The shape and position of the salt water wedge in coastal aquifers

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ABSTRACT The paper deals with the shape and position of the interface between fresh and saline groundwater in coastal aquifers. It is restricted to the one-dimensional case in steady state. The interface is assumed to be sharp; the effects of diffusion and dispersion are neglected. The paper deals with the following three cases: (a) confined groundwater, (b) phreatic groundwater and (c) semiconfined groundwater. The study clearly shows the influence of various geohydrological constants on the shape and position of the salt water wedge. The study results in graphs describing the horizontal distance between an observation well and the upper and lower ends of the salt water wedge as a function of the piezometric level of the fresh groundwater observed in that well. The results of the study are useful in predicting the effect of human activities, such as groundwater abstraction, on the length and position of the salt water wedge.

La configuration et la situation du biseau salé dans des nappes côtières
RESUME Cet article traite de la configuration et de la situation de l'interface eau douce-eau salée dans des nappes côtières. Il se limite au cas unidimensionnel en régime permanent. L'interface est supposée bien nette; les effets de diffusion et de dispersion ont été négligés. L'article traite les trois cas suivants: (a) nappe captive, (b) nappe libre et (c) nappe semi-captive. L'étude démontre clairement l'influence des divers paramètres géohydrologiques sur la configuration et la situation du biseau salé. Elle aboutit à des graphiques donnant la distance horizontale entre un puit d'observation et les extrémités supérieures et inférieures du biseau salé en fonction du niveau piézométrique de l'eau douce mesuré dans ce puit. Les résultats de l'étude serviront à la prévision de l'effet des activités humaines, comme le prélèvement des eaux souterraines, sur la longueur et la position du biseau salé.

NOTATION

$c$ hydraulic resistance of semi-pervious layer [T]
$C, C_1, C_2$ integration constants
D  aquifer thickness below reference level [L]
E  \( [p - (1 + a)h_s]/\alpha = \text{equilibrium depth [L]} \)
f  rate of natural recharge [L T^{-1}]
h  \( h_f - h_s \)
h_f  piezometric level of the flowing fresh groundwater above reference level [L]
h_s  piezometric level of the stagnant saline groundwater above reference level [L]
H  depth of the interface below reference level [L]
k  permeability of the aquifer for fresh groundwater [L T^{-1}]
L  length of the salt water wedge [L]
p  height of the phreatic groundwater table above reference level [L]
q  groundwater flow per unit length [L^2 T^{-1}]
q_0  groundwater flow per unit length at \( x = 0 \) [L^2 T^{-1}]
x  ordinate [L]
x_0  ordinate, reference value [L]
x_l  ordinate of the lower end of the salt water wedge [L]
x_u  ordinate of the upper end of the salt water wedge [L]
X  ordinate value of observation well [L]
\alpha  \( (\rho_s - \rho_f)/\rho_f = \text{relative density difference} \)
\lambda  \sqrt{kDc} = \text{characteristic length [L]}
\rho_f  density of fresh groundwater [M L^{-3}]
\rho_s  density of saline groundwater [M L^{-3}]

INTRODUCTION

Salt water intrusion occurs in coastal areas all over the world. In many places it may cause serious problems.

Under natural conditions, the aquifer recharge discharges as fresh groundwater into the sea or ocean. In coastal areas, fresh groundwater flows over the saline groundwater which, in undisturbed conditions, is stagnant and in pressure equilibrium with the fresh groundwater. The transition from fresh to saline groundwater with depth is not a sharp one due to the effects of dispersion and diffusion. Yet it is and may often be schematized as a sharp interface. Due to pumping of the fresh groundwater the natural equilibrium is disturbed and saline groundwater moves inland until a new dynamic equilibrium is established. This is acceptable as long as no groundwater abstraction works are threatened by contamination with salty water.

This practical problem is generally recognized and the mechanism of this two phase groundwater flow problem is well understood. Yet many dramatic failures are known and can be predicted. Such failures can be due to various reasons, such as the absence of alternative sources which leads to overpumping of the fresh groundwater. Private interests, insufficient legislation or even political issues often make the technical solution of this geohydrological problem very complicated or even impossible.

