Evaluation of hydrogeological parameters in heterogeneous porous media

P. LACHASSAGNE, E. LEDOUX
Centre d'Informatique Géologique, Paris School of Mines, 35 rue Saint Honoré, 77305 Fontainebleau, France

G. de MARSLY
Professor at the University Paris VI and at the Paris School of Mines, Laboratoire de Géologie Appliquée, 4 pl. Jussieu, 75252 Paris Cedex 05, France

Abstract For most hydrogeological problems, and particularly in heterogeneous porous media, the global permeability (or transmissivity) of the aquifers is the most important parameter to be determined for long-term predictions. In practice, the most common methods used by hydrogeologists to determine these parameters consist in radial pumping or injection tests. Two approaches can thus be considered: the first consists of performing numerous short duration tests and then to calculate from these local values a global estimation of the transmissivity; the second consists of performing long-term pumping tests where the value of the effective transmissivity is obtained directly by classical methods of interpretation. The aim of this paper is to compare, by means of numerical simulations, the results of the two approaches for the estimation of the global transmissivity. Results appear to be identical as long as a few rules of interpretation are satisfied.

Estimation des paramètres hydrogéologiques au sein de milieux poreux hétérogènes

Résumé Dans la plupart des problèmes hydrogéologiques, et en particulier en ce qui concerne les milieux poreux hétérogènes, la perméabilité (ou transmissivité) globale est l'un des paramètres les plus importants à déterminer dans l'optique de la prévision de leur comportement à longue échéance. En pratique, les essais de pompage ou d'injection radiaux sont les techniques le plus couramment employées par les hydrogéologues pour déterminer ces paramètres. On distingue alors principalement deux approches différentes: la première consiste à réaliser de nombreux essais de courte durée répartis le plus régulièrement possible dans l'espace, puis à calculer à partir des valeurs locales qui en sont déduites une estimation globale de la transmissivité; la deuxième est axée sur la réalisation d'essais de pompage de longue durée pour lesquels une valeur de transmissivité globale est obtenue.
directement à l'aide des méthodes classiques d'interprétation. Le but du travail présenté ici est de comparer, au moyen de simulations numériques, les estimations de transmissivité globale obtenues à l'aide de ces deux approches au sein de modèles représentant des milieux poreux hétérogènes. Il apparaît que les résultats de ces deux méthodes sont identiques si l'on respecte quelques règles simples pour le dimensionnement et l'interprétation des essais.

INTRODUCTION

For most hydrogeological problems, and particularly in heterogeneous porous media, the permeability or transmissivity of the aquifers is the most important parameter to be determined. To do so, the hydrologist can measure a variety of quantities in the field but he can never directly measure a permeability: he has to rely on a "model" to interpret the measurements and calculate a permeability or transmissivity. A very large number of "models" is available, with varying underlying assumptions, for instance:

(a) rock sampling and laboratory measurements;
(b) flow tests (slug tests, injection or pumping tests);
(c) analysis of natural variations of water levels;
(d) inverse modelling;
(e) correlation with another parameter (e.g. electrical transverse resistance, or thickness, or grain size); and
(f) use of environmental or artificial tracers.

Each of these methods determines a parameter value which is representative of a certain volume of rock.

Generally, the most common approach used by hydrogeologists consists of radial pumping tests. These can be classified into two types as a function of their duration and, therefore, as a function of the volume of the aquifer affected during the test.

The first type consists of pumping tests of short duration. These give evaluations of local permeability (or transmissivity). These local values are highly variable in heterogeneous media and can be combined either by means of geostatistical methods which give a regionalized representation of the data and of their variability, or by averaging which leads to an estimation of the values of the hydrogeological effective parameters of the medium.

The second type consists of classical long duration pumping tests where a value of effective permeability is obtained directly by classical methods of interpretation. One can then try to determine which type of average of the local values is estimated by such methods. Attention here has been particularly focussed on this second approach.

Presented first are the modifications realized in a two-dimensional finite difference simulation code in order to make it suitable to probabilistic theory which says that, in a "macroscopically uniform" flow, the effective transmissivity of a medium is equal to the geometric mean of the local transmissivities. Based on this model, simulations of radial pumping tests were then performed for steady and transient response in heterogeneous randomly
Hydrological parameters in heterogeneous porous media. The consequences for the correct determination of the effective transmissivity of the aquifer will be given.

