUH by means of the Laguerre model gives complete attention to the constraint (11), one has to concentrate on optimization procedures. With the least-square formulation already mentioned, a quadratic programming procedure seemed to be justified (Wagner, 1972). At the first stage a linear time discrete model is used in order to see the merits of this procedure. Thus we state the model

$$Q_t = \sum_{j=0}^{N} H_j P_{t-j}$$

where $Q_t$ is the runoff at time $t$, $P_{t-j}$ is the rainfall at time $t - j$, $N$ is the memory of the considered system.

With the water balance one can conclude

$$\sum_{j=0}^{N} H_j = 1$$

Furthermore $H_j > 0$ ($j: 0 \rightarrow N$).
The coefficients \( H_i \) are calculated with the criterion

\[
\min w = \sum_{t=0}^{T} \left( Q_t - \sum_{j=0}^{N} H_j P_{t-j} \right)^2
\]  

(23)

where \( T \) is the length of the runoff series.

The formulation (21), (22) and (23) states a standard quadratic programming problem. This is seen by rearranging (23) to the form:

\[
\max \left[ w - \sum_{t=0}^{T} Q_t^2 \right] = 2 \sum_{j=0}^{N} H_j \left[ \sum_{t=0}^{T} P_{t-j}Q_t \right] - \sum_{i=0}^{N} \sum_{j=0}^{N} H_i H_j \left[ \sum_{t=0}^{T} P_{t-i}P_{t-j} \right]
\]

For the Laguerre formulation (17), (18) the same procedure can be followed if \( a_j(t; k) \) is substituted for \( P \) in (21) and \( ch_j \) for \( H \). Thus

\[
Q_t = \sum_{n=0}^{N} c_{n}a_n(t; k)
\]

Now the constraint is given by

\[
\sum_{j=0}^{N} (-1)^j ch_j = \frac{1}{2}
\]

A complication here is that not all coefficients \( ch_j \) need to be positive. This means that attention has to be given to the choice of the digital program used. Otherwise some formulae have to be rewritten to ensure that all coefficients are positive in the program.

**CASE STUDY OF A SEWER DISTRICT IN ENSCHEDE**

The town of Enschede is making a model of their sewer system to obtain information concerning the frequencies of storm water overflows into open water, and the effect of modifications in the system (e.g. the construction of retention reservoirs) on the behaviour of the system. The local Civil Engineering Department of the town, in cooperation with the Technical University of Twente constructed a model which consists of three elements:

1. sewer districts, out of which drainage is discharged into the other two types of elements,
2. retention reservoirs,
3. main sewers.

The case study which is described here, consists of a black box analysis of the sewer district ‘Varviksingel’, which covers an area of 140 ha. The slope of the principal sewer in the district varies from 1:250 to 1:1000, while the total difference in level in this sewer amounts to 16 m. In this district houses with sloping roofs are predominant, and it includes a small park and some playgrounds. The total surface area of roofs and pavements was unknown and therefore this percentage was estimated as the average.
runoff coefficient (0.235) of a number of events, assuming that for these events only these surfaces contributed to the sewer discharge with negligible losses.

One autographic raingauge was used, and at the downstream end of the sewer district the water level in the sewer was measured with a pressure transducer, which was connected to a recorder. The discharge values could be found from the charts of this recorder. The data were extracted from the charts of the raingauge and water level recorder for analysis, using a 5-min time interval. During the data processing the values for the discharge, which were valid for stationary flow, were corrected for the slope of the water level. As the sewer system is of the mixed type, a correction for baseflow was also applied.

For this case study it was obvious that the lack of some vital information, (percentage roofs plus pavements, only one raingauge) did not allow a detailed analysis such as the comparison of models or the comparison of the effect of various ways of subtracting the losses on the modelling results for subsystem 2 as mentioned in the introduction. Therefore only a black box analysis was carried out, approximating the rainfall histogram, the outflow curve and the IUH by a set of orthonormal Laguerre functions. Six events were used for identification. For the analysis a computer program was used which describes the rainfall histogram and the outflow curve with 20 Laguerre functions, while the IUH was iteratively described by \( k (k = 5, 6, 7, \ldots, 12) \) Laguerre functions. For every value of \( k \) the resulting set of linear equations, which describe the relation between the coefficients of the approximations, was solved using least squares. To select the optimal number of functions for the IUH one can demand that the area below the IUH must be as close to unity as possible and, if necessary, that oscillations be as small as possible.