Though the mechanism of this two phase groundwater flow problem is well understood the problem at hand in a particular area can be only fully assessed after gathering relevant geohydrologic data, such as the structure of the aquifer and its geohydrological constants,
including the density distribution of the groundwater. The observed piezometric levels complete the picture to quantify the groundwater flow.

This paper deals with some interesting properties of the interfaces between fresh and saline groundwater which, to the author's knowledge, have never been presented explicitly. As a result of this analysis it is possible to draw conclusions about the position of the salt water wedge under different conditions of outflow in coastal aquifers from piezometric levels observed in the fresh groundwater.

This paper is restricted to the case of one-dimensional steady-state flow in aquifers where Darcy's law applies, and where the interface is assumed to be sharp. Successively the confined, the phreatic and the semi-confined case will be dealt with.

**CONFINED GROUNDWATER**

**Description**

Figure 1 shows the profile of confined groundwater where fresh groundwater flows above stagnant saline groundwater. The reference level is chosen at the top of the aquifer. The three unknowns are: $H$, $h_f$ (or $h$) and $q$. The three basic equations read:

$$q = -kH \frac{dh_f}{dx}$$  \hspace{1cm} (1)

$$q = q_0$$  \hspace{1cm} (2)

$$h = h_f - h_s = \alpha (h_s + H)$$  \hspace{1cm} (3)

Combination of these equations leads to the differential equation:

$$H \frac{dH}{dx} = -\frac{q_0}{\alpha k}$$  \hspace{1cm} (4)

with solutions for $H$ and subsequently for $h$:

![Confined groundwater: definition of symbols.](image-url)
In the problem under consideration, as depicted in Fig. 2, the boundary condition is \( x = 0, H = 0 \), so that

\[
H = \sqrt{(-2 \, q_0 \, x/ak + C)} \quad (5)
\]

\[
h = \alpha[h_s + \sqrt{(-2 \, q_0 \, x/ak + C)}] \quad (6)
\]

\[
q = q_0 \quad (7)
\]

Expression (8) for \( H \) is parabolic with its axis horizontally along the top of the aquifer. Consequently, expression (9) for \( h \) is also parabolic with its axis horizontally at the level, \( \alpha h_s \).

The length \( L \) of the salt water wedge can be readily calculated from expression (8); \( L \) is a critical value of \( x \) when \( H \) equals the aquifer thickness, \( D \). \( L \) is given by the following expression:

\[
L = \frac{-\alpha k D^2}{2 \, q_0} \quad (11)
\]

Beyond \( x = L \), where \( H = D \) and

\[
h = \alpha(h_s + D) \quad (12)
\]

there is no interface and \( h \) is described in the same coordinate system by:

\[
h = \alpha h_s + \alpha D/2 - q_0 \, x/kD \quad (13)
\]

Note that for \( x = L \), both (9) and (13) yield:
Salt water wedge in coastal aquifers

\[ h_L = \alpha (h_s + D) \]  \quad (14)

\[ \frac{d(h)}{dx} \bigg|_L = \frac{-q_0}{kD} \]  \quad (15)

Figure 3 shows both the parabola of (9) and the straight line of (13). The applicable parts have been drawn in solid bold lines while the fictive parts are shown as dashed bold lines.

![Figure 3](image_url)

**FIG.3** \( h \) vs. \( x \) for confined groundwater.

The following remarks can now be made.

(a) Table 1 summarizes which of the relevant parameters or combinations thereof occur in the above expressions.

**TABLE 1  Parameters occurring in the expressions for confined groundwater**

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Expression: for:</th>
<th>(8)</th>
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<th>(11)</th>
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</table>

(b) From Table 1 it appears that the shape of the interface is independent of the depth and the thickness of the aquifer. For a given situation with fixed values of \( \alpha \) and \( k \), the outflow, \( q_0 \), is the only parameter under human control. The length \( L \) is inversely proportional to the outflow \( q_0 \) (expression (11)).

(c) According to expression (13), for \( x > L \), \( h \) in the reach upstream of the salt water wedge is described by a straight line through the point \( P \) (\( x = 0, h = \alpha (h_s + D/2) \)) in Fig.3. This point is independent of the outflow \( q_0 \); \( q_0/k \) affects only the slope of the straight line.

