Analysis of a nonlinear rainfall–runoff model and its application to the Mošteník experimental basin in Czechoslovakia

OTO MENDEL  Institute of Hydrology and Hydraulics, Bratislava, Czechoslovakia

Abstract. The Mošteník experimental basin is part of the Váh River basin in Czechoslovakia. Besides an analysis of a nonlinear model, the paper discusses selected results from the basin, including its basic characteristics and problems of hydrograph separation, calculations of the runoff coefficient and antecedent precipitation index, and their application for determination of the effective rainfall. The effective rainfall is used as an input to the model; the output is compared with measured hydrographs and the results analysed taking into consideration the size, vegetation and other characteristics of the basin.

L'analyse d'un modèle non-linéaire de l'écoulement provenant des précipitations et son application sur le bassin expérimental Mošteník en Tchécoslovaquie

Résumé. Le rapport présente quelques résultats de la recherche expérimentale sur le bassin et on décrit les caractéristiques fondamentales. Le bassin expérimental Mošteník est un petit tributaire de la rivière Váh en Tchécoslovaquie. Outre avec l'analyse du modèle non-linéaire le rapport traite les problèmes de la séparation des hydrogrammes, les calculs du coefficient d'écoulement et de l'indice des précipitations pluvieuses antécédentes et leur application pour la détermination des précipitations effectives. On utilise la pluie effective comme entrée dans le modèle mathématique. La sortie est comparée avec des hydrogrammes mesurés et les résultats sont analysés au point de vue de la grandeur, de la couverture végétale et des autres caractéristiques du bassin.

The Mošteník experimental basin was established in 1958 by the Institute of Hydrology and Hydraulics of the Slovak Academy of Sciences at Bratislava with the aim of extending the knowledge of the physical bases of the hydrological processes in both natural conditions and those affected by man. The Mošteník is a left-hand tributary of the River Váh near the town of Považská Bystrica. The total area of the basin (17.2 km²) is divided into eight sub-basins with areas ranging from 0.0864 to 12.6 km².

The basin is a part of the Strážov highlands which characterize the hilly and partially also the midmountain zone of Slovakia. From the geological point of view the investigated area and its environment are a part of the Klippen Belt.

About 40 per cent of the basin, i.e. 7 km², is forested (monocultures of pines and spruces), 45 per cent is agricultural land and 15 per cent is pasture.

The long term average annual rainfall in the basin is 780 mm, with the maximum of 115 mm in June and the minimum of 31 mm in March. The long term average monthly temperatures are within the range -3.3°C (January) to 17.1°C (July). The average monthly relative air humidity is 62 per cent (lowest average value, April 1974) and 93 per cent (highest average value, December 1974). The velocity and direction of the wind are affected by local orographic conditions in the small sub-basins.

The primary aim of the study was described by Balco (1959) and the experimental results obtained in particular stages are described by Mendel (1978, 1979a, 1980a).

At present the research is focused on the study of the rainfall–runoff regime, the areal distribution of the liquid and solid precipitation, the effect of the micro and mesorelief on the course of the meteorological elements, and the basic hydrological balance of the forested and unforested basins. Considering the formation of runoff on the basis of theoretical and experimental research carried out on the runoff area
by means of soil physical characteristics and a mathematical model of the infiltration
process, the effective components of precipitation were determined which are formed
by a rainfall of a definite intensity with regard to its duration and the known ante-
cedent soil moisture, as published by Mendel et al. (1979). Another important result
is the physically solved distribution of precipitation into the particular components
of the water balance equation with special regard to the determination of hypodermic
runoff which so far has received little attention within the framework of hydrological
research (Mendel et al., 1980).

The choice of the place and the significance of both the experimental and represent-
ativa basins as well as the perspectives for the future with concern to the solution
of the topical problems of the hydrology in natural conditions were dealt with pre-
viously by the author (Mendel, 1979b, 1980b).

Parallel to the solution of these problems, much attention was also devoted to the
mathematical modelling of runoff from rainfall by using a nonlinear rainfall—runoff
model with a system of nonlinear reservoirs. The question of using a linear and non-
linear rainfall—runoff model by means of a system of nonlinear reservoirs for larger
basins has been already dealt with by several authors, e.g. Svoboda (1975), Mendel
et al. (1977), Drako (1979); due to the fact that these studies gave little information
on the behaviour of such models in small or very small basin areas, the decision was
made to apply these models to small experimental basins. This problem was also
dealt with by Ševčíková (1979).

