Snowmelt simulation models in relation to space and time

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ABSTRACT In runoff models snowmelt is usually determined by a degree-day approach. The computed melt of a particular day may be erroneous, but since peaks from groundwater flow are the result of several days' melt, runoff peaks are accurately estimated. For small basins where overland flow occurs and considerable streamflow variations over the day are observed, snow processes have to be simulated through the use of more complete snow models. In this paper the resolution in time required for snowmelt computations in order to compute runoff peaks accurately and the resolution which can be obtained by different snow models are discussed. The effect of time dependence of meltwater input for different forms of runoff is quantified using analytical approaches as well as conceptual runoff models. Subsequently, different snow models are chosen for different kinds of river basins.

INTRODUCTION

Snow surface melt, percolation of meltwater through a snowpack, and melt flux to the ground surface can be described in many ways.
Depending on the degree of sophistication of the description, the melt-related processes are quantified with a different resolution in time. The resolution of the melt rate required for determining the runoff with a certain accuracy depends on how rapidly and how strongly the basin output reacts to changes in input.

We discuss first the resolution in time which can be given by different snow models. A snow model is defined as a mathematical description of melt-related processes which gives the flux of melt water at the bottom of the snowpack as output. The response of river flow to snowmelt is then considered. The time of concentration for different kinds of basins is computed to determine the period over which input in the form of snowmelt affects runoff variations. Mathematical runoff models are used to calculate runoff for snowmelt varying in time in different ways. Based on these simulations the time resolution of snowmelt models required for estimating runoff from different basins is determined.

SNOW MODELS

It is possible to differentiate three types of snow models as was done by Morris (1982), namely (a) those based purely on air temperature, degree-day methods, (b) those based on the energy budget for computing surface melt, and (c) those based on the energy budget for the snow and including percolation through the snowpack. Actually there are intermediate forms of models, for example simplified energy budget calculations or lumped descriptions of processes within the snowpack. The intention here is to determine to what extent the full description of the melt processes can be simplified while still preserving accurate peak value and accurate timing of runoff.

A physically-based mathematical model of the processes in a snowpack was first given by Colbeck (1972). The most extensive study concerning point energy and mass balance for snow covers is probably the one reported by Anderson (1976). A model including point energy balance and liquid flow within the snowpack was reported by Morris & Godfrey (1979). Morris (1982) compared this full distributed model with the point energy balance method and the degree-day method. Given correct radiation data the energy balance method was found to be superior to the degree-day method, and for computing snowmelt on a daily basis found to be as good as the full distributed model. However, the energy balance method is less accurate in periods of alternate melting and refreezing of liquid water within the snowpack.

PROCESSES WITHIN THE SNOWPACK

The processes within the snowpack that influence timing of melt flux to the bottom of the snow cover after snowmelt has been initiated at the snow surface are (a) refreezing (b) redistribution of liquid water and (c) meltwater percolation. From a kinematic approach Colbeck (1972) showed that the rate of propagation of a certain melt flux through ripe snow is proportional to the melt intensity to the 2/3 power. Meltwater travels faster through the snow cover the more intense the melt is.
As melting proceeds vertical drains, which can be observed on the snow surface, develop. The meltwater flows preferentially through these drains. The time lag from the production of surface melt to the arrival of melt water at the bottom of the snow cover is reduced. For a 0.2-0.8 m thick snow cover the time lag after vertical drains have developed is usually less than 1 h.

In initially dry snow, the propagation rate is slow and mostly determined by the time needed to increase the water content above the irreducible value. It may take more than a day of surface melt before any water leaves a moderately deep snowpack. During a cold night liquid water in the top layer of the snow cover freezes to ice. It is only after several hours of surface melt that the snowpack above the refreezing front is again saturated above the irreducible value. Once all liquid water which has refrozen during a cold night has again melted, the time before meltwater reaches the ground does not exceed more than one or a few hours. Martinec (1985) reported a 2 h retarding effect of a 95 cm deep snowpack and Bengtsson (1981) a 1 h retarding effect of a 50 cm snow cover.