**AVERAGING PERMEABILITIES**

One may first recall that the averaging of permeabilities is not necessarily a straightforward operation: it is well known, for instance, that when different parallel layers of a porous medium with permeability $K_i$ and thickness $e_i$ are assembled, the average permeability $K$ of the medium is the harmonic mean ($\Sigma e_i/K = \Sigma (e_i/K_i)$) if the flow is orthogonal to the layers, or the arithmetic mean ($\Sigma e_iK = \Sigma (e_iK_i)$) if the flow is parallel to the layers. In a more general way, if $N$ measurements of permeability $K_i$ are available in an aquifer, we will call the harmonic mean $K_h = N/\Sigma (1/K_i)$, the arithmetic mean $K_a = (1/N) \Sigma K_i$, and the geometric mean $K_g = \exp (1/N \Sigma \ln K_i)$, i.e. the arithmetic mean taken in the log space. One always has the following inequality between the three means: $K_h < K_g < K_a$. It is also possible to define a unique averaging formula, as, for example, in Journel et al. (1986), the power averaging formula:

$$K^m = (1/N) \Sigma K_i^m$$

with $m = -1$ for the harmonic average, $m = +1$ for the arithmetic average, and $m \to 0$ for the geometric average (at the limit), but all other possible values of $m$ will also produce an average.

The issue to be discussed is the type of average which is obtained from different tests integrating the permeability over a large volume of rock, and also the type of average which is needed to predict flow or transport.

It is, however, necessary to first discuss the type of distribution that is generally observed in the field. Several authors, e.g. Krumbein (1936), Law (1944), Walton & Neil (1963), Farengolts & Kolyada (1969), Ilyin et al. (1971), Jetel (1974), Freeze (1975), Rousselot (1976), Neuman (1982), have observed that permeabilities are generally lognormally distributed in a given formation; this means that if the logarithm of the permeability is taken, then this magnitude has, in general, a Gaussian frequency if a large number of measurements is available. If we now consider that the permeability in space is a random function, i.e. that in two different locations, the permeabilities will in general be different, and that they are independent of each other, then some theoretical results can help to determine the type of average needed.

Matheron (1967), for instance, has shown that:

(a) if the flow is "macroscopically uniform" (parallel flow lines on the average), whatever the number of dimensions of the space, the distribution of the permeability and its spatial correlation, the average permeability always ranges between the harmonic mean and the arithmetic mean of the local permeabilities;

(b) if in addition, the probability density function of the permeability is lognormal and unvarying by rotation, in two dimensions, the average permeability is exactly equal to the geometric mean;
(c) it is not possible to define an average permeability in the steady state for radial flow.

Similarly, Gelhar (1976), Bakr et al. (1978), Gutjahr et al. (1978), have given linearized approximations of the average permeability in uniform flow for a lognormal distribution function of the permeability:

- in one dimension: \( K = K_g (1 - \alpha_2^2/2) \),
- in two dimensions: \( K = K_g \),
- in three dimensions: \( K = K_g (1 + \alpha_2^2/6) \),

where \( K_g \) is the geometric mean and \( \alpha_2 \) the variance of ln \( K \).

A last result concerns the dependence or independence of the different permeability measurements in space. If a large number of permeability measurements is available from different locations in space in the same formation, it is easy to observe that the permeability measurements are not independent: close to a location where a high permeability has been measured, the probability is greater that one will measure a high value again, and vice versa. However, this dependence is a function of the distance between the measurement points; after a certain distance, the measurements appear, in general, to become independent. The theory of geostatistics, developed by Matheron (1971), is a means of estimating regionalized variables from local measurements and of quantifying this spatial dependence.

It can be shown that the above results on the averaging of permeabilities still apply even if they are spatially correlated: the geometric mean is the correct average for two-dimensional uniform flow, and has to be weighted as a function of \( \alpha_2 \) for other dimensions, to the first order.

