A physically-based model of the formation of snowmelt and rainfall-runoff

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ABSTRACT This paper presents the development of a physically-based model of snowmelt and rainfall-runoff formation. The model describes the processes of snow-cover formation, snow melting, infiltration into frozen and thawed soil, surface flow and flow in the stream network. The model has been applied to the Sosna River basin (area 16 300 km$^2$). In order to describe the processes of runoff formation the basin is divided into 200 finite elements according to topography, soil type and land-use. It is assumed that some characteristics are statistically distributed within these finite elements, so that it would be possible to take into account the mesostructure of basin characteristics. A method for taking these distributions into account is proposed. The results of tests of the model are presented.

INTRODUCTION

For a number of years work has been carried out at the Water Problems Institute of the USSR Academy of Sciences aimed at developing a system of physically-based hydrological models allowing us to predict components of the water balance for river basins (primarily river runoff) on the basis of meteorological data and basin parameters.
These investigations have resulted in the construction and verification (on the basis of experimental data) of models describing the following processes:

(a) snow-cover formation and snow melting (the model describes snow accumulation, changes of its physical characteristics, heat and moisture transfer in snow, meltwater formation and its vertical movement in the snowcover);

(b) heat and moisture transfer during soil freezing (the equations of heat transfer in the "snow-soil" system and of moisture migration to the freezing front are solved);

(c) water infiltration into frozen soil (the equations of heat and moisture transfer in frozen soil are solved, taking into account phase transformations and the presence of super-cooled soil moisture);

(d) vertical moisture transfer in non-frozen soil and soil moisture evaporation (the equations describing isothermal moisture transfer in soil taking into account the water losses through the root system and water interchange effects at the soil surface, are solved).

(e) overland flow (one or two-dimensional kinematic wave equations are solved);

(f) subsurface flow (the processes of formation of subsurface runoff and subsurface flow are described);

(g) groundwater movement and the interaction of surface and groundwater on the river slope and in the river channel (two-dimensional Boussinesq equations are solved, combined with the St Venant equations);

(h) unsteady flow in river channel system (the St Venant or kinematic wave equations are used).

The range of hydrological phenomena that can be described using the above models is wide and as a rule it proves expedient to use certain modifications of the model system for the solution of particular problems. These versions allow us to simulate some of these processes, taking into account the type of initial data and the required accuracy and detail of the calculations.

Consider now the results of application of the model describing snowmelt and rainfall-runoff to the Sosna River basin (area 16 300 km²).

BASIN SCHEMATIZATION AND CALCULATION OF SURFACE RUNOFF AND CHANNEL FLOW

The basin of the Sosna (a tributary of the Don) is situated in the forest-steppe zone and consists mainly of arable lands (some 80% of the basin area is ploughed). Ravines and gorges occupy 8% of the basin area and forests 2%. The main soil types are podzolled and lixiviated chernozem and grey forest soils. The groundwater table lies mainly at a depth of 20-25 m, which allows us to neglect the interaction of surface and groundwaters. The spring flood in the Sosna depends on the antecedent meteorological conditions and varies within a wide range from 7 to 160 mm with a coefficient of variation of 0.55. Meltwater losses are mainly determined by the hydrothermal regime of the soil during the autumn-winter period, on which the formation of an impermeable layer in the soil depends. The volume
of rainfall floods in the Sosna basin is considerably less than that of the spring flood. However, once in 3-5 years their maximum discharges turn out to be comparable to those of spring floods. The volume of rain falling during snowmelt can constitute 5-10% of the volume of meltwater.

For a number of years the State Hydrological Institute has been carrying out large-scale experimental studies which have allowed us to obtain detailed information on the characteristics of the Sosna basin.

The Sosna River basin schematization was made in such a way that it could provide an adequate account of the physico-geographical, soil and topographical characteristics of the basin and at the same time minimize the number of grid points for numerical solution of the equations in order to save computer time. Since computation of surface flow required a finer spatial grid than computation of other processes, the type of surface flow model and topographical peculiarities of the basin turned out to be of primary importance in the schematization.

Overland and channel flows were calculated using a one-dimensional kinematic wave equation, solved by a finite element or integration method (Kuchment et al., 1983). The channel system was divided into reaches which served as finite elements, taking into account the configuration of the stream network and the required accuracy in numerical integration of the kinematic wave equation. Then the basin was divided into hillslopes along which one-dimensional overland flow to the corresponding finite element of the channel system could occur. On these slopes quadrangular sections with uniform slope values, roughness and soil characteristics (finite elements of overland flow) were chosen. All in all, 200 finite elements were distinguished in the basin. The channels were considered to have a rectangular cross-section with the width and roughness constant for every finite element (Fig.1).