In order to compute runoff with accurate timing it is important to determine how much water refreezes during cold periods. It was shown by Bengtsson (1982a) that for periods less than a day, the penetration depth of the refreezing front and the amount of meltwater needed to resaturate the snowpack to irreducible liquid content are proportional to the square root of the number of negative degree-hours after cessation of surface melt. The refreezing method was successfully tested aginst daily melt and runoff data from Luleå, Sweden, and was found capable of describing runoff from small plots with a resolution of 2-4 h. Thus, if a resolution of no better than a few hours is required, only the time lag introduced by the refreezing of liquid water needs to be considered.

SURFACE MELT

If all the terms of the energy balance of a snow surface are correctly quantified, surface melt is also determined correctly. The different terms in the point energy balance of the snow surface vary over the day. The solar radiation exhibits a very large variation over the day and is not well related to the air temperature. Only in a forest with a dense canopy is it physically sound to relate snowmelt to air temperature alone by, for example, the degree-day method (Bengtsson (1976)). In open areas the surface melt can be correlated to air temperature and solar radiation absorbed by the snowpack. Therefore, a parametric approach including air temperature and solar radiation should be capable of determining snowmelt with short time resolution. Such a formula was suggested by Bengtsson (1984).

\[ m = C T_a + (1 - \alpha) R_s / F \]  \( \text{(1)} \)

where \( C \) = a temperature index e.g. degree-day or degree-hour coefficient, \( \alpha \) = albedo of the snow, \( R_s \) = incoming solar radiation, \( T_a \) = air temperature and \( F \) = latent heat of fusion.

Results from an application of the solar radiation degree-day
method are shown in Fig. 1. It is possible to compute snowmelt with a high time resolution. Of course, the coefficient C and the albedo must be known. The value of C appropriate for equation (1) will be lower than the corresponding value for a simple temperature index model without solar radiation.

As is evident from its name, the degree-day method is not meant to be used over periods shorter than one day. Even on a daily basis, net energy input to the snow surface cannot be accurately correlated to air temperature only. The degree-day method can only give an accurate estimate of snowmelt for periods extending over some days. In order to obtain a time resolution shorter than one day the solar radiation must be accounted for.

INFILTRATION AND SUBSURFACE FLOW

In the following sections we discuss the extent to which river discharge in different kinds of river basins reacts to time dependent meltwater input. Infiltration processes, subsurface flow and overland flow are considered.

A theory for predicting the response times of unsaturated and saturated hillslopes, based on kinematic wave equations, was reported by Beven (1982). This theory was applied using parameters typical of Swedish forests on till soils. For hills with a slope of 0.1 and extending over a length exceeding 25 m, the time of concentration for snowmelt rates was computed to be at least 1-2 weeks.

The effect of day-to-day variations of snowmelt was simulated by allowing daily infiltration rates to alternate every second day between 0 and 30 mm. In Fig. 2 the computed variation in groundwater recharge and subsurface runoff from a 25 m long hillslope is shown. Although every second day there is no infiltration at all, the predicted runoff is steady, only varying between 14.6 and 15.4 mm
Snowmelt simulation methods

Intensity mm day$^{-1}$

FIG. 2 Calculated groundwater recharge and runoff from a typical Swedish forested hillslope of till soil when quasi-steady state conditions prevail with alternating infiltration of 0 and 30 mm every second day.

day$^{-1}$ over the day. Even for shallow slopes with fairly permeable soils the subsurface runoff response is the effect of infiltration over many days.

OVERLAND FLOW

During snowmelt the ground may be frozen, at least to some extent. The hydraulic conductivity of the soil is then reduced by several magnitudes. The top layer of the soil may become saturated. When there are cracks in the soil runoff can take place in subsurface routes. When overland flow occurs at the bottom of the snow cover it is a much slower process than overland flow on bare ground. The flow at the saturated base of the snowpack can be described by Darcy's law. The time of concentration or travel time from the most distant point to the bottom of a hillslope is simply (Colbeck, 1974):

$$t_c = \frac{nL}{KI}$$  \hspace{1cm} (2)

where $t_c =$ time of concentration, $L =$ length of hillslope, $I =$ hill slope, $K =$ saturated hydraulic conductivity of the snow, $n =$ porosity of the saturated basal layer of the snowpack.

