Long term sediment yields from short data records

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Abstract. A method is described of determining the long term sediment transport at a site when only a few years of sediment data are available. The site selected should be an established flow gauging station with a reasonable number of years of flow records. The sediment data are plotted against the ratio of annual runoff to long term mean annual runoff, and an interpolation made to determine the sediment transport \( Q_{si} \) when the water runoff equals the long term mean. By using a frequency distribution applicable to ephemeral rivers (the two parameter gamma distribution) and some sediment data from rivers in the semiarid parts of Southern Africa, it is shown that the long term sediment yield is equal to \( Q_{si} \) to a first approximation. The technique is particularly applicable to rivers draining arid and semiarid areas because the chances are small of striking a year close to average in only a few years of measurement.

Evaluation de la production à long terme de sédiments à partir de relevés de courte durée

Résumé. Il existe une méthode pour évaluer les transports solides à long terme pour un site donné quand les données acquises sur la sédimentation ne datent que de quelques années seulement. Le site choisi devrait être une station de jaugeage aménagée ayant des relevés d’écoulement qui datent de plusieurs années déjà. Les données sédimentaires sont alors reportées en fonction du rapport du ruissellement annuel au ruissellement moyen à long terme, avec une interpolation faite pour évaluer le \( Q_{si} \) des transports solides lorsque le volume ruisselé se trouve égal au ruissellement moyen à long terme. En utilisant une fréquence de distribution appropriée aux rivières intermittentes (la distribution gamma à deux paramètres) et quelques données relatives aux transports solides provenant des rivières des régions semi-arides de l’Afrique Australe, il est montré que le transport solide à long terme est égal à \( Q_{si} \) en première approximation. Cette méthode s’applique tout particulièrement aux rivières des régions arides et semi-arides, car les chances de tomber sur une année proche de la moyenne après un nombre limité d’années d’observation, sont minimes.

NOTATION

- \( n \) : Power in sediment transport versus annual runoff relationship;
- \( P \) : probability density for annual water runoff;
- \( Q_s \) : annual sediment transport \([t]\);
- \( Q_{si} \) : mean annual sediment transport \([t]\);
- \( Q_{sl} \) : sediment transport \([t]\) for a year when runoff equals mean annual runoff;
- \( V_* \) : relative annual runoff, equal to annual water runoff divided by mean annual runoff;
- \( \alpha \) : reciprocal of coefficient of variation, squared, of annual water runoff series;
- \( \Gamma \) : gamma function.

INTRODUCTION

In arid and semiarid climates the runoff is a very small fraction of the annual rainfall. For example in the dry tropics of Southern Africa the runoff is about 25 mm for river basins with 600 mm annual rainfall, and only 10 mm where the annual rainfall is 500 mm. A large fraction of the rain is lost by evapotranspiration. Annual runoff volumes fluctuate very widely indeed, depending on whether the yearly rainfall was an above the average or below the average amount. This fluctuation in runoff directly affects the yearly amounts of sediment that are transported. The weight of sediment
transported in a single season may be either very much larger than average or very much smaller than average. Only rarely is the amount close to the long term average sediment yield.

In making reconnaissance measurements for an engineering project it is usual to have one year, and in some circumstances a few years in which to establish data on sediment transport. The chances of striking a runoff year close to the long term average are small. A simple method is suggested of using data from a few years of measurement at a particular station to establish the long term mean sediment transport at that station.

**ANNUAL RUNOFF VOLUMES**

The two parameter gamma distribution (Yevjevich, 1972) is used commonly to describe the frequency distribution of annual volumes of water runoff. The distribution is particularly suited to arid and semiarid climates because it caters for very large coefficients of variation. Mitchell (1974) has used the two parameter gamma distribution to describe annual runoff events for rivers in Southern Africa. The frequency $P$ of the yearly runoff volumes is

$$\frac{\Delta P}{\Delta V_*} = \frac{1}{(1/\alpha)^\alpha \Gamma(\alpha)} V_*^{(\alpha-1)} \exp(-\alpha V_*)$$

in which $V_*$ is the relative water runoff, equal to the annual runoff divided by the long term mean runoff; $\alpha$ is the reciprocal coefficient of variation squared; $\Delta P/\Delta V_*$ is the probability density; and $\Gamma$ represents the gamma function (Dwight, 1961).

The distribution is shown in Fig. 1 for six values of the coefficient of variation. Values of 1.2—1.6 are typical of the semiarid areas mentioned in the Introduction. Three 70 year series of synthetic water runoff events (Fig. 2), obtained from the gamma distributions and a random number generator, illustrate the difficulty of finding long term mean values (both of runoff and sediment yield) from only a few years of data when the coefficient of variation is large. The chance of striking a year when the runoff is close to the long term mean runoff is small.

**SEDIMENT TRANSPORT**

Field measurements of suspended sediment transport have been made for the last few years at established gauging stations in the semiarid parts of Southern Africa (Ward, 1979). Three to four years of sediment data were available for the Gwai basin (14 400 km$^2$) and the Umsweswe basin (1990 km$^2$). Both rivers had small values of mean annual runoff (22 mm Gwai, 52 mm Umsweswe) and large values for the coefficient of variation of the yearly runoff events. In an average runoff year the Gwai and the Umsweswe carry 50 per cent of their annual sediment loads in 2.9 days and 3.5 days respectively (see Leopold et al., 1964, Table 3.8 for comparison with rivers in the United States). Preliminary work at these sites is described in Ward (1977).

