Fit of ice motion models to observations from Variegated Glacier, Alaska

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**ABSTRACT** We test how well models of ice deformation and sliding can be fit to the progressive evolution of geometry and velocity measured on Variegated Glacier during its normal phase of flow between surges. If there were no sliding motion, the observed changes require a progressive softening of the ice. Alternatively, if the creep law of ice were constant in time, a progressive increase in sliding rate is implied. Although the inferred sliding rate apparently varies directly with shear stress and inversely with estimated effective normal stress, no single quantitative relationship can be identified which can explain both the spatial pattern and time evolution.

**INTRODUCTION**

The distribution of velocity in a glacier or ice sheet is presumably determined by (a) the instantaneous geometry of the surface and bed, (b) properties of the ice, and (c) interactions at the boundaries. If the last two of these are known or by some means calculable from the first, we may hope to relate velocity to geometry alone. This is the presumption of existing ice-flow models.
models, which commonly assume that (b) and (c) are described by spatially and time independent ice creep and basal sliding laws which respond to stress calculated from (a). The stress may be calculated from the geometry in various ways: from local ice depth, cross-section shape, and slope (Nye, 1952, 1960; Waddington, 1981; Mahaffy, 1976); from more complicated methods that account for the geometry by longitudinal averaging (Bindschadler, 1982; Echelmeyer, 1983; Kamb & Echelmeyer, 1986); or by a coupled analysis of the complete profile length (Budd & Jensen, 1975; Budd, 1975; Langdon & Raymond, 1978; Hutter et al., 1981). These models differ principally in the degree that longitudinal stress gradients suppress spatial variation in the velocity. It is expected that the sliding law is influenced not only by basal shear stress but also by an effective overburden stress reduced by water pressure (Liboutry, 1979). Therefore, to complete a geometrically determined ice-flow model, it is necessary to determine the distribution of basal water pressure from the geometry.

In this paper, we are concerned with identifying a model of the above sort that fits the evolution of velocity and geometry observed on Variegated Glacier, Alaska during its normal phase of flow between surges (Bindschadler et al., 1977). The glacier surged in 1964-1965 and again in 1982-1983 (Kamb et al., 1985). The observations considered here extend over the time interval 1973 to 1981 and have been described in detail by C. Raymond & W. Harrison in a manuscript submitted elsewhere (henceforth denoted R & H). A unique opportunity is provided for testing ice-flow models because large, progressive changes in both geometry and velocity occurred.

Based on an initial analysis, R & H reported that the observed changes in velocity and geometry could be explained in terms of standard ice deformation models with a residual presumably arising from basal sliding. While the inferred sliding rate appeared to increase with increasing basal shear stress and decrease with increasing effective normal stress, no quantitative sliding law emerged from their analysis. Here we carry this analysis forward to show in more detail the quantitative aspects of the testing of various models and the possible conclusions. Our approach is similar to that of Bindschadler (1983), who compared the fit of various hypothetical sliding laws to the distribution of basal velocity estimated for summer 1973. We shall be concerned with changes that occurred over the full eight-year interval of the measurement sequence. In order to simplify the basal water flow element in our model tests, we restrict attention to velocity averaged over the winter seasons when transient inputs of water were absent.

FLOW MODELS

We consider flow models of the form:

\[ u = K_d c^n h + K_b \tau^n \]

In Equation (1), \( u \) is the center line surface velocity; \( h \) is surface normal ice depth; \( \tau \) is basal shear stress; \( N \) is effective normal stress at the bed; \( K_d \) is an ice-flow parameter controlling internal deformation; \( K_b \) is a sliding parameter; \( n, m \) and \( p \) are constant power parameters. By relating \( u \) to the mean velocity over depth and/or valley cross section (Nye, 1965), Equation (1) is straightforwardly incorporated into continuity models used for large scale ice flow modelling.

The first term on the right of Equation (1) represents the contribution from internal shearing over the depth \( h \) predicted from Glen's creep law with \( K_d = 2A/(n+1) \) (Paterson, 1981). There are several assumptions behind this standard form, but it is generally accepted as a good approximation (Raymond, 1980). Since Variegated Glacier is temperate (Bindschadler et al., 1976), it is not necessary to account for temperature-induced depth variation of flow law
parameter $A$.