Dunne et al. (1976) and Bengtsson (1981) have tested the Darcian theory. Calculated peaks were generally accurate, but there was an overestimate of the time lag between peak rates of melt and runoff. It is suggested that this discrepancy is due to channelling of water beneath the snow cover.

When modelling runoff from study plots at WREL, Luleå, Sweden, Bengtsson was able to obtain correct timing of the peaks when the effective hydraulic conductivity of the basal layer of the snowpack was chosen to be as high as 0.1-0.2 m s$^{-1}$. For an area of 50 m length with a 2% slope the time of concentration is then about 3 h and for a slope of 10% about $\frac{3}{2}$ h. Melt intensity variations clearly affect the timing of the runoff.
Subsurface flow in soil cracks is similar to overland flow. The flow velocity is independent of infiltration rate and flow depth. If the hydraulic conductivity is chosen as $10^{-3} \text{m s}^{-1}$ and the macro-porosity as 0.3, the time of concentration for 50 m length is 9 days for a slope of 2% and 2 days for a 10% slope. This means that in flat areas the subsurface runoff in soil cracks is usually the averaged result of some days of snowmelt. However, for a 0.5 km² large meadow of very shallow slope in Luleå, Bengtsson (1982b) estimated, from runoff measurements, the time of concentration to be only about 10 h. Hydrochemical measurements and infiltration studies showed that meltwater ran off in cracks just beneath the ground surface.

RIVER ROUTING

Before the stream water reaches the outfall of a drainage basin, the peaks of lateral input to the streams as overland or subsurface flow are attenuated. The time of concentration of the river system within a basin can be estimated by tracing a kinematic wave through the river system from far upstream in a small stream down to the outfall. In a basin of 20 km², the time of concentration is about 10 h or less, while the time of concentration for the river system of a large basin of more than 100 km² is one to several days.

RESPONSE OF CONCEPTUAL MODELS

Runoff from river basins is forecasted on an operational basis using mathematical models which are conceptually reasonable. An example of a very well tested model is the Swedish HBV model (Bergström, 1976). The part of the model transferring snowmelt to river flow is in principle a form of linear reservoir. The runoff response of the HBV model to meltwater input is described by reservoir time coefficients (the inverted values of the coefficients are used in the model).

For values typical of Swedish forests, the reservoir coefficients should be about 1-50 days. Some optimized reservoir coefficients used in HBV applications are shown in Table 1. The coefficients do not seem to depend on the area of the river basin but on small scale conditions. It should be noted that for large basins a simple form of river routing is performed in the HBV model, so the total basin response is not explained by the time constants given in the table.

The HBV model was applied on a hypothetical basin in order to test the sensitivity of runoff computations to variations in melt input. Parameter values typical of Swedish basins were chosen. The melt input was varied between 0 and 20 mm day⁻¹, giving a mean daily input of 10 mm day⁻¹. As long as the storage within the basin was computed to be small, the computed runoff was hardly affected by melt input variations at all. When the upper part of the conceptual reservoir started to contribute to runoff, variations in input were recognized as variations in runoff. Computed hydrographs with different melt input are shown in Fig.3.

If in a quasi-steady situation the daily input was assumed to
TABLE 1  Time reservoir coefficients optimized for use in the HBV model applied to different Scandinavian river basins. The coefficients are determined as the sum of the inverted values of the two reservoir coefficients given by Bergström (1976)