Every significant runoff event during a four year period was sampled by resident field observers and the daily discharges of suspended sediment determined. This was facilitated by the fact that the rivers flow significantly during only the three to four months following the monsoon and not during the rest of the year. Estimates of bed load transport were made using Einstein’s bed load calculation (Einstein, 1950), and added to the suspended load transport to give annual total tonnages. Details of the measuring procedures, sampling techniques, river basins etc. are given in Ward (1979).

Values of annual sediment tonnages were plotted against $V_*$ the relative annual water runoff (Fig. 3). $V_*$ is the ratio between the annual water runoff and the long
term mean runoff. This long term mean quantity was reasonably well known, because measurements were carried out at established gauging stations with about 20 years of records.

The logarithmic plot of Fig. 3 shows that simple power relationships of gradient slightly less than 1.0 fit the data quite well. Although the data points are too few in number for the form of this relationship to be determined satisfactorily, the data are important because they include large runoff events of rare occurrence (a 40 year runoff season was measured at the Umsweswe site). The relationships are certainly in accord with physical intuition, since the concentrations of suspended sediment in a river during a large runoff year will be less than in a small runoff year. This statement is reasonable when it is recalled that the sediment availability, rather than the water transporting ability, will limit the sediment discharge during years of large water runoff. In general the relationships may be written,

\[ Q_s = Q_{s1} V_*^n \]  

(2)
in which \( Q_s \) is the annual sediment transport \([t]\); \( Q_{s1} \) is the sediment transport \([t]\) for a year when \( V_* = 1.0 \); and \( n \) is a constant.
FIGURE 2. Seventy year sequences of flow events generated from the two parameter gamma distribution. Comparison of rivers from semiarid areas with rivers from temperate areas.

FIGURE 3. Annual sediment transport per unit area of basin versus ratio between annual runoff and long term mean runoff.
LONG TERM AVERAGE SEDIMENT YIELD

The amount of sediment transported during a particular year has been shown to depend upon $V_*$. Consider a very long series of annual events, arranged in order of increasing $V_*$. The contribution of events in the range $V_*$ to $(V_* + \Delta V_*)$ to the total sediment transport of the whole series is given by the product of the number of events in the category times the amount of sediment transported at runoff value $V_*$. The contribution of events in this range to the long term mean sediment transport $\langle Q_s \rangle$ is given by the product $\Delta P$ times the amount of sediment transported $Q_s$. These two functions are shown in Fig. 4.

$$\langle Q_s \rangle = Q_s \Delta P$$  \hspace{1cm} \text{(3)}

$$= \frac{Q_s V_*^n}{(1/\alpha)^{\alpha} \Gamma(\alpha)} V_*^{(\alpha-1)} \exp (-\alpha V_*) \Delta V_*$$  \hspace{1cm} \text{(4)}
Integrating over the range $V^*_s = 0$ to $V^*_s = \text{some very large value}$, gives the long term mean sediment transport $\langle Q_s \rangle$:

$$
\langle Q_s \rangle = \frac{1}{Q_{s1}} \frac{(1/\alpha)^2 \Gamma(\alpha)}{\Gamma(\alpha)} \int_0^\infty V^*_s^n V^*_s^{(\alpha - 1)} \exp(-\alpha V^*_s) \, dV^*_s
$$

When $n = 1$ this reduces to the first moment of equation (1) (see Fig. 1) which is equal to 1.0 by definition. The result is to be expected on physical grounds because if the sediment transport increases linearly with the water runoff then a year whose runoff equals the long term mean sediment transport will also be a year whose sediment transport equals the long term mean sediment transport. For other values of $n$ equation (5) is evaluated by numerical integration, care being taken to use very small increments of $V^*_s$ for values of $V^*_s$ close to zero for small values of $\alpha$. The ratio $\langle Q_s \rangle/Q_{s1}$ given in Table 1 for five values of the coefficient of variation, is the ratio between the long term mean sediment transport and the sediment transport during a year when $V^*_s = 1.0$. Thus if the sediment transport $Q_{s1}$ during a year when the water runoff equals the mean annual runoff can be found, then the long term mean sediment transport is determinable by multiplying by the ratios given in Table 1. Inspection of Fig. 2 shows that for any three successive years of runoff there is a reasonably large chance of finding one event greater than average and one event smaller than average. $Q_{s1}$ may therefore be found by interpolation, using the three years of sediment measurements. $\langle Q_s \rangle$ may then be found by multiplying by the ratios of Table 1. In practice, for design purposes, the assumption that $\langle Q_s \rangle = Q_{s1}$ is almost certainly valid because the two quantities never differ from one another by more than 20 per cent (Table 1). This error is no larger than the error with which sediment transport data can be measured in ephemeral rivers.

As an example, consider the Limpopo River at Beitbridge. The basin area is 196 000 km$^2$ and the mean annual runoff 11 mm. The Hydrological Branch, Ministry
of Water Development, Salisbury, Rhodesia has 20 years of flow gauging records for this station (opened in 1959). Three years of sediment transport data are available, see Table 2 and Ward (1979). Figure 5 shows a linear interpolation on the basis of the relative annual water runoff. At a value $V_* = 1$ the annual sediment transport is $3.4 \times 10^6$ t. To a first approximation this is equal to the long term average sediment yield.

CONCLUSION

A method is presented of finding the long term sediment yield at sites where only a few years of sediment transport data have been measured and where a reasonably large number of years of river discharge records are available. The annual sediment transport amounts are plotted against the relative annual water runoff $V_*$ and an interpolation made to find the transport $Q_{s1}$ at a runoff value $V_* = 1.0$. To a first approximation these values $Q_{s1}$ are shown to be equal to the long term mean sediment transport.

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