OVER RECHARGE EXPLORATION OF GROUNDWATER AND LAND SUBSIDENCE
IN TAIYUAN CITY, SHANXI PROVINCE, CHINA

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Abstract
Land subsidence was found in Taiyuan, an industrial city in
Shanxi plateau of North China, since the early 1970s after pre­
cise leveling. The maximum subsidence was up to 1.232m in 1982.
The configuration of subsidence isoline conformed basically
with the isopotential line of groundwater drawdown. The subsi­
dence centre situated at the front verge of Taiyuan alluvial
fan where cohesive soil interlayer became more and thicker.

By the history and existing situation of ground water deve­
lopment in Taiyuan, associated with the hydrogeological condi­
tion, using mathematic model and records of long term observa­
tion, this paper proved that the land subsidence in Taiyuan
was due to intensive development over recharge. This paper
stressed that attention should be paid to keep the balance of
yield and recharge when developing ground water from alluvial
fan of an industrial city. Otherwise, land subsidence was
liable to occur even in inner land far away from seashore.

I.
Taiyuan is a city of heavy industry in North China. The ground
water resources there were relatively abundant. At its north
is the Shan Lan spring with a flow of 0.55m³/s. At its south
is the 'Never Old Spring' of the famous scenic spot — Jin Ci
Temple, with a flow of 2m³/s. The water had been used for
agricultural irrigation since the time of War State (2000B.C.)
for the production of well known Jin Ci Temple rice. After
the 1950s the extraction of ground water increased rapidly as
a result of development in production and prosperity in popu­
lation. Take the pumpage in 1950 as 1, then in 1960 it incre­
sed to 19.7, in 1970, up to 32, in 1978, up to 48.7 and in
1982 up to 100. Hence the ground water level of Taiyuan was
lowered continuously as a whole and formed a cone of depression
with an area of 290km²(Fig.1). The centre of the cone was at
the west of the city. The depression was as deep as 50-60 m.
In 1979-1980 precise leveling of second and third order were
conducted twice by Taiyuan Institute of Building Design and
Municipal Surveying Team and resurveyed in 1982. An isoline
map of land subsidence was produced (Fig.2) which showed that
the maximum subsidence was 1232mm. It occurred in Wu Jia Pu at
the south of the city. Area with subsidence over 1000mm was
7.1km² and that with subsidence deeper than 200mm was 245km².
The area of drawdown cone conformed basically with that of land
subsidence. The centres of drawdown and land subsidence depict­
ed that land subsidence was correlated with ground water ex­
loitation. The centres were not coincided with each other.
There was a distance about 5.5km between these two centres, which showed moreover that land subsidence was related to the variation of alluvium distribution. Analyses of these two aspects are emphasized in the following paragraphs.

II. Taiyuan is located at the north end of central Shanxi basin extending in the direction of north to south. The total length of the basin is more than 100km and the maximum width in the central part is 25km. The depression is more than 1000m in depth. An alluvial fan of 29x10km$^2$ in area and 300m in thickness has been deposited at the gorge as the Fenhe river flowed from the West Mountain of Taiyuan into the basin. Taiyuan city lies on the central part of the fan (Fig.3).

The length of Fenhe river up to Taiyuan city is 196km, covering an area of 7640km$^2$. The average flow for many years was 16.16m$^3$/s. The maximum flow was 1480m$^3$/s during the rainy season of July, August and September, then the river water was muddy with sandy particles up to 0.524ton/m$^3$. From December to January and February of the next year it is an arid period, the water then is very clear and being drained from ground water. Great quantity of river water seeps into underground when Fenhe river flows through the exposed Ordovician limestone stratum with an area about 1200km$^2$ of West Mountain, where the river becomes a dry valley after the rainy season. But spring water gushes from both sides of the limestone nearby the gorge (see Fig.3), then water appears again in the river bed and the
flow increases along its path. The flow reaches its maximum amount, 9m$^3$/s, at the gorge. After the construction of reservoir at the upstream of Taiyuan in 1964, the flow reduced to 5.5m$^3$/s. In 1965 the first water supply yielding 2.5m$^3$/s was built in the river bed of Fenhe river gorge for which shallow wells were drilled in gravel layer (not more than 40m in thickness) of the river bed. The flow reduced further to 3m$^3$/s, which fully denoted that the flow of clear water in Fenhe river came from the drainage of karst water.