(d) The straight line of expression (13) lies everywhere at a height \( \alpha (h_s + D/2) \) above the straight line:

\[ h = -q_0 \frac{x}{kD} \]  \quad (16)
which is drawn as a fine line in Fig.3. This straight line would apply over the full length in the case of outflow of fresh groundwater into a fresh water lake. In that case there is no saline groundwater and no interface. The height $\alpha(h_s + D/2)$ can thus be considered as an outflow resistance due to the presence of the saline groundwater.

(e) The above solutions are based on the Dupuit-Forchheimer approach which ignores the vertical component of flow. Therefore, the solution is approximate, but it is quite acceptable for the greater values of $x$, where the slope of the interface is small. For the smaller values of $x$ the solution is not acceptable. The correct solution has been given by Glover (1959). His solution for $h$ is the same as in expression (9), where in his case $h_s = 0$. His solution for $H$, reworked in the symbols of the present paper, reads:

$$H = \left[ \frac{-2q_0}{\alpha k}(x - \frac{q_0}{2\alpha k}) \right]^\frac{1}{2}$$

which means that all values of $x$ in (8) are increased by $-q_0/2\alpha k$ or, in other words, the salt water wedge as a whole is displaced a length $-q_0/2\alpha k$ in the direction of the open water. The curve of expression (17) is also drawn in Fig.2; it is the dashed line a distance $-q_0/2\alpha k$ left of the parabola of expression (8), drawn as a solid line. Note that the distance $-q_0/2\alpha k$ is exactly half the value of $H$ at $x = 0$.

Use and interpretation of piezometric level data

The problem described here is characterized by the length $L$ of the salt water wedge. Unless intensive sampling is undertaken in a number of boreholes or a geophysical survey is performed this length cannot be measured directly. The easiest data to measure are the piezometric levels. So the question arises how to derive the length $L$ from the piezometric level $h_x$ of the fresh groundwater measured in an observation well at the location $x = X$, or rather from $h_x$.

The relationship between $L$ and $h_x$ can be obtained by eliminating $q_0$ from (11) and (9) for $L > X$ and from (11) and (13) for $L < X$. For $L > X$ (and $\alpha h_s < h_X < \alpha(h_s + D)$):

$$h_x = \alpha[h_s + D/(X/L)]$$

and for $L < X$ (and $h_X > \alpha(h_s + D)$):

$$h_x = \alpha[h_s + \frac{1}{2}D(1 + X/L)]$$

For the sake of completeness, mention should be made of the assumption that there is no groundwater abstraction at any place $x < L$ in the case $L > X$, nor at any place $x < X$ in the case $L < X$.

The result, expressions (18) and (19), is presented graphically in Fig.4. From this figure one can determine the position of the saltwater wedge, as characterized by the length $L$ as a function of the observed value of $h_x$ in an observation well at $x = X$.

The sensitivity of this relationship can be expressed by $|dh_x/dL|$,
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which is:

\[ \frac{1}{2} \alpha DX \frac{L}{L}^{-\frac{3}{2}} \quad \text{for } L > X \quad (20) \]

\[ \frac{1}{2} \alpha DX \frac{L}{L}^{-2} \quad \text{for } L < X \quad (21) \]

This means that the sensitivity of this relationship increases with decreasing L, as is clearly shown in Fig.4. It means also that the sensitivity increases with increasing length X or, in other words, the more inland the observation well the more accurately the position of the salt water wedge can be determined, provided that the ratio \( \alpha \) and the aquifer thickness D are constant and known with sufficient accuracy. Remarkably, the variables \( k \) and \( h_s \) do not appear in (20) and (21).

**PHREATIC GROUNDWATER WITH RECHARGE**

**Description**

Figure 5 shows the profile of phreatic fresh groundwater having natural recharge \( f \) above stagnant saline groundwater. The reference level is selected as the piezometric level of the stagnant saline groundwater. The three unknowns are \( H, h \) and \( q \). The three basic equations read:

\[ q = -k(H + h) \frac{dh}{dx} \quad (22) \]

\[ \frac{dq}{dx} = f \quad (23) \]

\[ h = \alpha H \quad (24) \]

These are combined to produce the differential equation:

\[ H \frac{dH}{dx} = -\frac{f x + C_1}{(1 + \alpha)\alpha k} \quad (25) \]

with solutions for \( H \) and subsequently for \( h \) and \( q \):
FIG.5 Phreatic groundwater: definition of symbols.