The second term on the right of Equation (1) is intended to represent basal sliding over a wet bed. The form is consistent with various proposals based on experimental results of ice friction (Budd et al., 1979) and theoretical analysis of sliding over hard beds (Lliboutry, 1979; Fowler, 1986). Actual beds, in particular the bed of Variegated Glacier, may be mantled by debris (Engelhardt et al., 1978; Harrison et al., 1986). Sliding over a soft bed would depend on the rheology of the un lithi fied subglacial material (Boulton et al., 1974) and has not been analysed extensively, but a similar form should be adequate, at least for narrow stress ranges. The parameter $K_b$ is related to the structure of the bed: for example, the micro-topography of a hard bed or thickness of deformable debris. Although the form of a sliding law is not well established, that in Equation (1) appears to be a reasonable supposition with present knowledge.

The parameters of the model are $n$, $K_d$, $m$, $p$, and $K_b$. The input to the model is the geometry, for example, in the form of longitudinal profiles of bed elevation, cross-section shape (in the case of valley glaciers), and ice surface elevation. To implement the model it is necessary to calculate $h$, $\tau$, and $N$ from the geometry. To calculate $h$ is trivial. The details of how we calculated $\tau$ and $N$ for Variegated Glacier from available data are given by R & H. We summarize their work here.

To calculate $\tau$ we use a longitudinal averaging scheme designed to account for longitudinal stress gradients (Kamb & Echelmeyer, 1986). The drag from the valley sides is taken into account by a shape factor (Nye, 1965) which depends on ice surface elevation (Bindschadler, 1982).

To determine $N$, it is necessary to calculate the overburden stress and the basal water pressure. Overburden stress can be calculated straightforwardly from ice thickness. We estimate water pressure using the theory of Röthlisberger (1972) for steady state water flow through a basal tunnel. We are therefore implicitly assuming that a basal tunnel exists and that the pressure in it controls the pressure over wide areas of the bed. The discharge characteristics of Variegated Glacier are consistent with the presence of a basal tunnel in the late summer (Humphrey et al., 1986; Kamb et al., 1985), and we choose the parameters in the Röthlisberger theory to match measurements of basal pressure in boreholes (Kamb & Engelhardt, in press). During much of the summer, the behavior of Variegated Glacier is strongly affected by short time scale phenomena (Kamb & Engelhardt, in press; Raymond & Malone, 1986; Humphrey et al., 1986; Harrison et al., 1986), and conditions are often not steady state. To avoid the theoretical difficulties associated with transient water inputs (Spring & Hutter, 1981) and their effects on the motion (Iken, 1981), we restrict our analysis to winter motions. During winter hydraulic conditions are probably near steady state. However, we have no definite evidence that basal tunnels actually survive through winter.

Clearly, there are several levels of uncertain assumptions; the determination of $N$ is an especially weak link in our testing of Equation (1) against Variegated Glacier data. Similarly, it is a weak link in all ice-flow models that attempt to account for sliding and, therefore, introduce some sub-model of basal water flow.

FIT TO VARIEGATED GLACIER

The question and the data

We now ask if there are parameters $n$, $K_d$, $m$, $p$, and $K_b$ such that Equation (1) and our methods for calculating $\tau$ and $N$ correctly predict the evolution of Variegated Glacier. If so,
how well are the parameters constrained? How compelling is the particular form of Equation (1)?

We are concerned with the profiles of bed elevation and cross section shape, which define a constant bed, and with the profiles of surface velocity and elevation for each of the eight winters 1973-74 to 1980-81. The data are given in detail by R & H. Surface elevation and velocity were typically measured at 1/4 km to 1/2 km spacing over the distance from about 4 km to 16 km from the glacier head. Characteristics of the data and the relevant quantities derived from them are given in Table 1. The number of samples indicated refers to a spatial spacing of 1/4 km. Because of interpolation to establish relevant quantities at identical positions, longitudinal averaging in calculation of \( \tau \), and similar longitudinal interaction in calculating \( N \), these samples are not independent.

