A simplified model for estimating glacier ablation under a debris layer

MASAYOSHI NAKAWO
Department of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo 060, Japan

SHUHEI TAKAHASHI
Kitami Institute of Technology, Kitami, Hokkaido 090, Japan

ABSTRACT In order to predict the ablation of glacier ice under a debris layer, a simple model is proposed. Required data for the prediction are global radiation, air temperature, the degree-day factor around the area to be investigated, albedo of debris-free ice, and critical thermal resistance of the debris cover. It is shown that this latter variable can be estimated from the four former variables, which are comparatively easy to measure or estimate. Regarding the physical properties of the debris layer per se, its albedo and thermal resistance need to be given. The predictions of this model compare favourably with field observations.

NOTATION

- $A$: atmospheric radiation flux, W m$^{-2}$
- $C$: conduction heat flux through debris layer, W m$^{-2}$
- $C_0$: heat used for melting ice per unit time for debris-free ice, W m$^{-2}$
- $C_p$: specific heat of air at constant pressure, $1.0 \times 10^3$ J kg$^{-1}$ deg$^{-1}$
- $E$: evaporation heat flux at top surface of debris layer, W m$^{-2}$
- $E_0$: evaporation heat flux at debris-free ice surface, W m$^{-2}$
- $e$: saturation vapour pressure as a function of temperature, Pa
- $e_a$: mean vapour pressure in air, Pa
- $e_S$: vapour pressure at top surface of debris layer, Pa
- $e_0$: vapour pressure at debris-free ice surface, 610 Pa
- $F$: radiation heat flux at top surface of debris layer, W m$^{-2}$
- $F_0$: radiation heat flux at debris-free ice surface, W m$^{-2}$
- $G$: global radiation flux, W m$^{-2}$
- $G^*$: $G(\alpha_o - \alpha)/C_0$
- $H$: sensible heat flux at top surface of debris layer, W m$^{-2}$
- $H_0$: sensible heat flux at debris-free ice surface, W m$^{-2}$
- $h$: thickness of debris layer, m
- $h_0$: thickness of debris-free ice surface, W m$^{-2}$
- $K_m$: thermal conductivity of debris layer, W m$^{-1}$ deg$^{-1}$
- $K_r$: $4\sigma(273)^3$, 4.615 W m$^{-2}$ K$^{-1}$
- $k$: degree-day factor for debris-covered ice, C/T$_a$, W m$^{-2}$ deg$^{-1}$
- $k_o$: degree-day factor for debris-free ice, C$_O$/T$_a$, W m$^{-2}$ deg$^{-1}$
- $k^*$: $k/k_o$
- $L_e$: latent heat of evaporation, $2.494 \times 10^6$ J kg$^{-1}$
**INTRODUCTION**

Most large glaciers in the Nepal Himalaya are covered with debris in their ablation area (Moribayashi & Higuchi, 1977; Fujii & Higuchi, 1977). It is essential, therefore, to understand the effect of debris cover on ablation of glacier ice for predicting the water supply from mountain glaciers of the Himalaya.

Applying a simple model, Nakawo & Young (1981) showed that the ablation under a given debris layer can be estimated from meteorological variables. There are two difficulties, however, in using the model practically to estimate the mean value of ablation under a debris layer extending over a wide area. One is the difficulty of determining directly the thermal resistance of the layer in the field, which is one of the essential parameters of the model. The other relates to the fact that detailed information on meteorological variables is necessary for the estimation.

In regard to the former difficulty, Nakawo & Young (1981) put forward a method by which a reasonable estimate of the thermal resistance of a debris layer can be obtained in the field. This method was successfully employed in analyzing experimental data (Nakawo & Young, 1982). In this paper, Nakawo & Young's model is simplified in order to overcome the latter difficulty for its practical use in the field.