METHODS AVAILABLE FOR PERMEABILITY MEASUREMENTS

First briefly reviewed here will be the most common methods currently used to estimate the permeability or transmissivity of a formation in the field; also discussed will be the size of the domain investigated by the tests, and the type of average obtained by the test, if the tested domain is heterogeneous.

Rock sampling and laboratory measurements

If "undisturbed" rock samples or cores are taken from a formation, it is usually assumed that the size of the sample is of the order of the Representative Elementary Volume (Bear, 1972; de Marsily, 1986), and that the permeability measured in the laboratory on the sample is representative of a "local" permeability in situ. If the sample is layered, its anisotropy can be determined by performing the test in two orthogonal directions in order to obtain the horizontal and vertical permeabilities.

Now suppose that \( N \) samples have been taken from a single well over the vertical in the formation; can one predict the average permeability (or estimate the transmissivity) of this formation? First assume that a continuous recording of the permeability over the well is available (a fully cored reservoir where all sections of the core have been tested in the laboratory, for
instance). Then the local average horizontal permeability (or transmissivity) around the well will be given by the arithmetic mean of the local horizontal measurements. This is because, in the immediate vicinity of the well, the flow is indeed orthogonal to the direction of the well, and the various layers act strictly in parallel. Similarly, the average vertical local permeability (e.g. for the estimation of upconing) will be given by the harmonic mean of the vertical permeabilities. But a more general estimate of the average permeability within the whole formation may be wanted, and not just at the well bore. If the formation is strictly layered with uniform and continuous layering, the arithmetic and harmonic means will indeed be respectively the correct averages; but if this is not the case, and the formation is made of lenses of rocks of different permeabilities, with a complex geometry and interweaving, the arithmetic mean will overestimate the average horizontal permeability, since the lenses are not strictly in parallel, but sometimes the flow has to go from one lens to the next, in series. Similarly, the harmonic mean will underestimate the average vertical permeability. For uniform flow, one may then apply the theoretical results and estimate the average permeability using the geometric mean, weighted by the appropriate function of the variance of In $K$, depending on the dimensionality of the flow of interest (in general, the flow is three dimensional). This should be done separately both for the horizontal and the vertical permeabilities.

Now if the sampling is not continuous, an estimation technique taking into account the spatial correlation of the permeability will be needed. Indeed, a sample must be considered representative of an area and, in some way, the averaging must be weighted by this area. This is particularly true if the sampling intervals are not uniform. One simple way to make such an estimation is to use kriging (de Marsily & Ahmed, 1987) for the mean, i.e. to look for an optimal set of weights $\lambda_i$ which will give the unbiased and optimal estimate of the mean $\bar{Z}$ of the magnitude $Z$ by $\bar{Z} = \sum \lambda_i Z_i$, $Z_i$ being the measurements. The variogram of $Z$ must first be obtained from these measurements. Now to estimate, respectively, the average horizontal and vertical permeabilities at the well bore, $Z$ will be taken as $K$ or $1/K$; kriging will thus give directly the arithmetic and harmonic means. To estimate the permeabilities within the formation, one takes $Z = \ln K$, and determines directly by kriging the geometric mean. One can then apply the weighting functions depending on the variance of ln $K$ and the dimensionality of the flow, as before (note that the threshold of the variogram is precisely the variance of $Z$). It is thus clear that the variogram of the permeability must be calculated not only for $K$, but also for $1/K$ and ln $K$. Note that kriging, using the appropriate $K$, $1/K$ or ln $K$, can also produce an estimate of $K$ along the well bore and not only the mean.

Another thing to remember about discontinuous sampling is that, in highly pervious media, where the low permeability layers are thin strata distributed in the medium and difficult to sample, the vertical permeability will generally be overestimated, whereas for impervious media, where the high permeability layers are thin strata (or fractures), the horizontal permeability will generally be underestimated, unless geophysical techniques are used to locate these anomalies.
Pumping tests