<table>
<thead>
<tr>
<th>Basin</th>
<th>Area (km²)</th>
<th>Time coefficient (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L. Tivsjön</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Nolsjön</td>
<td>18</td>
<td>4</td>
</tr>
<tr>
<td>Stabby</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Stormyra</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Solmyren</td>
<td>28</td>
<td>12</td>
</tr>
<tr>
<td>Gimbalsbyn</td>
<td>2178</td>
<td>21</td>
</tr>
<tr>
<td>Kultsjön</td>
<td>1109</td>
<td>3</td>
</tr>
<tr>
<td>Malgomaj</td>
<td>1862</td>
<td>3</td>
</tr>
<tr>
<td>Ströms Vattudal</td>
<td>3851</td>
<td>6</td>
</tr>
<tr>
<td>Filefjell</td>
<td>154</td>
<td>6</td>
</tr>
</tbody>
</table>

FIG. 3 Simulated runoff from a hypothetical basin using the HBV model with 0 or 20 mm daily snowmelt input. Alternate every second day between 0 and 20 mm, the runoff was computed to fluctuate between 8.5 and 11.5 mm day⁻¹. For the hypothetical basin that was modelled it seems that melt input must be known with a resolution better than a few days in order to model intense runoff accurately.

OBSERVATIONS OF BASIN RESPONSE

Martinec (1985) found it useful to define the time lag for snowmelt
as the shift between the daily temperature fluctuations and of the resulting runoff. For alpine basins varying in size between 2.65 and 4000 km$^2$ he found variations in time lag ranging from 4 to 12 h only. The time lag of the largest basin was sometimes observed to be 24 h. The short time lags seem to refute the long response times suggested by the previous theoretical discussion, but it should be noted that time of concentration is different from lag time. Groundwater runoff, as shown for example in a study by Bengtsson (1982b), is immediately influenced by changed infiltration rates (the lag time is short), but the magnitude of the influence is weak (the time of concentration is long).

At BRW, Bensbyn Research Watershed, in Luleå, Sweden, a large snow hydrological research programme has been carried out. Repeated diurnal melt cycles resulted in diurnal peak flows in the late afternoon when a large meadow close to the outfall was the main area contributing to runoff, and during the night when mainly forested areas contributed. Practically all meltwater from the meadow ran off as overland flow or fast subsurface flow in soil cracks within the day the melt occurred. The time of concentration for the forested area was estimated as several days, depending on the melt rate, but the time lag, as used by Martinec, was found to be less than 12 h.

SNOW MODELS IN RUNOFF COMPUTATIONS

In the sections on snowmelt it was concluded that surface melt with resolution shorter than one day can be obtained only if solar radiation is accounted for. Unless timing of runoff from a very short impermeable surface has to be computed with an hourly resolution, percolation of meltwater through the snowpack need not be described in detail. It is sufficient to account for refreezing of liquid water within the snow. Using the degree-day method on a day-to-day basis, surface melt is not computed correctly for each day, but the average melt over some days can be correctly estimated.

The time of concentration of subsurface flow is at least several days, even if the soil above the groundwater surface is initially at a high degree of saturation. Day-to-day variations of infiltration rates are hardly recognizable in the subsurface runoff variations. Overland flow, even on hillslopes of low gradient, is the result of melt over the previous hours only. When meltwater runs off in subsurface routes just below the ground, the time lag is in the order

<table>
<thead>
<tr>
<th>Type of runoff</th>
<th>River basin:</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Large</td>
<td></td>
</tr>
<tr>
<td>Overland</td>
<td>Energy balance</td>
<td>Complete model</td>
</tr>
<tr>
<td>Subsurface</td>
<td>Degree-day</td>
<td>Solar radiation</td>
</tr>
<tr>
<td>Groundwater</td>
<td>Degree-day</td>
<td>Degree-day</td>
</tr>
</tbody>
</table>

TABLE 2 Snow models to be used for different runoff conditions
of days, but day-to-day variations of infiltration rate influence runoff much more than does the occurrence of aquifer flow.

The stream and river system within a basin has a delaying and attenuating effect on peak flows. In large basins diurnal melt variations are not recognizable as river flow fluctuations even if overland flow is predominant.

The snow models which seem appropriate for application to different runoff conditions and to basins of different size are given in a synthesized form in Table 2. River basins here are rather arbitrarily described as large or small, depending on how fast input to the small streams of the river system is recognized at the outfall of the basin.

REFERENCES