In 1972 the second water supply was built, that means additional 40 deep wells were drilled about 10km downstream of the first water supply, or at the head of Taiyuan alluvial fan which was more than 150m in thickness. Its capacity was 2.6m$^3$/s. The yield was more than 5000m$^3$/day by single artesian well. The author had taken part in the verification of ground water resources before the pipeline was laid. According both to profiles (Fig.4,5) provided by hydrogeological investigation and the existence and decrease of clear water flow of Fenhe river, the relation between the recharge of ground water in alluvial fan and the drainage of West Mountain karst water was recommended as expressed in Fig.6. After being recharged with the river seepage the West Mountain karst water drained downstream along the valley and was blocked by the gravel layer of alluvial fan (its permeability was far less than that of limestone at the gorge). Most of the karst water drained upwards towards the valley and became clear water flow of Fenhe river, with only a small portion of it flowed further on to recharge the alluvial fan. This inference could be drawn from three facts as follows:

(1) After the accomplishment of the first water supply, an observation hole No.K of 250m in depth was drilled 206m into the limestone. The water level in limestone raised
with the increase of depth (Table 1). That showed the karst water had an upward component of flow velocity.

(2) As the water table and flow were steady, cone of drawdown did not expand and its influenced extent was not more than 500m in the first water supply, which indicated that the recharge source was nearby and the amount of recharged water was greater than the yield of 2.5m$^3$/s.

(3) Pumping tests of nine wells conducted during the exploration of the second water supply indicated that the total flow was only 34000m$^3$/day. But the water level was unsteady throughout the pumping period of one month and the cone of drawdown extended over 10km.

Table 1

<table>
<thead>
<tr>
<th>borehole depth (m)</th>
<th>26</th>
<th>48</th>
<th>79</th>
<th>120</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>water level (m)</td>
<td>-0.56</td>
<td>+0.8</td>
<td>+1.03</td>
<td>+1.15</td>
<td>+1.26</td>
</tr>
</tbody>
</table>

The amount of recharge to alluvial fan from karst water was determined as less than 34000m$^3$/day. As the recharge to alluvial fan was so small that the author proposed to call off the construction of the second water supply but to enlarge the first water supply instead. However, this proposal was not accepted at that time. Now, after ten years, not only the second water supply was put in operation but also thousand wells were drilled in the urban district and south
suburbs. The incessant over extraction caused water table drawdown of the whole alluvial fan.

III.

In order to confirm that the lowering of water level was the result of extraction over recharge the following mathematical model was recommended: assume the aquifer was in the shape of rectangle and its length and width were L and B respectively. The aquifers were simplified as a single layer, homogeneous and isotropic.

The recharge end at the north was maintained constant flow and the drainage end at the south was kept constant water level. Both laterals of aquifer were impervious. There were \( N \) wells in the aquifer to be pumped with constant flow \( Q \), then problem (I) is given as follows:

\[
\begin{align*}
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} &= \frac{p}{T} \frac{\partial h}{\partial t} \\
\text{at } t = 0; \quad h &= h_0 x/L \\
\text{at } x = 0; \quad h &= 0 \\
\text{at } x = L; \quad \frac{\partial h}{\partial x} &= -h/L \\
\text{at } y = 0; \quad \frac{\partial h}{\partial y} &= 0 \\
\text{at } y = B; \quad \frac{\partial h}{\partial y} &= 0 \\
\text{at } x = x_i, \quad y = y_i; \quad \int_0^r \frac{\partial h}{\partial r} \, dr &= \frac{Q_i}{2\pi n_i} \\
\text{where } h \text{ is water head} \\
L \text{ is the length of aquifer} \\
B \text{ is the width of aquifer} \\
T \text{ is transmissivity} \\
S \text{ is storage coefficient} \\
r \text{ is radius of well} \\
\text{suffix } i \text{ is the serial number of well} \\
x, y \text{ are coordinates} \\
t \text{ is time}
\end{align*}
\]