**CRITICAL THERMAL RESISTANCE**

It has long been recognized that a thin debris layer accelerates the ablation rate of the underlying ice, whereas a thick layer retards it. There is a critical thickness $h_c$ at which the ablation rate for debris-covered glacier ice is the same as for debris-free ice. Since it is convenient to express $h$ in terms of a thermal resistance $R$, $R_c$ corresponding to $h_c$ is discussed in this section and also its dependence on $\alpha_0$, $\alpha$, and various meteorological variables.
According to the model given by Nakawo & Young (1981), the energy-balance equation at a debris surface is expressed as

\[ C = F + H + E \]  \hspace{1cm} (1)

where

\[ F = (1 - \alpha)G + A - \sigma(T_s + 273)^4 \]  \hspace{1cm} (2)

\[ H = \beta u_a(T_a - T_s) \]  \hspace{1cm} (3)

\[ E = \beta L_{e_{u}} \frac{0.623}{p c_p} (e_a - e_s) \]  \hspace{1cm} (4)

All the flux terms are taken positive downward. The model also involves the following assumptions:

\[ C = T/R = Lf\rho r \]  \hspace{1cm} (5)

which means that the temperature profile in a debris layer is in a steady state and the conduction heat solely contributes to ice ablation. Similar expression can be given for debris-free ice.

\[ C_o = F_o + H_o + E_o \]  \hspace{1cm} (1)'

\[ F_o = (1 - \alpha_0)G + A - \sigma(T_o + 273)^4 \]  \hspace{1cm} (2)'

\[ H_o = \beta u_a(T_a - T_o) \]  \hspace{1cm} (3)'

\[ E_o = \beta L_{e_{u}} \frac{0.623}{p c_p} (e_a - e_o) \]  \hspace{1cm} (4)'

\[ C_o = Lf\rho r_o \]  \hspace{1cm} (5)'

Using the approximation of a binomial expansion of the last terms of equations (2) and (2)' considering that \( T_s, T_o \ll 273 \), subtracting equation (2)' from (2), and (3)' from (3) give

\[ F - F_o \approx (\alpha_0 - \alpha)G - K_f T_s \]  \hspace{1cm} (6a)

\[ H - H_o = -\beta u_a T_s \]  \hspace{1cm} (6b)

since \( T_o = 0^\circ C \) and \( T_s \) is in \( ^\circ C \). If \( e_s \) is assumed to be a saturation vapour pressure at \( T_s \), the relationship

\[ e_s = e(T_s) \]

can be expanded by a Taylor series, and taking its first order (i.e. assuming a linear increase of \( e_s \) with \( T_s \)),

\[ e_s \approx e(T_o) + e'(T_o)(T_s - T_o) \]
where $e'$ represents $de/dT$ at temperature $T_0$. This would give a good approximation for $e_S$ for low $T_S$, i.e. for a thin debris layer, which is usually wet at its surface. This approximation, however, would lead to an underestimate of $e(T_S)$ for high $T_S$, i.e. for a thick layer. Nakawo & Young (1981, 1982) showed, however, that $e_S$ is smaller than the saturation vapour pressure at temperature $T_S$ when $T_S$ is large. In other words, a thick debris layer was dry at the surface, and $e_S < e(T)$. The underestimate of $e(T_S)$ for a high $T_S$, would therefore result in giving a more realistic $e_S$ than a precise value of $e(T_S)$ would. It is considered, therefore, that the above approximation will not be far from reality, although for assessing $E$ accurately a detailed study would be necessary on the process of moisture transport through a debris layer. From equations (4) and (4)', assuming this linear relation between $e_S$ and $T_S$,

\[ E - E_0 = -\beta_L u a \frac{0.623}{c_p} e'(T_0) T_S \]  

(6c)

since $e = e_S(T)$. Combining equations (1), (1)', (6a), (6b) and (6c), and replacing $T_S$ by $C R$ (from equation (5)),

\[ C - C_0 = G(\alpha_0 - \alpha) - CR[K_r + \beta u a + \beta_L u a \frac{0.623}{c_p} e'(T_0)] \]  

(6)

C must be equal to $C_0$ when $R = R_c$, hence

\[ R_c = \frac{G(\alpha_0 - \alpha)}{C_0[K_r + \beta u a + \beta_L u a \frac{0.623}{c_p} e'(T_0)]} \]