\[
H = \left[ \frac{-fx^2 - 2 C_1x + C_2}{(1 + \alpha)k} \right]^{1/2} \tag{26}
\]

\[
h = \left[ \frac{\alpha(-fx^2 - 2 C_1x + C_2)}{(1 + \alpha)k} \right]^{1/2} \tag{27}
\]

\[
q = fx + C_1 \tag{28}
\]

Boundary conditions for the problem under consideration, as depicted in Fig.6, are: \( x = 0, H = 0, q = q_0 \) (\( q_0 \) is negative) so that:

\[
H = \left[ \frac{-fx^2 - 2 q_0 x}{(1 + \alpha)k} \right]^{1/2} \tag{29}
\]

\[
h = \left[ \frac{\alpha(-fx^2 - 2 q_0 x)}{(1 + \alpha)k} \right]^{1/2} \tag{30}
\]

\[
q = fx + q_0 \tag{31}
\]

Both (29) and (30), for \( H \) and \( h \), represent ellipses with their horizontal axis at the level \( h_S \).

The length \( L \) (see Fig.6) can be readily calculated from (29). \( L \) is that critical distance \( x \) for which \( H = D \), the thickness of the aquifer below the level \( h_S \) (see Fig.6). The result is:

\[
L = \frac{-q_0/k \pm \sqrt{[\frac{(q_0/k)^2}{(1 + \alpha)k} - (1 + \alpha) (f/k)D^2]}}{f/k} \tag{32}
\]

In this problem only the negative sign needs to be considered; the positive sign is applicable together with the negative value in the case of an island of infinite length perpendicular to the plane of drawing.

Beyond \( x = L \), where \( h = D \), there is no interface and the
phreatic groundwater table is described, in the same coordinate system, by:

\[ h = \frac{aD}{2} - \frac{(2q_0 + fx)x}{2(1 + a)kD} \]  

(33)

Note that for \( x = L \) both (30) and (33) yield:

\[ h_L = \frac{aD}{2} \]  

(34)

\[ \left( \frac{dh}{dx} \right)_L = \frac{\sqrt{(q_0/k)^2 - (1 + a)\alpha(f/k)D^2}}{(1 + a)D} \]  

(35)

Figure 7 shows both the ellipse of (30) and the parabola of (33). The applicable parts have been drawn in solid bold lines and the fictive parts in dashed bold lines.

The following remarks can now be made.

(a) Table 2 summarizes which of the relevant parameters or combinations thereof occur in the above expressions.

(b) From Table 2 it appears that the shapes of the interface and the phreatic groundwater table are independent of the thickness of the aquifer. For a given situation with fixed values of \( a, k \) and \( f \),
the outflow $q_0$ is the only parameter under human control, and $D$ is a
determinant of $L$.

(c) According to expression (33), for $x > L$, the phreatic
groundwater table, as described by $h$, is a parabola through
point $P$ ($x = 0, h = \frac{1}{2}D$) in Fig. 7. The point $P$ is independent
of the outflow $q_0$ and the natural recharge $f$, rather of $q_0/k$ and
$f/k$.

(d) The parabola of expression (33) is displaced by a height
$\frac{1}{2}D$ above the parabola:

$$h = \frac{-2q_0 + fx}{2(1+\alpha)kD}$$

which is shown by a fine line in Fig. 7. This parabola would apply
over the full length in the case of outflow of fresh groundwater
into a fresh water lake involving transmissivity of the aquifer as
$(1+\alpha)kD$. In that case there is no saline groundwater and no
interface. Again, height $\frac{1}{2}D$ can be considered as an outflow
resistance due to the presence of the saline groundwater.

(e) As in the confined case, the solutions given by (26) and (27)
are acceptable for the larger values of $x$. For the values of $x$ near
zero the interface is displaced by $-q_0/2ak$ in the direction of the
open water. The offset curve is shown as a dashed line in Fig. 6
and is consistent with the Glover solution for low values of $x$.