Local flow tests Slug tests or injection tests over a short packed section of an aquifer can be considered as giving a local value of the permeability over the section; if the portion of the well that is tested is long compared with its diameter, the test already gives an arithmetic average over that section. However, only the horizontal permeability will be obtained, and not the vertical one. Tests to build a special triple packer system to estimate the vertical permeability in a borehole by forcing fluid vertically from one chamber to the next inside the formation have so far proved unsuccessful (BGS, 1988). The average permeability is, indeed, the arithmetic value since the flow is forced to be orthogonal to the well bore. If the whole aquifer thickness is tested by successive slug or injection tests in successive packed sections, one can estimate the local average permeability by the arithmetic mean of the measurements; if incomplete testing has been done, kriging in the mean using $K$ for $Z$ can also be applied. If the average permeability of the formation is required, and not just the value at the well bore, kriging should be done with $\ln K$, and possibly weighted; however, it is preferable in that case to have very short test sections, since a long test section produces an arithmetic average over that section which cannot be transformed into a series of local values to be used for estimating the geometric mean.

Classical pumping tests A very interesting case is that of classical pumping tests, in fact the most common method used by hydrogeologists to evaluate the hydrogeological parameters of a studied domain.

The aim of this study was to verify if the classical methods for the interpretation of such tests both in the steady and the transient state were efficient when used for the characterization of highly heterogeneous porous media. An attempt has therefore been made to represent, with a mathematical groundwater simulation code, heterogeneous porous media on which to perform as many pumping tests as needed.

(a) Calibration of a two-dimensional finite differences model for use with probabilistic theory: The first study was to verify if the different simulation codes commonly used at the Centre d'Informatique Géologique do represent in an accurate way flow in a two-dimensional porous medium, i.e. that for a "macroscopically uniform" flow, the effective transmissivity of a medium is equal to the geometric mean of its local transmissivities (see above).

Both a finite elements and a finite differences simulation code were therefore tested (de Marsily et al., 1978). In order to calculate the effective permeability (or transmissivity) of the domain represented by the grid, reproduction of the Darcy experiment was chosen. Thus different prescribed head boundaries were assumed at the two opposite limits of the square grid studied (Fig. 1) with zero flow boundaries assumed at the other two. The square grid consisted of square elements of ten metres side.

A transmissivity value taken from a randomly generated log-normal distribution was applied to each element. This distribution is defined by its geometrical mean and its standard deviation. The effective transmissivity $T_{ef}$
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of the discrete block medium is then calculated by measuring the steady state flow rate $Q$ through the model:

$$T_{ef} = \frac{Q}{iA}$$

where $i$ is the hydraulic gradient between the two extremities of the grid and $A$ is the area of the section of the model perpendicular to the flow.

Thus both the equivalent transmissivity and an estimation of the round-off and computing errors can be inferred from the measurements of the inflow and outflow along the two opposite prescribed head boundaries of the grid.

Making the assumption that the flow is "macroscopically uniform", the calculated effective transmissivity can then be compared with the geometric mean of the local transmissivities for a set of realizations using various standard deviations. It clearly appears (Fig. 2) that the numerical models
deviate significantly from the theory proposed by Matheron for high values of the standard deviation: the finite elements code tends to overestimate the effective permeability while the finite differences model shows a tendency to underestimate it.

Thus such models cannot be used within the scope of this study because of their respective systematic bias. An attempt was therefore made to correct the bias of the finite differences code. In fact this type of model presents an explicit way of computing the interblock transmissivities which are used for the determination of the flow rate exchanged between two adjacent elements. This method consists of calculating the harmonic mean of the transmissivities of the two considered blocks. The former expression was replaced by the power averaging formula described earlier. A value of \( m \) which could minimize the bias observed was then sought. In fact, a calculation with an \( m \) value greater than \(-1\) (harmonic mean) will increase the local flow rate exchanged between each element of the grid, the global flow rate through the model, and thus its effective permeability.

\[ \text{Fig. 2 Results of the "macroscopically uniform" flow test for the finite difference and finite element models (one set of 170 realizations).} \]
transmissivity.

It appeared experimentally (Fig. 3) that an $m$ value of $-0.23$ is the more accurate to minimize the discrepancy between the effective permeability curves and the theoretical results. As in the preceding cases with the biased models, the dispersion of the calculated effective transmissivity values around the geometric mean of the local transmissivity distributions increases with their standard deviation. This phenomenon could be explained by the effect, for high values, of the standard deviation of a channelling phenomenon: the majority of flow would take place in particular pathways (Tsang, 1989).