Since the scaling factor also has an influence on the result, the program that was used executed the whole identification procedure for four different scaling factors as follows: 1 time unit of the Laguerre functions corresponds with 5, 10, 20 or 40 min of real time, respectively. Of these four possibilities the case that one time unit of the Laguerre functions corresponds with 5 min of real time was selected. This scaling factor yielded satisfactory results for all cases except one (event 5 of Table 1), which demonstrated a disturbance in the tail of the IUH. For this event a scaling factor was tried where 1 time unit of the Laguerre functions corresponds with 2½ min real time. The disturbance remained however. It is possible that for this event the raingauge did not represent well enough the rainfall on the catchment. A further difficulty was that the rainfall and discharge recorders were not synchronized. From the resulting IUHs it appeared that there was a considerable translation in the system, this translation was estimated as the average value of the translations which were found for the six IUHs individually. Characteristics of the six rainfall events are given in Table 1. The small negative losses which were found for events 4 and 6, which both started with wetted surfaces because of antecedent rainfall, were distributed proportionally over the rain-
FIGURE 3. IUHs of Varvikingel.
FIGURE 4. IUHs of Vanyakigal.
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Surfaces were wetted because of antecedent rainfall

FIGURE 5. Simulation of the discharge from the event on 9 June 1974, using the IUH of the event on 5 May 1973.

Although some encouraging simulation results are obtained for discharge waves, caused by severe thunderstorms, using the average IUH of Fig.4, assuming that some grass verges also contributed runoff, the uncertainty of some basic factors, such as the total surface of roofs and pavements, as well as the contribution of the pervious area fall histogram. The positive losses which were found for the remaining four events were assumed to be initial losses.

As the resulting IUHs differed rather in their peak value they were divided into two sets of three, according to the magnitude of their peak value (Figs.3 and 4). Comparing Figs.3 and 4 a distinct nonlinear behaviour of the system can be observed, as heavier rainfall yields sharper higher IUHs. As a test of the identification procedure that we followed, the discharge from event 6 was predicted, using the IUH of event 4. The simulated discharge was found to be in good agreement with the measured discharge (Fig.5).
TABLE 2. The characteristics of the rainstorms at Lelystad

<table>
<thead>
<tr>
<th>Date of event</th>
<th>Total rainfall [mm]</th>
<th>Loss [mm]</th>
<th>Effective rainfall [mm]</th>
<th>Catchment area</th>
</tr>
</thead>
<tbody>
<tr>
<td>28-6-73</td>
<td>2.11</td>
<td>1.37</td>
<td>0.74</td>
<td>Residential quarter</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.56</td>
<td>0.55</td>
<td>Shopping centre</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.89</td>
<td>0.22</td>
<td>Parking lot</td>
</tr>
<tr>
<td>22-7-73</td>
<td>5.92</td>
<td>0.89</td>
<td>5.03</td>
<td>Residential quarter</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.66</td>
<td>4.26</td>
<td>Shopping centre</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.22</td>
<td>2.70</td>
<td></td>
</tr>
</tbody>
</table>


- - - following a φ-index or

- - - - following a percentage of the rainfall intensity.
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A CASE STUDY CONCERNING CATCHMENT AREAS IN LELYSTAD

Some illustrative results

Some well equipped urban catchments have been instrumented in Lelystad (Kraijenhoff van de Leur and Zuidema, 1969; van den Berg, 1976; van den Berg et al., 1977).

The models mentioned in the section ‘modelling’ are applied in this case study. Results are presented for two rainstorms (Table 2) to illustrate what may be expected from comprehensive research.

The rainfall intensities of these events computed from the recordings of the rain-gauges are presented in Fig.6. The total time, here a half or one hour, to which the computations are applied, doesn’t involve the whole recession curve, so the effective rainfall from the whole event may be higher.

The unit hydrograph has been computed by the four models for the rainstorm of 22 July 1973 on the discharge of the residential quarter. The losses have been distributed following a Horton model (Fig.7).

In the Laguerre model the unit hydrograph has been produced by five Laguerre functions with the next coefficients: 0.205, −0.234, −0.202, −0.081, 0.082 and \( k = 4.67 \) DT (DT = 2 min). Compare this with the Laguerre coefficient of the Nash-1 model: \( 1/k = 0.213 \).