DESCRIPTION OF SNOWCOVER FORMATION AND SNOWMELT PROCESSES

In order to describe snow-cover formation and snowmelt the following system of equations was solved:

\[
\frac{\rho_i \partial I}{\rho_w \partial t} + \frac{\partial \theta}{\partial t} = \frac{\partial K(\theta, I)}{\partial Z}
\]

\[
C_{ef} \frac{\partial T}{\partial t} = \frac{1}{\partial Z} \lambda \frac{\partial T}{\partial Z} + \rho L \frac{\partial I}{\partial t}
\]

where \(C_{ef} = \rho_i C_i (1 - p_s) + \rho_w C_w \theta\).

\(\theta\) and \(I\) are the specific volumes of liquid water and ice in the snow respectively, \(T\) is the temperature, \(P_s\) the snow porosity, \(K\) the hydraulic conductivity and \(\lambda\), the thermal conductivity. \(\rho\) denotes density, \(C\), heat capacity and \(L\), the latent heat of fusion of ice. \(Z\) is the vertical coordinate and indices \(w\) and \(i\) denote corresponding characteristics of water and ice.
At the lower boundary the heat flux into the soil was defined as:

\[ q_s = \lambda_s \frac{T_g - T_s}{Z_s} \]  

where \( \lambda_s \) is a weighted average coefficient of thermal conductivity; \( T_g \) is the temperature at the soil surface, \( T_s \) the temperature in the lower layer of the snow and \( Z_s \) the distance between the points \( T_g \) and \( T_S \). The snow water yield was considered equal to the hydraulic conductivity of its lower layer, which was calculated with the help of an empirical formula, taking into account density, water retention capacity and saturated hydraulic conductivity. The system of equations was solved using an explicit finite-difference scheme. The calculations take movement of the snow upper boundary due to melting, snow evaporation and snowfall into account, as well as changes in snow-cover depth and density caused by its compaction under gravitational forces (Kuchment et al., 1983).
INFILTRATION CALCULATIONS

In order to compute heat and moisture transfer in frozen and thawed soil, a system of equations was solved with the help of an implicit finite-difference scheme:

\[
\begin{align*}
\frac{\partial \theta}{\partial t} + \frac{\rho_i}{\rho_w} \frac{\partial I}{\partial t} &= \frac{\partial}{\partial Z} (K \frac{\partial \psi}{\partial Z} - K) \\
C_{ef} \frac{\partial T}{\partial t} &= \frac{\partial}{\partial Z} \left( \lambda \frac{\partial T}{\partial Z} \right) + \rho_w C_w (K \frac{\partial \psi}{\partial Z} - K) \frac{\partial T}{\partial Z} + \rho_L \frac{\partial I}{\partial t}
\end{align*}
\]

where \( C_{ef} = \rho_w C_w \theta + \rho_i C_i I + \rho_g C_g (1 - \rho) \).

\( \psi \) is the capillary potential of the soil moisture and the index \( g \) refers to the soil matrix.

The heat flux and snow water yield were used as upper boundary conditions. If there was no snow on the soil surface, the heat flux from the atmosphere and precipitation intensity were used as upper boundary conditions. The moisture flux, equal to the coefficient of hydraulic conductivity in the lower soil layer, and the heat flux \( q_e \) were defined at the lower soil boundary:

\[
q_e = -\lambda \frac{T_e - T_N}{Z_e}
\]

where \( T_N \) is the temperature of the lower soil layer, \( T_e \) the soil temperature at a depth of 3.2 m (this temperature is practically unchanged during the winter period), \( Z_e \) the distance between the layers at which \( T_N \) and \( T_e \) are measured.

Empirical formulae were used to make approximate calculations of hydro- and thermo-physical characteristics from the soil parameters porosity (\( P \)), volumetric density (\( \rho_B \)), maximum hydroscopicity (\( \text{MH} \)), field capacity (\( \text{FC} \)), saturated hydraulic conductivity (\( K_0 \)) and specific thermal conductivity of soil components (\( C_i \)). Thus, for a non-frozen soil the relation between soil capillary pressure and hydraulic conductivity was calculated using the equations:

\[
\psi(\theta) = \psi_{\text{MH}} \left( \frac{\text{MH}}{\theta} \right)^n
\]

\[
K(\theta) = K_0 \left( \frac{\theta}{\rho} \right)^{n+2}
\]

\[
n = \ln(\psi_{\text{MH}}/\psi_{\text{FC}})/\ln(\text{FC}/\text{MH})
\]

where \( \psi_{\text{MH}} \) and \( \psi_{\text{FC}} \) are values of \( \psi \) at the soil moisture values \( \text{MH} \) and \( \text{FC} \).