We test various combinations of parameters by calculating the root mean square residual \( \sigma \) to Equation (1), or allow certain parameters to be free and choose them to minimize \( \sigma \).

### Table 1 Characterization of input data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1973-1974 (53 samples)</th>
<th>all years (341 samples)</th>
<th>1980-1981 (50 samples)</th>
<th>accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min.</td>
<td>Max.</td>
<td>Mean</td>
<td>Min.</td>
</tr>
<tr>
<td>( h(m) )</td>
<td>100</td>
<td>392</td>
<td>282</td>
<td>78</td>
</tr>
<tr>
<td>( \tau (10^5 \text{ Pa}) )</td>
<td>0.63</td>
<td>1.63</td>
<td>1.23</td>
<td>0.63</td>
</tr>
<tr>
<td>( N(10^5 \text{ Pa}) )</td>
<td>8.8</td>
<td>12.1</td>
<td>10.4</td>
<td>8.5</td>
</tr>
<tr>
<td>( u(\text{m d}^{-1}) )</td>
<td>0.003</td>
<td>0.184</td>
<td>0.094</td>
<td>0.003</td>
</tr>
</tbody>
</table>

* Includes only the random error from surveying surface markers and the propagation of these errors into derived quantities. Does not include possible error in ice depth from echo sounding (c. ± 20 m) which in the course of time at any one location would introduce a systematic error.

Deformation assuming no sliding

Figure 1 illustrates the fit of the model to the data assuming that there was no sliding in any of the winters we consider (i.e., \( K_d = 0 \)). To construct Fig. 1, we found the value of \( K_d \) which minimized \( \sigma \) for each value of \( n \).

If we consider only the data from 1973-74, we find the best fit to correspond to \( n = 3 \). If we choose \( n = 3 \), as is consistent with accumulated evidence from lab and other field measurements (Hooke, 1981; Paterson, 1981), the best value of \( K_d \) is \( 1.53 \times 10^{-4} \text{ bar}^{-3} \text{d}^{-1} \). We refer to this combination of parameters as model 1 (see Table 2, line 2). The direct comparison of the velocity predicted from model 1 and measured in 1973-74 is shown in Fig. 2.

When we examine only those data from 1980-81, a value of \( n = 4 \) provides the best fit and the ice appears to be softer than in 1973-74 (i.e., \( K_d \) is larger for the same \( n \)). The softening is also illustrated in Fig. 3, which shows velocity in 1980-81 is distinctly larger than predicted by model 1.
Fit of ice motion models to observations

For comparison purposes, symbols in Fig. 1b show combinations of $n$ and $K_d$ that have been proposed from other sources. These creep laws fall in between the results from 1973-74 and 1980-81. R & H noted that either of these creep laws gives a reasonable fit to the overall data set, which is evident in lines 1 and 14 of Table 2.

If we stick to the assumption of no sliding ($K_d = 0$), then we have an apparent softening of the ice with time such that the average properties are similar to those selected from other glacier data by Paterson (1981; Table 3.3, 0°C). However, the progressive feature of the changes in Variegated Glacier indicate that we are not dealing with random errors that scatter around "true values". Either there was (a) a real progressive softening of the ice, in which case we would have to learn how to predict the changes in $K_d$; or (b) a progressive increase in sliding rate, which we would like to explain using the second term in Equation (1).
Ice softening

The following schematic for a cyclic variation in ice rheology associated with the surge cycle seems possible. During surge motion the ice is subjected to large variable longitudinal strains (Kamb et al., 1985; Raymond et al., in press). These surge induced strains and the subsequent annealing after surge termination could disrupt c-axis fabrics developed under surface parallel shearing during the previous normal phase of motion and leave the ice with a fabric unfavorable for shearing. Subsequently, during the next quiescent phase of normal motion, shearing in the basal ice would gradually redevelop a fabric favorable for shear deformation and soften the ice progressively.