The way in which the losses are assumed to be distributed is indicated by two examples of different unit hydrographs in Fig.8. The computation of the unit hydrograph by quadratic programming for the case represented in Fig.7 consumed far more computer time than other runs without a result if the losses are distributed as a percentage of the net precipitation. To get an idea of the differences between the three catchment areas their unit hydrographs have been drawn in Fig.9 for the rainfall of 28 June 1973 and the losses distributed as a percentage. The translation and attenua-
A mutual comparison of the different methods
The Nash model in combination with the method of moments is a simple one, and tends mostly to produce a significant, however, inaccurate result. The error in the calculation of second and higher moments may be considerable because the data of the recession curve include errors. Around the optimal combination of the parameters \( n \) and \( k \) other combinations will show a slightly poorer result. So the values of the parameters spread wide. The product \( n.k \), the centre of gravity, is however a more constant parameter. A provisional result when \( n \) is confined to real, whole numbers greater than 1 is given in Table 3.

For small catchment areas like those considered here, with a rapid response to rainfall, one will often find that \( n \) equals about 1. The highest value of the unit hydrograph is restricted now to the first time interval, for physical reasons this is improbable.

The identification method of the Fourier transform yields poor results, mainly because instabilities in the unit hydrograph increase continuously with time. The best result is represented in Fig.7. This shortcoming is implied in the theory of Fourier transforms.

The provisional results suggest that the identification method with Laguerre functions is in general superior to both foregoing methods.
Model validation
It has been stated that the application of models in essence deals with curve fitting. Some runoff hydrographs have been calculated and now we are interested in the rate of success as the third phase in modelling.

The determination of a so-called model efficiency coefficient analogous to the coefficient of determination (Nash and Sutcliffe, 1970) is a suitable resource to that purpose (see Fig.8). A criterion that is related to that coefficient is the cross-correlation coefficient between the calculated and recorded runoff hydrograph for increasing lags (Fig.9) (van den Berg, 1976; Yevjevich, 1972).

CONCLUSIONS

Investigations have to be carried out on the distribution of the losses with time.

Nonlinearity was found for the sewer district Varviksingel and this emphasizes the
necessity to get information about the degree of nonlinear behaviour of various types of urban catchments.

For the Lelystad data the following results were obtained:

1. All four applied models simulate only the beginning of the recession curve properly.
2. The computed unit hydrographs are sensitive to a change in the assumed distribution of the losses.
3. The Nash model in combination with the method of moments is a simple one, and tends mostly to produce a significant, however inaccurate result.
4. The Fourier transform method is a poor method to use to compute a unit hydrograph.
5. The identification with Laguerre functions is an expansion of the Nash model.
6. The identification based on quadratic programming may produce ill shaped unit hydrographs but the computed and recorded runoffs are in good agreement in the sense of a least squares criterion.
7. The cross-correlation coefficient with lag zero between computed and

FIGURE 10. *Above:* the runoff hydrograph of the parking lot on 28 June 1973 from 11.40 h to 12.10 h. The time interval 1 min. The dashed line represents the runoff computed with the unit hydrograph in Fig. 9.

*Below:* the runoff hydrograph of the residential quarter on 22 July 1973 from 14.00 h to 15.00 h, time interval 2 min. The dashed line represents the runoff computed with the unit hydrograph in Fig. 7, from the method with Laguerre functions. The pulled lines are the registered runoff hydrographs. For the diagrams of the rainfall of these events, see Fig. 6.
Calculation of instantaneous unit hydrographs in an urban area

TABLE 3. The parameters of the Nash model in two Lelystad catchments

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>k [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Runoff from residential quarter</td>
<td>1</td>
<td>9–11</td>
</tr>
<tr>
<td>Runoff from parking lot</td>
<td>1</td>
<td>5–7</td>
</tr>
</tbody>
</table>

recorded runoffs, the model efficiency coefficient, and the autocorrelation coefficient of the residuals are useful to model validation.

(8) A time interval of one or two minutes is optimal to compute unit hydrographs for the catchment areas in Lelystad.

To apply the identification methods on recorded and accurate data, as available in Lelystad, one may expect that there will be a few cases in which a method fails for divergent reasons.

A comprehensive study of the Lelystad data with an evaluation and an accuracy analysis to validate the described methods is a promising continuation of these results.

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