For frozen soil we have:

\[
\psi(\theta, I) = \psi_{\text{MH}} (1 + C_i I)^2 \left( \frac{\text{MH}}{\theta} \right)^m \left( \frac{\rho - I}{\rho} \right)^{m-1}
\]
\[ \kappa(\theta, I) = \kappa_{\theta} \theta^{m+2} / (1 + C_1 I)^2 \]

where \( C_1 = 8 \). For the known values of total moisture content and soil temperature \( T \), the ice content in the soil is determined as:

\[ 4.6(1 + C_1 I)^2 (P - I)^{m-1} \frac{M H}{w(p_i/p_w)} + T = 0 \]

**SUB-GRID EFFECTS**

Since computers have limited storage and small-scale geographical maps are usually not available, definition of the basin characteristics at the spatial grid points used in the model allows the background spatial variations of these characteristics only to be described. On the other hand, experience in calculating the processes of snowmelt and rainfall-runoff formation shows that the meso-structure of the relief and small-scale variations in soil characteristics on the sub-grid scale can play an important role in runoff formation. Relatively small depressions on the hillslope, not depicted on the maps, can form zones of water accumulation and cause considerable losses in the runoff. Significant losses of overland runoff can occur on narrow strips of soil with high permeability or on plots with saturated hydraulic conductivity less than that at the grid points, if such plots occur in sub-grid areas. Actual flow values can be considerably larger than computed ones. Sub-grid scale effects are of great importance in the description of snowmelt runoff formation. Disregard of relief depressions, where snow can accumulate and zones of no flow can appear, can entail significant errors both in the volume and form of the hydrograph. Such errors can also be explained by neglect of plots with different depths of freezing, mesoscale variability of hydraulic conductivity and other soil constants.

The problem of parametrization of sub-grid effects can be presented in a form of a two-stage task:

(a) selection of sub-grid processes or variations of factors which affect hydrological parameters to be calculated at the grid points;

(b) expression of sub-grid parameters in a form of characteristics assigned to grid points.

Experience in applying lumped-parameter models is useful for the solution of these problems. It allows us to take into account the sub-grid effects in the following way:

(a) the type of function describing the statistical distribution of a basin characteristic is assumed to be one and the same for all the area elements; only the mean value of the characteristic (determined on the basis of gridpoint values) changes for every element.

(b) only the distribution of hydraulic conductivities is taken into account in the computation of rainfall-runoff. (It is assumed that this distribution can be approximated by a two-parameter
A physically-based snowmelt model

(c) two-parameter \( \gamma \)-distributions of snow water equivalent and depth of freezing were used as well as definition of the areas of no flow to calculate snowmelt runoff;

(d) the Popov equation was applied to determine water losses in no-flow areas

\[
\rho_n(t) = \rho_0 \left[ 1 - \exp\left(\frac{R_B - R_M}{\rho_0}\right) \right]
\]

where \( \rho_0 \) is an empirical parameter, \( R_B \) is the equivalent depth of snow-cover water yield and \( R_M \) is the equivalent depth of meltwater absorbed by the soil.

RESULTS OF TESTS OF THE MODEL

To test the model standard hydrometeorological observations and data from field investigations were used. Computations of spring floods started on 1 March. Initial information included the following data:

(a) snow water equivalent, measured at the end of February;

(b) soil temperature and moisture in a 1 m layer, also measured at the end of February

(c) air temperature and humidity, cloudiness, wind velocity, solid and liquid phase precipitation, measured every 6 h for the whole observation period.

The basin was divided into three zones according to the type of soils. The soil-hydrological constants, adopted on the basis of agro-hydrological reference books and field experimental data for every zone, are shown in Table 1.

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>( P_B ) (g cm(^{-3}))</th>
<th>( MH )</th>
<th>FC</th>
<th>( P )</th>
<th>( K_O ) (cm s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grey forest soils</td>
<td>1.30</td>
<td>0.04</td>
<td>0.25</td>
<td>0.51</td>
<td>3 \times 10^{-3}</td>
</tr>
<tr>
<td>Lixiviated chernozems</td>
<td>1.10</td>
<td>0.10</td>
<td>0.32</td>
<td>0.54</td>
<td>5 \times 10^{-4}</td>
</tr>
<tr>
<td>Typical chernozem</td>
<td>1.10</td>
<td>0.12</td>
<td>0.34</td>
<td>0.55</td>
<td>5 \times 10^{-4}</td>
</tr>
</tbody>
</table>

The coefficient of variation of snow water equivalent for every element was calculated using the following empirical formulae:

\[
C_{vs} = [(2.5 + 0.023 S)/S]^2 \quad \text{at} \quad S < 80 \text{ mm} \tag{9}
\]

\[
C_{vs} = 0.00038 S + 0.32 \quad \text{at} \quad S > 80 \text{ mm} \tag{10}
\]

where \( S \) is snow water equivalent (mm), averaged for every element of
the area. The relation \( C_{yH} = 16/H \) (where \( H \) is the average depth of soil freezing) was used to determine the coefficient of variation of soil freezing depth. The coefficient of variation of \( K_0 \) was taken to be equal to 1.6.