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Expression: for: $H$ (29) $h$ (30) $L$ (32) $h$ (33) $h_L$ (34) $(dh/dx)_L$</th>
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<tr>
<td>$D$</td>
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<td>$\alpha$</td>
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<td>$q_0/k$</td>
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<td>$f/k$</td>
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</table>

Use and interpretation of piezometric level data

As in the confined case, a relationship between $h_X$ and $L$ is possible.
For $x = L$ and $H = D$, expression (29) can be expressed as:

$$D^2 = \frac{-fL^2 - 2q_0 L}{(1+\alpha)ak}$$

whereas, for $X < L$, and according to expression (30):

$$h_X = \left[\frac{\alpha(-fx^2 - 2q_0 X)}{(1+\alpha)k}\right]^{\frac{1}{2}}$$

and, for $x > L$, using expression (33):

$$h_X = \frac{\alpha D}{2} - \frac{2q_0 X + fx^2}{2(1+\alpha)kD}$$
Eliminating $q_g$ from expressions (37) and (38) respectively (37) and (39) yields:

for $X < L$ (and $0 < h_X < aD$):

$$h_X = \left[ \frac{\alpha^2 D^2}{L} + \frac{\alpha}{1 + \alpha} \cdot \frac{f}{k} (XL - X^2) \right]^{\frac{1}{2}}$$

(40)

and for $X > L$ (and $h_X > aD$):

$$h_X = \frac{aD}{2} (1 + \frac{X}{L}) + \frac{1}{2} \frac{f}{k} \frac{XL - X^2}{(1 + \alpha)D}$$

(41)

again assuming that there is no groundwater abstraction at any place $x < L$ in the case $L > X$, nor at any place $x < X$ in the case $L < X$.

The expressions (40) and (41) do not allow a general presentation as in Fig.4 for the expressions (18) and (19) for the confined case. The parameter $f/k$ must be chosen and moreover, contrary to the confined case, where $X$ and $L$ occurred only in the ratio $X/L$, $X$ and $L$ occur separately in (40) and (41). Therefore the following example has been chosen for presentation in Fig.8: $\alpha = 0.025$, $D = 100$ m, $f/k = 10^{-3}$ m day$^{-1}$/10 m day$^{-1} = 10^{-6}$ and $X = 200, 500, 1000, 1600.78$ and 2500 m. The value 1600.78 m needs explanation: it is that value of $X$ for which $h = aD$ at $L = X = -q_0/f = D/[(1 + \alpha) \alpha/(f/k)]$. For $X = 2500$ m the curve has been drawn only for that part where $-q_0 > fX$ or $X$ is smaller than the distance from the coastline to the groundwater divide.

The sensitivity of the relationships described by (40) and (41) is expressed by $|dh_X/dL|$. As appears clearly from Fig.8 this sensitivity increases with decreasing $L$. It appears also that the sensitivity increases with increasing length or, in other words, the more inland the observation well the more accurately the position of the salt water wedge can be determined, provided that $\alpha, D$ and $f/k$ are constant and known with sufficient accuracy.

**SEMICONFINED GROUNDWATER**

**Description**

Figure 9 shows the profile of semiconfined groundwater: fresh
groundwater flowing above stagnant saline groundwater. The reference level is chosen at the top of the aquifer.

The three unknowns are H, \( h_f \) (or \( h \)) and q. The three basic equations read

\[
q = -kH \frac{dh_f}{dx} \\
\frac{dq}{dx} = \frac{p - h_f}{c} \\
h = h_f - h_s = \alpha(h_s + H)
\]

Combining these equations to produce a differential equation:

\[
H \frac{d^2H}{dx^2} + \left( \frac{dH}{dx} \right)^2 - \frac{H - [p - (1 + \alpha)h_s]/\alpha}{kc} = 0
\]

A particular solution of this differential equation is:

\[
H = E = [p - (1 + \alpha)h_s]/\alpha
\]

where E is the "equilibrium depth". This term arises since when \( H = E, h_f = p \). Then, in the absence of horizontal flow in the aquifer and vertical flow in the semipervious layer, the stagnant fresh groundwater is in static equilibrium with the stagnant saline groundwater.