Based on the no sliding assumption, the accumulated shear strain over the eight-year interval of measurements can be found roughly as follows. The shear strain rate averaged over depth is the surface displacement divided by depth. On average this was about 2 (Table 1) with local values between 1 and 3. The corresponding accumulated shear strain at the bed would be larger by a factor of \( n + 1 \) or 4, which corresponds to shear strains of order 10 near the bed. This amount of strain was ample for significant fabric development (Kamb, 1972) and acceleration of the creep rate (Duval, 1981) assuming the initial fabric was not already accommodated to shearing. Similarly, the strains were not so large that a steady state fabric would have necessarily been reached through the ice column (Hudleston, 1977) and a continuing evolution could have occurred.

Because of sensitivity of ice rheology to water content (Duval, 1977), one must consider whether there could be an interaction of water production and drainage in the ice that could have led to water accumulation at the grain scale between surges.
These concepts indicate a rheological softening is not unreasonable. We will not pursue this line of reasoning here in detail, but bring it up as a potentially viable alternative to the path followed below.

**Apparent sliding behavior**

Now we shall assume the second of the above alternatives and investigate choices of the parameters \( m, p, \) and \( K_b \). The contribution to the motion that is assigned to sliding depends crucially on the choice of creep law parameters (\( n \) and \( K_d \)). We assume sliding rate was smallest in 1973-74 and therefore choose model 1 as a preferred representation of the creep law. In order to examine the sensitivity of our conclusions about sliding, we choose a second creep law identified as model 2 with \( n = 3 \) and \( K_d = 1.13 \text{ bar}^{-3}\text{d}^{-1} \). This particular value of \( K_d \) is the maximum value that leads to a prediction of no locations of negative basal sliding rate in 1973-74. In comparison, the value of \( K_d \) in model 1 is one that produces a mean residual of 0 and predicts equal amounts of negative and positive sliding. These relationships are evident in Fig. 2. In a sense the difference in models 1 and 2 reflects the spread that is allowed by the 1973-74 data. Model 2 eliminates negative sliding which, on the time scale of many months, is unrealistic.

For each of these creep models 1 and 2, we allowed \( m, p, \) and \( K_b \) to vary to find the combination which minimized the residuals for data from 1980-81 with the results identified as

![Figure 3: Comparison of velocity for winter 1980-81 as measured and predicted by flow models. (See Table 2 for identification of models.)](image)
TABLE 2 Fit of model parameters

<table>
<thead>
<tr>
<th>n</th>
<th>10^{-4} K_d bar^{-m} d^{-1}</th>
<th>m</th>
<th>l</th>
<th>10^{p} K_b bar^{-p} m d^{-1}</th>
<th>\sigma_{73-74} (cm d^{-1})</th>
<th>\sigma_{all} (cm d^{-1})</th>
<th>\sigma_{80-81} (cm d^{-1})</th>
<th>model i.d.</th>
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<tr>
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<td>-</td>
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<td>12.4</td>
<td>22.3</td>
<td>2</td>
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<tr>
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<td>[5.1]</td>
<td>[1.0]</td>
<td>[0.20]</td>
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<td>[4.13]</td>
<td>1b</td>
</tr>
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<td>1.53</td>
<td>5.0</td>
<td>1.0</td>
<td>[0.21]</td>
<td>8.9</td>
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<td>[3.]</td>
<td>[140]</td>
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<td>[4.86]</td>
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<td>[1.4]</td>
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<td>8.0</td>
<td>[4.22]</td>
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<td>[5.]</td>
<td>[4.]</td>
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<td>[4.91]</td>
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<td>16</td>
<td>[1.81]</td>
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<td>0.</td>
<td>7.4</td>
<td>7.3</td>
<td>[4.0]</td>
<td></td>
</tr>
</tbody>
</table>

* Paterson (1981, Table 3.3)

** Glen (1955, -0.02°C)

[ ] enclose residuals that were minimized and the parameters that were free to be adjusted in the minimization.