The method for using a nonlinear rainfall—runoff model was elaborated by Svoboda
(1975). The model consists of a series of $N$ equal reservoirs with $P$ being the input
into the first reservoir and $Q$ being the output from the last reservoir in the series of $N$
reservoirs. The water capacity $V$ in each reservoir is in relation to the runoff from this
$Q$ reservoir; this relation is expressed in the power from

$$Q = B \cdot V^{EX}$$  \hspace{1cm} (1)

where $Q$ = runoff from the reservoir, $V$ = capacity corresponding to this runoff, $EX =$
parameter of the transformation function, and $B =$ coefficient expressing the trans-
formation function $(B = Y/X^{EX})$; $X$ and $Y$ are the coordinates of the point through
which all lines $Q = f(V)$ at an arbitrary exponent $EX$ must pass. The position of this
point can also be determined by the coordinate $Y$ which represents a definite value of
the discharge $QCHN$ and the ratio $X/Y = BK$ which expresses the time constant of
the linear reservoir, as shown in Fig. 1.

The equation (1) and the equation of continuity (2) from the base of the solution

$$(P - Q) \, dt = dV$$  \hspace{1cm} (2)
Using the method developed by Klémek (1960) for the simplified solution of a flood wave through a reservoir it then holds that
\[(P_{I+1} - Q_{I+1}) \cdot \Delta t = V_{I+1} - V_I\]  \hspace{1cm} (3)
where the indexes indicate the computation interval and \(P\) is the inflow into the reservoir.

From equations (1) and (3), it follows that
\[Q_{I+1} = P_{I+1} + \frac{1}{B^{1/EX} \cdot \Delta t} \cdot (Q_{I+1}^{EX} - Q_{I+1}^{EX})\]  \hspace{1cm} (4)
The approximation method can be used in the solution of this general equation.

In the graphical solution it is assumed that the inflow into the reservoir is expressed as a function of time \(P = f(t)\) and that the runoff from the reservoir is a function of the water capacity \(Q = f(V)\) above the crest of the weir. During the passage through the reservoir the instantaneous balance is expressed by equation (2).

If we consider the basins as a hydrological system that can be substituted by a cascade of nonlinear reservoirs of the same size, then the parameters of the model are \(QCHN, BK, EX, DT, N, ITR\), where

\[QCHN = \text{definite value of the discharge \([m^3/s]\),}\]
\[BK = \text{time constant of the system \([h]\),}\]
\[EX = \text{exponent of the transformation function,}\]
\[N = \text{number of reservoirs,}\]
\[ITR = \text{parameter of translation \([h]\),}\]
\[DT = \text{time step \([h]\).}\]

These parameters can be determined only when the input and output values are of high quality. For the analysis and estimation of the reliability of the nonlinear runoff model selected hydrographs were available of the runoff from the Rybárík (0.119 km\(^2\)), Lesný (0.0864 km\(^2\)), Cingelová (0.22 km\(^2\)) and Lacková (12.6 km\(^2\)) sub-basins. For this purpose it was necessary to evaluate the basic rainfall and runoff data, to select the discharges and their corresponding rainfalls as well as the average total rainfall measured during the preceding 30 days. The values of direct runoff were determined from the chosen discharge waves by separation of the hydrograph components. Also computed were the runoff coefficients and the antecedent soil moisture of the basin with rainfall during the chosen time interval.

Under the assumption of a linear change of the runoff coefficient during the rainfall and using the values of the antecedent soil moisture, the effective rainfall was calculated and the data were used as input values in the nonlinear model.

Regarding the separation of the direct runoff of the hydrographs, we are aware of the first inaccuracy occurring in the calculation of the effective rainfall because this represents precisely the runoff volume. The choice of the separation method depends in many cases on the subjective experience of the researcher.

From the values of the direct runoff \(h_0\) and the total causal rainfall \(h_z\), according to the known relation, the runoff coefficient
\[\bar{\phi} = \frac{h_0}{h_z}\]  \hspace{1cm} (5)
was calculated.

The preceding saturation of the basin area by the rainfall was expressed by the index
\[IPZ = k_1 d_{z_1} + k_2 d_{z_2} + \ldots + k_i d_{z_i} + \ldots + k_n d_{z_n}\]  \hspace{1cm} (6)
in which $d_{zi} =$ rainfall values during the $i$th day prior to the causal rain, $k_i =$ climatic coefficient $< 1$ (in our conditions $k = 0.9$), $i =$ number of days $(5, 10, 15, 20, 25, 30)$ with the time step of 1 day.