Differential equation (45) can now be written as:

\[
H \frac{d^2H}{dx^2} + \left( \frac{dH}{dx} \right)^2 - \frac{H - E}{kc} = 0
\]

Altogether, there are 13 solutions to this differential equation, (see van Dam & Sikkema (1982) and Sikkema & van Dam (1982)); five for \( E > 0 \), three for \( E = 0 \) and five for \( E < 0 \). As the present paper deals with the outflow of fresh groundwater in coastal aquifers, we will consider only those cases for which \( h_f > p \) and
thus, as $H > 0$, $E < 0$. Among the solutions for $E < 0$ there is only one for which $q = 0$ where $H = 0$, namely:

$$H = 3E/2 + (x - x_0)^2/6kc$$

with the corresponding solutions for $h$ and $q$.

$$h = \alpha[h_s + 3E/2 + (x - x_0)^2/6kc]$$

$$q = -\frac{\alpha(x - x_0)^3}{18kc^2} - \frac{\alpha(x - x_0)E}{2c}$$

Expression (48) describes a parabola with a vertical axis and its peak at $x = x_0$, $H = 3E/2$.

The parabolic form is preserved irrespective of the sign of $E$. Figure 10 illustrates the variation in parabolic form for the following conditions: (a) $E > 0$, (b) $E = 0$, (c) $E < 0$ and $-E < h_s < p$, (d) $E < 0$ and $-E = h_s = p$, (e) $E < 0$ and $-E > h_s > p$.

In each part of this figure the equilibrium depth, $E$, is drawn in a dash-dot line and the interface, $H$, is drawn in a solid bold line, i.e. that part of the parabola where both $x > x_0$ and $H > 0$. The dashed parts of the parabola are not applicable.

The case shown in Fig.10(a) where $E > 0$ does not apply to our problem. For the case shown in Fig.10(b), where $E = 0$, the interface follows the parabola on the right-hand side of the figure and the top of the aquifer in the left-hand side. The semipervious layer contains fresh groundwater. When $E < 0$ and $-E < h_s < p$ (Fig.10(c)), the interface follows the parabola on the right-hand side of the aquifer and then moves to the equilibrium depth, which, in this case, is within the semipervious layer. When $E < 0$ and $-E = h_s = p$ (Fig.10(d)), the interface follows the parabola in the right-hand side of the aquifer and then moves to the level of $h_s$ and $p$, which are equal in this case. The semipervious layer contains stagnant saline groundwater only in the left-hand side. This is the case of a coastal plain, where the groundwater table ($p$) is at mean sea level ($h_s$). This particular case is explored further. The aquifer and the semipervious layer need not to extend to $x = -\infty$, but at least to the place where $H = 0$, and $q = 0$.

Finally, for $E < 0$ and $-E > h_s > p$ (Fig.10(e)), the interface follows the parabola on the right-hand side of the aquifer and then moves to the top of the semipervious layer. The semipervious layer contains on the left-hand side only saline groundwater flowing vertically upwards. This upward flow implies horizontal flow of the saline groundwater in the underlying part of the aquifer. This conflicts with the assumption of stagnant saline groundwater made in the beginning of this paragraph and is not developed further in the paper.

The parabola of expression (48) is characterized by its curvature and by its elevation. The curvature:

$$\frac{d^2H}{dx^2} = \frac{1}{3kc}$$

depends on the product $kc$ only. The elevation depends on $E$, i.e. on $\alpha$, $p$ and $h_s$ only. For a given semiconfined profile the shape is
FIG. 10 Position of interface for (a) $E > 0$, (b) $E = 0$, (c) $E < 0$ and $-E < h_S < p$, (d) $E < 0$ and $-E = h_S = p$, and (e) $E < 0$ and $-E > h_S > p$, in semiconfined groundwater.
fixed and the elevation depends on the equilibrium depth. The position of the interface in the horizontal direction, described by $x_u$ and $x_1$, depends on the magnitude of the flow of fresh groundwater at the upstream end of the profile.

The length $L$ of the salt water wedge in a semiconfined aquifer of thickness $D$ can be calculated from expression (48), as the difference of the values of $x_u - x_0$, for which $H = 0$, and $x_1 = x_0$, for which $H = D$. For $H = 0$:

$$x_u - x_0 = \sqrt{(6 \, k_e - 3 \, E/2)} = 3 \sqrt{-k_c E} = 3 \lambda \sqrt{-E/D} \tag{52}$$

for $H = D$:

$$x_1 - x_0 = \sqrt[6]{k_c \cdot (D - 3 \, E/2)} = 3 \lambda \sqrt{2/3 - E/D} \tag{53}$$