Models 1b and 2b in lines 5 and 9 of Table 2. The pattern of residuals depending on \( m \) and \( p \) is shown in Fig. 4. Both creep laws give a minimum in the residual pattern at roughly the same values of \( m \) and \( p \). In view of the flatness of the minimum, either case is consistent with \( m = 5 \) and \( p = 1 \), which define models 1a and 2a listed in Table 2, lines 6 and 10. The difference in creep laws of model 1 and 2 is manifested in models 1a,b in comparison to 2a,b primarily as a difference in \( K_b \), which represents a play off of the partitioning between internal shearing and basal sliding. The comparison of velocities predicted by models 1a and 2a and measured in 1980-81 is shown in Fig. 3. The addition of the sliding components embodied in models 1a and 2a greatly reduces the discrepancy in comparison to predictions from models 1 and 2 without sliding. The reduction in residuals is also seen by comparison of lines 2 and 6 or lines 4 and 10 in Table 2, which show a 80% reduction in variance.
To incorporate the time progression of year-by-year glacier states, we did a similar minimization of residuals using the data from all winters. When we allowed all of $m$, $p$, and $K_b$ to vary, the best fit was found for $m = 5$ and $p = 3$ to 4, as shown in lines 8 and 12 of Table 2, which define models 1c and 2c. However, this was not identifiably better than the fit with $m = 5$ and $p = 1$ as shown in lines 7 and 11 of Table 2, which define models 1d and 2d. Thus the values of $m$ and $p$ are not inconsistent with those deduced from the 1980-81 data. Comparison of models 1d and 2d with 1a and 2a shows, however, that the values of $K_b$ are smaller than those found from the 1980-81 data considered alone. This difference is related to the trade-off of the partitioning between internal shearing and basal sliding mentioned above. Our assumptions about the creep law set this trade-off so that there is no or little sliding in the early years.

We also allowed the trade-off between internal shearing and sliding to be set by fixing $n = 3$, $m = 5$, $p = 1$ and varying both $K_d$ and $K_b$ to minimize the residuals to the data from all years. The result defines model 3 listed in line 13 of Table 2. Since $K_b$ turns out to be very small, model 3 has essentially no sliding. The creep law parameter $K_d$ in model 3 is set at a level intermediate between the best fits for 1973-74 and 1980-81. Although the fit of model 3 to the data from all years is relatively good, its predictions systematically overestimate velocity in the early years and underestimate velocity in the later years.
DISCUSSION

Based on the foregoing results it is tempting to suggest a sliding law of the form \( u_b \sim \frac{\tau^5}{N} \). The powers \( m \sim 5 \) and \( p \sim 1 \) appear to be insensitive to the major uncertainty about the value of \( K_d \). However, the following factors argue against such a strong conclusion. The minima in the residual patterns (Fig. 4) are broad. Fits of similar goodness are possible with a range of \( m \) and \( p \). Because of the non-linearity of the flow model being fit and the interdependence of sample points through data smoothing and longitudinal averaging in calculation of \( \tau \) and \( N \), we have not been able to deduce a statistically based estimate of errors in the estimation of \( m \) and \( p \) or a criterion for rejecting (or accepting) the proposed form of the model. However, an uncertainty range in \( m \) of \( \pm 2 \) to \( 3 \) and in \( p \) of \( \pm 3 \) to \( 5 \) is not unreasonable in view of the residual contour pattern. The results from model 3 also indicate that the sliding term is not successful at predicting the velocity changes from 1973-74 to 1980-81.

Figure 5 further illustrates the uncertainty of fit. The samples of \( u, \tau, N \) triplets are plotted versus \( \tau \) and \( N \) using different symbols depending on the value of the basal sliding contribution \( u_b \) estimated from measured quantities as \( u - K_d \tau^m \) assuming model 2 for the internal shearing. If there were a unique relationship between \( u_b, \tau \) and \( N \) the different symbols ought to fall along the same lines. However, this is not the case. Instead, the different symbols are scattered across a range of \( \tau \) and \( N \) values, indicating the presence of other factors influencing the basal sliding velocity. This suggests that the relationship between \( u_b, \tau \) and \( N \) is more complex than the simple power-law form implied by the model.