Solution of problem (I) is:

\[
h = h_0 x/L - \frac{L}{TwB} \sum_{n=1}^{\infty} \sum_{m=1}^{(2n-1)} \left( \frac{2}{L} \cos \frac{(2n-1)\pi (x-x_0)}{L} \cdot \cos \frac{(2n-1)\pi (y-y_0)}{2} \right) \left( 1 - e^{-\frac{T}{2}(\frac{2n-1)^2\pi^2}{L^2}} \right) \\
+ \frac{1}{L} \sum_{n=1}^{\infty} \sum_{m=1}^{(2n-1)} \frac{1}{\cos \frac{(2n-1)\pi (y-y_0)}{2} \cdot \cos \frac{(2n-1)\pi (x-x_0)}{L} \cdot \cos \frac{m\pi x}{B} } \\
\frac{Q_i}{2\pi n_i} \\
\text{The flow at the drainage surface } (x=0) \text{ of aquifer is} \\
Q = T \int_0^B \frac{\partial h}{\partial x} \mid_{x=0} \, dy \\
\text{Differentiate } h \text{ of (I) for } x \text{ and substitute it into the above equation, then integrate, get} \\
Q = T B (h_0/L) - \sum_{i=1}^{N} \frac{Q_i}{n_i} 
\]
The first item on the right side of (3) represents quantity of recharge $Q_r$. This equation illustrates that when $N_f = 0$, or when no well is pumping, the flow of drainage of ground water

$$Q_d = TB(h, \alpha) = Q_r$$  \hspace{1cm} (4)

When the discharge of pumping equals to that of recharge, or

$$TB(h, \alpha) = \frac{N_p}{(2\pi) \alpha} Q_d$$  \hspace{1cm} (5)

From equations (3), (4), (5) it is known that the reason why the water level of aquifer keeps steady during extraction is because the flow of extraction is less than that of recharge. It also proves that the flow of extraction is out of the flow of recharge and takes place of the flow of natural drainage. When the flow of extraction equals to that of recharge there will be no more natural drainage but the cone of drawdown will still be steady. If the flow of extraction increases, then unsteady drawdown would be induced as a result of the consumption of ground water storage. The unsteady drawdown’s may be counted from the steady cone of $h(x, y)$, the expression of which could be derived from (1) by letting $t \rightarrow \infty$. Then the problem of unsteady stage due to over recharge development should be modified as

$$\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} = \frac{q}{t} \frac{\partial S}{\partial t}$$

$$t = 0 \quad S = 0$$

$$x = 0 \quad \frac{\partial S}{\partial x} = 0$$

$$x = L \quad \frac{\partial S}{\partial x} = 0$$

$$y = 0 \quad \frac{\partial S}{\partial y} = 0$$

$$y = \beta \quad \frac{\partial S}{\partial y} = 0$$

$$x = x_j \quad y = y_j \quad \int_{x_j}^{x} \frac{\partial S}{\partial y} \mid_{y = y_i} \: dq = \frac{Q_i}{2 \pi \alpha} \quad i = 1, 2, 3, \ldots, N_p$$

where $j$ is the serial number of well increased during over recharged extraction

$\sum_{j=1}^{N_p} Q_j$ is the flow of those wells

$N_p$ is the total number of wells

The solution of problem (II) is

$$S = \left( \sum_{j=1}^{N_p} Q_j \right) / 2\pi \alpha$$

where

$$S_j(x, y, N_p, x_j, y_j, Q_j)$$

$$= \frac{2}{2\pi \alpha} \sum_{j=1}^{N_p} Q_j \left\{ \frac{1}{2\pi} \left( 1 - e^{-\alpha \pi x_j^2 / \alpha} \right) \cos \frac{\alpha y_j}{\alpha} \right\}$$

As mentioned above the drawdown’s in (6) is counted from the surface of steady cone. If “$s$” is counted from natural water level, then the drawdown of steady cone “$s$” should be added, thus
If drawdown of a certain point (such as a certain observation well) in aquifer is studied, then \((x,y)\) is a fixed value, so the above equation could be expressed by a simple linear equation as

\[ S = a + bt \quad (9) \]

By (9) it is obvious that when the extraction is exceeded, any point in aquifer will have drawdown in linear relation with time and the slope of \(s-t\) line is in direct proportion to the yield of over recharge.