so that $L = x_1 - x_u = 3 \lambda [\sqrt{2/3 - E/D} - \sqrt{-E/D}] \tag{54}$

See Figs 10(c), (d) and (e), and 10(b) where $E/D = 0$. Beyond

$$x - x_0 = 3 \lambda \sqrt{2/3 - E/D} \tag{55}$$

where

$$h = a(h_s + D) \tag{56}$$

there is no interface and $h$ is described, in the same coordinate system, by:

$$h = \frac{3}{\lambda} aD\left[1 + h_s/D - \sqrt{(2/3 - E/D)}\right] \exp\left[-\frac{x - x_0 - 3\lambda \sqrt{2/3 - E/D}}{\lambda}\right] \tag{57}$$

$$+ \frac{3}{\lambda} aD\left[1 + h_s/D + \sqrt{(2/3 - E/D)}\right] \exp\left[+\frac{x - x_0 - 3\lambda \sqrt{2/3 - E/D}}{\lambda}\right]$$

Note that for $x - x_0 = 3\lambda \sqrt{2/3 - E/D}$ both expressions (49) and (57) yield:
Use and interpretation of piezometric level data

Like in the confined and in the phreatic case, the value of \( h_x \), measured in an observation well at \( x = X \), is a good indicator for the position of the salt water wedge. In the semiconfined case however the shape and length of the wedge depends on the geohydrological constants \( k_c \), \( D \) and \( E \) only. In contrast to the confined and the phreatic case, the groundwater flow \( q \) (which is 0 at \( x_u \)) does not determine the shape of the wedge, but only its position in the horizontal direction.

Expressions (49) and (57) contain the unknown \( x_0 \) which is of no interest in our solution. Instead, interest focuses on \( x_u \) and \( x_1 \) in expressions (52) and (53). Therefore \( x_0 \) in expressions (49) and (57) is eliminated as follows:

\[
x_0 = x_u - 3 \lambda \sqrt[6]{-E/D}
\] (60)

and

\[
x_0 = x_1 - 3 \lambda \sqrt[6]{2/3 - E/D}
\] (61)

So, for example, substitution of expression (60) into expression (49) yields, for \( x = X \):

\[
h_X = \alpha \{h_s + 3 E/2 + [X - x_u + 3\lambda \sqrt[6]{-E/D}]^2/6 k_c\}
\] (62)

and substitution of expression (61) into expression (57) yields for \( x = X \):

\[
h_X = \frac{1}{2} \alpha D [1 + h_s/D - \sqrt[6]{2/3 - E/D}] \exp[-(X - x_1)/\lambda] + [1 + h_s/D + \sqrt[6]{2/3 - E/D}] \exp[+(X - x_1)/\lambda]
\] (63)

Expressions (62) and (63) describe the relationship between \( h_X \) and \( X - x_u \), and that with \( X - x_1 \), to define the extremes of the salt wedge with respect to the location of the observation well. The length \( L \) of the wedge also follows from expression (54). Thus, Fig. 11 can now also be used to specify \( (X - x_1)/\lambda \) and \( (X - x_u)/\lambda \) as a function of the measured value \( h_X \) in terms of the particular example.

As in the confined case and in the phreatic case the assumption of no groundwater extraction was made for the appropriate reaches of
the aquifer \((x < x_1 \text{ in case } x_1 > X \text{ and } x < X \text{ in case } x_1 < X)\). As in the confined case and in the phreatic case the sensitivity, expressed by

\[
\left| \frac{dh_x}{d(X - x)} \right| \text{ and } \left| \frac{dh_x}{d(X - x_1)} \right|
\]

increases with increasing values of \(h_x\), provided that the geohydrological constants \(k_c\), \(D\) and \(E\) are constant and known with sufficient accuracy.

CONCLUDING REMARKS

The shape and position of the salt water wedge in coastal aquifers depend very much on the type of aquifer.

In confined and phreatic groundwater the outflow of fresh groundwater governs the length of the salt water wedge. In semiconfined groundwater, the outflow of fresh groundwater governs the position of the salt water wedge, the shape of which is invariant in a given system.

Piezometric levels measured in the fresh and saline groundwater in coastal areas enable us to determine the shape and position of the salt water wedge accurately, at least under the conditions described in this paper. The sensitivity of these relationships to uncertainties in the values of the geohydrological constants involved must still be investigated.

REFERENCES