![Fig. 5 Scatter plot of basal sliding velocity \( u_b \) (defined by the difference between measured surface velocity and that predicted by model 2), basal shear stress \( \tau \), and basal effective normal stress \( N \). Symbols indicate \( u_b \) values on a \((\tau, N)\) plane. Solid lines indicate contours of \( u_b \) predicted from model 2a which would theoretically separate the different symbols. Velocity units are \( \text{m d}^{-1} \).](image)

FIG. 5 Scatter plot of basal sliding velocity \( u_b \) (defined by the difference between measured surface velocity and that predicted by model 2), basal shear stress \( \tau \), and basal effective normal stress \( N \). Symbols indicate \( u_b \) values on a \((\tau, N)\) plane. Solid lines indicate contours of \( u_b \) predicted from model 2a which would theoretically separate the different symbols. Velocity units are \( \text{m d}^{-1} \).
to clump into patterns. There is no obvious clumping, and, indeed, one has the impression of total disorganization. However, when contours of constant $u_b$ are predicted from $K_b \tau^n / N^p$ using model 2a, one sees there is some pattern in the samples with relatively high velocity and, consequently, some basis for the fit. The appearance of randomness arises largely from the low sliding velocity samples which are spread throughout the diagram. This feature arises partly from our assumption of low sliding velocity in 1973-74 even though there were locations of quite high $\tau$ and low $N$ at that time.

Figure 5 also shows that the problems of fitting of Equation (1) to the data do not stem from an inadequate mathematical form for the sliding term. Figure 5 shows there is no strong pattern whatsoever, and there is little basis for picking some other mathematical form as preferable.

Incompatibility between the spatial and temporal patterns is illustrated in Fig. 6, which shows variation of quantities with time at two representative locations. The closeness of fit of model 2a to observations at these two locations is fairly typical of other locations, although there are zones of better fit (Km 7 to 12) or worse fit (up glacier from Km 6.5). (‘Km’ denotes distance in km from the head of the glacier.) It is evident in both cases that the change in velocity between 1973-74 and 1980-81 is underpredicted. This underprediction occurs at almost all locations. In other words, when the models are tuned to fit the spatial pattern at one time, the implied sensitivity of $u$ to changes in $\tau$ and $N$ is inadequate to explain the time changes.

In agreement with R & H we feel that the strongest conclusion we can realistically draw from the analysis presented here is qualitative, namely that sliding velocity increases with $\tau$ and decreases with $N$. One possible explanation for the lack of a clearly defined sliding law is that $K_b$ in Equation (1), or the corresponding parameter in other formulations, varies with position, which is in fact not unreasonable in view of possible differences in basal structure along the glacier length. If this is true, we must be additionally suspicious of the values of $m$ and $p$ found above, since they arose primarily out of the spatial pattern of velocity (e.g., the fit to data from 1980-81). A second possibility is that the estimation of effective normal stress $N$ from the Röthlisberger (1972) theory gives a false picture of how $N$ varies in space and time. Resolution of these issues apparently requires extensive sampling of the basal environment.

**CONCLUSIONS**

We may summarize our conclusions about the progressive year-to-year changes in winter velocity of Variegated Glacier as follows:

1. The ice creep is consistent with Glen's creep law with power $n$ of 3 to 4.
2. The velocity changes are inconsistent with shear deformation alone without sliding, assuming a constant, time-independent ice creep law.
3. If there were no sliding during any of the winter intervals, the anomalous velocity changes would require a progressive softening of the ice with time.
4. If the anomalous velocity changes are assumed to arise entirely from sliding, the sliding rate increases with basal shear stress and decreases with a theoretical effective normal stress.
5. The implied sliding velocities cannot be explained by a spatially independent sliding law.
FIG. 6 Evolution of velocity predicted from models 2 and 2a (curves) in comparison to measured winter velocity (heavy bars) at two selected locations identified by distance $x_p$ from the head of the glacier. Curves labeled R and H represent a creep law with $K_d$ chosen to give exactly zero sliding in 1973-74 as explained by R & H.
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