Fig. 8 shows a set of \(s-t\) curves drawn with observation data of ten years (1971-1981) of three observation wells, wells \(k_1, k_2, k_3\) were located at the second supply and another observation well \(k_1\) was in limestone at the first water supply situated in the valley of Fenhe river but out of the head of fan. From these curves we learned that the water level declined in observation wells \(k_2\) and \(k_3\) while the water level in well \(k_1\) kept steady. That shows the alluvial fan was under the condition of over recharged yield. For example, in well \(k_2\), the slope \(b\) of curve \(s-t\) could be classified into three stages and over recharged yield could be calculated approximately according to

\[ b = \left( \sum_{j=1}^{N} Q_j \right) / \Delta S \]

that is

- 1971~1974; \( b=1.5 \text{ m/3 years} = 0.5 \text{ m/year} \); \( Q_{\text{over}} = 12300 \text{ T/day} \)
- 1974~1979; \( b=1.07 \text{ m/3 years} = 1.78 \text{ m/year} \); \( Q_{\text{over}} = 14200 \text{ T/day} \)
- 1979~1981; \( b=4.3 \text{ m/3 years} = 4.3 \text{ m/year} \); \( Q_{\text{over}} = 34100 \text{ T/day} \)

Hence it is proved that the direct cause of cone of drawdown in alluvial fan is over recharged yield.

**IV.**

The coincidence of area of the extent of land subsidence and cone of depression reflected the relation of cause and effect between the two. The fact that the centre of drawdown was not coincide with the centre of land subsidence was related to the formation of alluvial fan. The head of alluvial fan was formed by sandy gravel layer as shown in Fig. 1. At the centre and tail of the fan the sand layer became finer and thinner and the interlayer of cohesive soil increased and became thicker. As counted from the geological log of 196m deep well in the centre of land subsidence the cohesive layer was 64%. The compressibility of cohesive soil was greater than that of sandy gravel layer. But further to the south, though the proportion of cohesive soil would become greater the drawdown would become less and so the land subsidence would decrease gradually.

With the consolidation equation expressed by compressibility strain proposed by a Japanese, Oaku Mura (1967)

\[ \frac{\partial^2 \varepsilon}{\partial z^2} = - \frac{1}{c_v} \frac{\partial^2 \varepsilon}{\partial t} \quad (10) \]

where \( \varepsilon \) is the strain of cohesive soil layer produced by load increase caused by pressure decline,

- \( c \) is consolidation coefficient of soil; \( c_v = k/n_v \gamma_w \)

The permeability coefficient of cohesive soil layer in allu-
vial fan $k=1-10^3$ m/day, so the value of $c_\nu$ should generally be $10^4\text{cm}^2/\text{yr}$.

Hence the initial and boundary condition of (10) could be written as:

$$
\begin{align*}
  t &= 0 ; \quad \varepsilon = 0 \\
  z &= 0 ; \quad \varepsilon = m_\nu pt/t \\
  z &= -M ; \quad \varepsilon = m_\nu pt/t,
\end{align*}
$$

where $m_\nu$ is the compressibility coefficient of soil layer

$p$ is pressure stress

$t$ is time interval between the start of loading and the accomplishment of calculation. Hence the range of $t$ in the formula is $0 < t \leq t_f$