In the calculation of $\phi$ and $IPZ$, the equations (5) and (6) were used. It is known that at the beginning of the rain the coefficient of runoff $\phi_P < \bar{\phi}$ and at the end $\phi_k > \bar{\phi}$ and that its initial value depends mainly on the antecedent soil moisture of the basin $IPZ$.

The method for calculating the effective rainfall was elaborated by Svoboda (1975). If we assume

$$\phi_P = f(IPZ)$$  \hspace{1cm} (7)

then $IPZ = 0$ when $\phi = 0$, and $IPZ = 100$ mm when $\phi_P = 1 = \bar{\phi}$.

These values represent the chosen limiting values of $IPZ$ which characterize the unsaturated or saturated basin area. At $IPZ > 100$ mm the initial coefficient of runoff remains unchanged and is equal to the mean runoff $\bar{\phi}$.

For the calculation of the values $0 < \phi_P < 1$ it holds

$$\phi_P = 0.01 \cdot IPZ \cdot \bar{\phi}$$  \hspace{1cm} (8)

If we assume that the function expressing the time changes of the runoff coefficient during the rainfall duration is symmetric than we are able to calculate the final value of the runoff coefficient $\phi_k$ from the known $\bar{\phi}$ and $\phi_p$-values

$$\phi_k = 2\bar{\phi} - \phi_P$$  \hspace{1cm} (9)

In the case that $\phi_k < 1$ then the absolute value of the initial coefficient $\phi_P$ can be computed from the relation

$$\phi_P = 2\bar{\phi} - 1$$  \hspace{1cm} (10)

so that $\phi_k = 1$.

The calculated coefficients $\bar{\phi}$, $\phi_P$ and $\phi_k$ which represent the beginning and end of the causal rain (duration of rain) can be drawn on an axis of coordinates and by their junction we then obtain the course of the change of the runoff coefficient at the time of the causal rain duration.

The causal rain is expressed by $h_{zi}$ hourly intervals which are drawn on an axis forming a definite angle with the horizontal axis. The $h_{zi}$-values are projected onto the horizontal axis. In the centre of the projected $h_{zi}$-values we then trace a normal line and its point of intersection with the straight line dependence from $\phi_P$ to $\phi_k$ then indicates the value of the runoff coefficient $\phi_i$.

By means of $\phi_i$ we then calculate for each $i$th time interval the values of the effective rainfall

$$h_{zei} = h_{zi} \cdot \phi_i$$  \hspace{1cm} (11)

The whole procedure is shown in Fig. 2.

In order to find out which $IPZ$-values represent best the characteristics of the saturated basin area, we compared the sum of the calculated effective rainfall values with the direct runoff $h_0$:

$$h_0 = \sum_{i=1}^{m} h_{zei}$$  \hspace{1cm} (12)

In the case when

$$h_0 - \sum_{i=1}^{m} h_{zei} \leq \text{min}$$  \hspace{1cm} (13)
we considered IPZ as best for expressing the preceding saturation of the basin area with regard to the considered number of days before the causal rainfall.

Due to the large amount of the processed data obtained in the investigated experimental basin areas we give as an example measured and calculated hydrographs (September 1976) in the Lesny sub-basin (Fig. 3). In the calculation of the effective rainfall values and their transformation using a nonlinear model the procedure described above was used by Ševčíková (1979). In this concrete case the total rainfall $h_z = 18.0 \text{mm}$, $IPZ = 5.265 \text{mm}$ (calculated for the time 15 days prior to the causal rain), $\phi = 0.01$, $h_0 = 0.17 \text{mm}$.

The most stable results were obtained in forested basins and the most variable results in agricultural basins.

The optimal parameters of the model were $N = 2.4$; $BK = 3.5$; $DT = 1$; $EX = 2.0 - 2.5$; $ITR = 0$; $B = \left[ \frac{N}{(BK/DT)} \right]^{EX}$. The effect of the number $N$ was almost unobservable. The obtained results were estimated besides the visual comparison also on the base of the agreement.

On the basis of the obtained results we conclude that the $\phi$ and $IPZ$-values are mostly affected, besides other characteristics of the basin area, by the vegetation which has a substantial effect on the runoff formation in small basin areas.

The results showed that the considered model can be used for the simulation of runoff both in large and small basin areas. The accuracy of the output however is strongly affected by the accuracy of the input (precipitation) with regard not only to the quantity of the rainfall but especially to its duration and its beginning. In the case of small basins a very short time step must be taken into consideration because this affects the accurate synchronization of the ombrograph and limnigraph; in the opposite case this may cause a great inaccuracy with regard to the very short time duration of the procedure.

REFERENCES