For estimating $C_o$, it is assumed that $C_o$ is proportional to $T$. The proportionality parameter $k_o$ is named the "degree-day factor", because it is the same as the proportionality parameter between the meltwater runoff and the integrated temperature. $C_o$ can then be replaced by $k_o T_a$. Hence,

\[ R_c = \frac{G(\alpha_0 - \alpha)}{k_o T_a[K_r + \beta u a + \beta_L u a \frac{0.623}{c_p} e'(T_0)]]} \]  

(7)

Equation (7) expresses the dependence of $R_c$ on various meteorological variables. It has to be noted, however, that $k_o$ will be a function of $\alpha_o$, $G$, $T$, $u$, and other variables such as cloud conditions (see e.g. Kuh, 1979; Braithwaite, 1980). In other words, $k_o$ is not really a constant, as will be discussed later.

The wind velocity $u_a$ is not difficult to measure but is relatively more difficult to estimate over a wide area of the glacier surface than the other variables, equation (7) can be reduced further as follows. Assuming $e_a$, like $e_S$, to be equal to

\[ e(T_0) + e'(T_0)(T_a - T_0) \]
Model for estimating ablation under debris

and combining equations (3)' and (4)',

\[ R_c = \frac{G(\alpha_o - \alpha)}{k_o (K_{Ta} + H_o + E_o)} \]  

(8)

It is assumed that long-wave radiation is approximately balanced, that is

\[ C_o \approx G(1 - \alpha_o) + H_o + E_o \]  

(9)

This assumption would not always hold, but should in the Nepal Himalaya during the monsoon season, when the dominant ablation takes place. Combining and rearranging equations (8) and (9), \( R_c \) can be expressed in the following form in which \( u_a \) is eliminated:

\[ R_c = \frac{G(\alpha_o - \alpha)}{k_o [(k_o + K_{Ta}) - G(1 - \alpha_o)]} \]  

(10)

The value of \( R_c \) can be determined directly by an experiment in which ablation rates are measured under debris layers with different thickness (different thermal resistances). In Fig.1, the value of \( R_c \) estimated by equation (10) is compared with the value from the direct method. The agreement between the two estimates is by no

![Comparison of \( R_c \) estimated from \( T_a \), \( k_o \) and \( G \) with values from \( h_c \) and \( K_m \).](image)

FIG.1 Comparison of \( R_c \) estimated from \( T_a \), \( k_o \) and \( G \) with values from \( h_c \) and \( K_m \). The horizontal bar corresponds to the standard deviation or range of each datum. \( K_m \) was not measured in Yoshida's experiment. The vertical bar represents a range of \( R_c \) corresponding to the range of \( K_m \) for his experiment. \( K_m \) for Magono & Kumai's data was obtained from a value of \( T_S \) through the procedure given by Nakawo & Young (1982). \( k_o \) for all the data was calculated by \( k_o = C_o/T_a \).

means excellent (note they are plotted in logarithmic scale).

Figure 1 shows, however, that equation (10) can give a reasonable estimate of the order of magnitude of \( R_c \) in spite of many assumptions and simplifications in deriving the equation. If
additional data on \( u_a \) were available, a better estimate would be obtained by using equation (7) than by using equation (10).

### ESTIMATE OF ABLATION UNDER A GIVEN DEBRIS LAYER

It is assumed that \( C_o \) and consequently \( r_o \) (from equation (5)'') can be obtained from a knowledge of \( k_o \) and \( T_a \). Equations (5) and (5)' and the definition of \( k \) and \( k_o \) give

\[
\frac{r}{r_o} = \frac{C}{C_o} = \frac{k}{k_o} = k^*
\]

Hence, one can estimate \( r \) or \( C \) if \( k^* \) is known. A method of estimation of \( k^* \) is given in this section.

From equations (6) and (7)

\[
\frac{C}{C_o} = \frac{C_o + G(\alpha_o - \alpha) - \alpha}{C_o + G(\alpha_o - \alpha)(R/R_c)}
\]

which reduces to

\[
k^* = \frac{1 + G^*}{1 + G^* R^*}
\]

For a large \( R^* \) in comparison with \( 1/G^* \), i.e. for a very thick debris layer,

\[
k^* \approx \frac{1}{R^*}
\]

It is suggested accordingly that \( k^* \) is inversely proportional to \( R^* \). Zhang & Bai (1980) proposed an empirical relation between \( k^* \) and \( h \).

\[
k^* \propto \frac{1}{h^b}
\]

They found that the value for \( b \) increased, approaching unity as \( C_o \) decreased, i.e. \( 1/G^* \) decreased. Since \( h \) is approximately proportional to \( R^* \), their results can be considered as in agreement with equations (13) and (14).