The solution of problem (10) and (11) is

$$
\varepsilon = \frac{m_\nu p}{T_i} \left[ T - \frac{\varepsilon}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \left( 1 - e^{-((2n-1)^2\pi^2 T_f)/T} \right) \frac{(2n-1)\pi^2}{M} \right],
$$

where $T = 4C_\nu t/M^2$, $T_i = 4C_\nu t_i/M^2$

Compressible value of cohesive soil layer

$$
S = \int_0^M \varepsilon dz.
$$

Substitute (12) into the above equation and integrate, then

$$
S = \frac{m_\nu p}{T_i} \left[ T - \left( \frac{\varepsilon}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} e^{-((2n-1)^2\pi^2 T_f)/T} \right) \right],
$$

Finally the degree of consolidation of cohesive soil layer could be derived,

$$
U = \frac{S}{m_\nu p}.
$$

The denominator of the above equation represents the final amount of subsidence.

Up to now, Taiyuan has not studied land subsidence systematically. There was neither case history of variation of subsidence with time nor essential information necessary for research work, such as the stratum structure of alluvial fan. Therefore it was difficult to check the amount of subsidence with (14). In order to confirm the cause of subsidence from the geological log of borehole at the centre of subsidence, we selected arbitrarily the 14th layer of clayey silt, 4.94m thick and study its consolidation coefficient and degree of consolidation. Hence we must determine $m_\nu$ first. It is assumed that the maximum total amount of subsidence 1.232m was the result of compressing cohesive soil layers with the compressibility of sand layer being neglected when the drawdown was 59.3m (5.93kg/cm$^2$), then the average compressibility coefficient was

$$
m_\nu = \frac{1.232}{1.235 \times 59.3} = 0.0018 \text{cm}^2/\text{kg},
$$

which was used for the 14th clayey silt instead of its real value.

Take the $c_\nu$ value from such a wide range as $c_\nu = 10^3 - 10^5 \text{cm}^2/\text{yr}$
and the results of corresponding degree of consolidation $U$ calculated by (14) and (15) are shown in Fig.9.

The order of quantity of both the degree of consolidation $U$ and the consolidation coefficient $c_{fr}$ was so reasonable that we may believe the maximum amount of subsidence was caused by drawdown.

Conclusions
It is proved by this paper that land subsidence may occur in alluvial fan of plateau basin under the condition of extensive drawdown caused by intensive development of groundwater, just like the case happened in silt layer along the seashore. Talyuan is a proper example from which the following instructions could be drawn.

The ground water storage accumulated during the prolonged geological age in large alluvial fan should not be extracted at random only based upon its superficial phenomena, such as the high water head and the artesian wells. If extraction were really necessary the routine of recharge, flow, drainage should be investigated in anticipation and its recharge be accurately estimated. Gain and loss should be strictly considered based on prediction of the dynamic equilibrium of ground water after extraction. For example, the loss to tourist resources and other disadvantages due to the elimination of springs and scenic spots caused by drawdown and on the other hand, the benefit to agriculture brought about by the reclamation of saline land should be compared and then conclusion could be drawn.

If extraction were determined to carry out, attention should be paid to the following:
(1) The yield should be critically controlled below the existing recharge in order to prevent unsteady drawdown.
(2) Extracting site should be in the head of alluvial fan...
where the aquifer is coarse grained, with less interlayer of cohesive soil.

(3) Do not extract from the tail of alluvial fan as the aquifer there is thin and the grains are fine. Not only the well yield is small but also large drawdown will be formed easily and compression of multi-layered cohesive soil will be induced so that land subsidence will occur. The example illustrated in this paper was an obvious one.

References
Chen Yu-sun et al., 1972,
The Second Stage of Xi Zhang, the Enlargement of Lan Cun, View on Groundwater Resources and Investigation Hereafter (1972)
Chen Yu-sun et al., 1973,
Existing Problems on Water Supply and Some Suggestions These two articles mentioned above were not published and being kept by Commanding Office of Water Supply Development Construction.
Hu Cong-gui, 1982,
Study of Land Subsidence and Water Resource of Taiyuan, Taiyuan Institute of Building Design
Oaku Mura Shin Rao,
Research of Land Subsidence in Niigata