For every element water losses were calculated for five or three values of snow water equivalent and depth of soil freezing, taking into account the corresponding distribution functions of these values over the area. Then the total water losses were determined for each of the elements (taking into consideration the proportion of the area corresponding to each value of the basin characteristics). The \( P_0 \) value was taken to be equal to 8 mm, the Manning roughness coefficient for the river slope \( n_s = 0.2 \text{ s/m}^{1/3} \). The time integration interval was one hour for the calculations of water losses during the period of snow melting. For the period of rainfall-runoff formation this interval was 15 minutes. Overland and channel flows were calculated at 6 h intervals.

Calculated and observed hydrographs were compared for six intermediate and one outlet site in the basin. Table 2 presents the results of comparison of total runoff and maximum discharges at the outlet point.

**TABLE 2** Runoff, precipitation and maximum discharges

<table>
<thead>
<tr>
<th>Year</th>
<th>Date of peak flood</th>
<th>Snow water equivalent for 1 March (mm)</th>
<th>Precipitation: snow (mm)</th>
<th>Precipitation: rain (mm)</th>
<th>Flood runoff: (a) (mm)</th>
<th>Flood runoff: (b) (mm)</th>
<th>Maximum discharge: (a) (m³ s⁻¹)</th>
<th>Maximum discharge: (b) (m³ s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>10 April</td>
<td>126</td>
<td>22</td>
<td>42</td>
<td>85 92</td>
<td>1790 2160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>2 April</td>
<td>187</td>
<td>23</td>
<td>17</td>
<td>48 20</td>
<td>1160 460</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>11 April</td>
<td>25</td>
<td>51</td>
<td>13</td>
<td>60 43</td>
<td>1340 1100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>5 April</td>
<td>107</td>
<td>55</td>
<td>51</td>
<td>165 147</td>
<td>4950 4200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>24 April</td>
<td>55</td>
<td>32</td>
<td>11</td>
<td>69 76</td>
<td>2400 1580</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1972</td>
<td>20 March</td>
<td>37</td>
<td>12</td>
<td>43</td>
<td>32 26</td>
<td>678 540</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>28 March</td>
<td>32</td>
<td>16</td>
<td>5</td>
<td>32 34</td>
<td>868 840</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>21 March</td>
<td>47</td>
<td>15</td>
<td>14</td>
<td>43 56</td>
<td>1650 1400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>2 April</td>
<td>100</td>
<td>17</td>
<td>19</td>
<td>14 21</td>
<td>149 290</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td>8 April</td>
<td>89</td>
<td>32</td>
<td>9</td>
<td>19 21</td>
<td>444 550</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) observed; (b) calculated.

Figure 2 shows observed and calculated water discharges for the flood of 1970. It illustrates the accuracy of the calculated hydrographs. Table 2 and Fig.2 show that on the whole the model gives a good agreement between actual and calculated water discharges, although for 1968 errors in snowmelt runoff calculations were considerable. However, it should be mentioned that these results were obtained without any calibration of the model parameters.

Repeated investigations of the sensitivity of the spring flood model parameters showed that, for the Sosna River, calibration of parameters can be reduced to determining the coefficients of
variation of snow water equivalent and the depth of soil freezing in the basin, as well as the saturated hydraulic conductivity and hillslope roughness.

In calculations of the rainfall-runoff hydrograph for the Sosna River the same values of roughness coefficients and soil constants were used as for the snowmelt runoff. The intensity of precipitation and initial soil moisture content were assigned on the basis of observation data at eight points. The time step for computation of overland flow for rainfall periods was 15 minutes and for dry
periods 2 h. Time steps for the integration of the equations of flow for the channel system were 6 h, as in the snowmelt runoff calculations. Computations of hydrographs for five floods showed that an acceptable accuracy was achieved by varying a unique calibration parameter, a coefficient of variation of saturated hydraulic conductivity, $C_{VK}$. Figure 3 shows a set of hydrographs with different $C_{VK}$ values.

![Graph showing hydrographs](image)

**FIG.3** Observed and calculated rainfall runoff hydrographs for the Sosna River at Elets (14 June to 15 July 1974) for different $C_{VK}$ values.

The results of model tests show that physically-based flow models which take into account statistical distributions of parameters in sub-grid elements are an effective instrument for describing river basin processes and allow us to make full use of the available a priori information. At the same time they provide an opportunity to include physically substantiated empirical parameters into the model. The use of such models in applied hydrology can greatly improve the reliability of hydrological calculations.

**REFERENCE**