Equation (13) is represented by the solid lines in Fig.2 in which \( k^* \) is plotted against \( R^* \) for particular values of \( G^* \) which are derived from experimental or field conditions as cited. Agreement between the theoretical predictions and the field data is quite good, although there is some scatter in the data.

There are other data available on \( k^* \) and \( R^* \) (Østrem, 1959; Loomis, 1970; Moribayashi & Higuchi, 1972; Fujii, 1977; Suizu, private communication). In their reports, however, no data on \( G^* \) are given. Nonetheless, they are plotted in Fig.3, in which the dependence of \( k^* \) on \( R^* \) given by equation (13) is also shown for various values of \( G^* \). The data of each set of observations are compatible with a predicted curve corresponding to one of the values for \( G^* \), and the variety of the results obtained by the different authors can be
explained in terms of $G^*$. 

FIG. 2 $k^*$ vs. $R^*$. Solid lines represent predictions by equation (13) for particular values of $G^*$ corresponding to each experiment. $R^*$ for Yoshida's data was calculated by $R^* = h/h^c$, i.e. assuming $K_m$ constant for every thickness.

FIG. 3 $k^*$ vs. $R^*$. Predictions by equation (13) are shown with fine solid lines for various values of $G^*$. $R^*$ for all the experimental data was calculated by $R^* = h/h^c$, i.e. assuming $K_m$ to be independent on $h$.

**DISCUSSION**

As was shown in the previous section, $C$, and hence $r$, can be obtained by knowing $T_a$, $k_0$, $G^*$ (or $G_\alpha$, and $\alpha_0$), and $R^*$ (or $R$ and $R_c$). It is assumed, in estimating $C$ or $r$, that $C_0$ is obtainable through $C_0 = k_0 T_a$. Many investigators found good correlations between $C_0$ and $T_a$, and the expression has been widely adopted. Takahashi et al. (1981) explained the good correlation in terms of correlations between $T_a$ and both $E_0$ and long-wave radiation balance in addition to a correlation between $T_a$ and $H_0$. Braithwaite (1981) also justified the expression by showing that the variation of $r_0$ is
mainly due to the variation of $H_Q$, despite the main heat source being $F_0$. In any event, this approach for estimating $C_Q$ would be useful for predicting ablation rate of glaciers in areas where meteorological data are limited such as the Nepal Himalaya, since $T_a$ can be extrapolated relatively easily from records at weather stations located at lower elevations.

In the extrapolation, however, one has to take carefully into account any local modification of the air mass which takes place depending on the distribution of debris-free ice surface, debris-covered area, and bare-rock area etc. (Braithwaite, 1980). The value of $k_Q$ should be evaluated from data accumulated for a long time, because it fluctuates widely over a short period, particularly when $T_a$ is close to 0°C (Takahashi et al., 1981). If it is determined from the data from, say, a few months period, substantial random errors tend to compensate each other.

Large glaciers in the Nepal Himalaya, such as Khumbu Glacier (Pushimi et al., 1980) and G2 glacier (Nakawo, 1979), are covered with thick debris in their ablation area, which extends for more than several kilometres in length and a few hundred metres in elevation. No debris-free area is found in the ablation area of those glaciers. The variables for debris-free ice surfaces, such as $r_Q$, $C_Q$, $a_Q$, are apparently meaningless at such a place on the glacier. These variables, however, are involved in the model only for providing a standard for each term for debris-covered ice. "Debris-free ice", therefore, can be hypothetical provided that $k_Q$ for the "debris-free ice" can be estimated by extrapolating its value from debris-free areas at higher elevations. The proposed model, therefore, could be adopted for estimating the runoff from glaciers in the Himalaya by studying $k_Q$ in various valleys and on mountain slopes.

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