![Graph showing finite differences model with power averaging formula](image)

**Fig. 3** Results of the "macroscopically uniform" flow test with the corrected finite difference model (85 realizations).

Further tests showed that this fitting was still valid when the direction of the flow, still being "macroscopically uniform", was modified with respect to the directions of the mesh.

The two-dimensional finite difference numerical model has therefore been corrected and now globally respects the probabilistic theory established by Matheron. Pumping tests in heterogeneous porous media will be simulated with this model.
(b) Simulation of pumping tests in heterogeneous porous media

Steady state: In steady-state conditions, the interpretation of a radial pumping test is made with the Dupuit (or Thiem) formula using the drawdown at two different distances from the axis, preferably two piezometers, or possibly the well and one piezometer.

In fact, when the steady-state flow rate $Q$ is known, measurements of the hydraulic gradient around the well allow calculation of the effective transmissivity $T_{ef}$ of the medium:

$$T_{ef} = \frac{Q \ln(r/R)}{(2\pi nAH)}$$

where $\Delta H$ is the difference in pressure head $(h - H)$ where $h$ and $H$ are measured at the distances $r$ and $R$ from the pumping well respectively.

Such pumping tests were reproduced with a grid (Fig. 4) representing a quarter of a confined heterogeneous aquifer satisfying the external boundary condition $h = H$ for $r = R$ for a pumping well situated at the centre of a circular island (the model was first verified by simulation with a heterogeneous transmissivity all over the domain studied).

![Fig. 4 Grid used for the steady state and transient pumping tests (1, 2, 3, 4 and 5: "names" of the rings).](image-url)
The heterogeneous aquifers studied were represented as for the preceding study by randomly generated transmissivity log-normal distributions.

The gradual evolution of the mean of the calculated equivalent transmissivities was first studied, i.e. the evolution of the flow rate through the model while progressively increasing the radial shape of the flow by modifying, from one set of simulations to another, the location of the internal prescribed head boundary representing the "pumping well" (Fig. 4).

Thus a gradual decrease was observed of the mean of the equivalent transmissivities from 1 to 0.83 and a correlated increase of the standard deviation from 0.019 to 0.250 when the well radius size decreased. This evolution was due to a mean drop in the produced flow rate. This phenomenon can be explained by an increase, when the radius of the well decreases, of the influence of the transmissivities of the few elements located in the immediate neighbourhood of the pumping well. In fact, a low transmissivity value situated on the pumping well element or in its immediate vicinity will greatly affect the produced flow rate while a relative greater value of transmissivity will not allow a higher flow rate to flow through the other part of the grid. The transmissivity of the pumping element seems thus to act as a limiting factor for the produced flow rate. One can, however, notice that the average transmissivity which is determined by such a test is close, even in the worst cases, to the geometric mean of the local transmissivities of the domain studied.

In the radial case, the influence on the calculated global transmissivities of the choice of the piezometers location was then studied, i.e. the place where the piezometric surface is observed. The effective transmissivity calculated with the arithmetic mean of the head computed on two concentric rings was therefore compared with the geometric mean of the local transmissivities situated between these two rings. The results for two lognormal distributions whose global transmissivities are respectively much lower and higher than their geometrical mean are presented on Fig. 5.

It appears that, first, there is no strong relation between the geometric mean of the transmissivities of the neighbouring observation wells and the effective transmissivity calculated with these two piezometers, and second, the greater the distance between the pumping well and the piezometers, the more accurate the evaluation of the effective transmissivity. In particular, if the drawdown at the well is considered, the transmissivity obtained is only representative of the value in the local area surrounding the well.

Thus, it seems that the best way to operate when performing a steady-state radial pumping test is to observe the piezometric surface at two locations which must be as far as possible from the pumping well.

**Transient state:** In transient state conditions, the interpretation of a radial pumping test is commonly done with the Theis identification method or with Jacob's logarithmic approximation formula. Either uses the temporal evolution, at the well or at piezometers, of the drawdown induced by the pumping rate.

Such pumping tests were first reproduced with the same grid (Fig. 4) as for the radial steady state study. However, the proximity of the prescribed head boundaries limited the possibility of interpretation of long duration tests.
as well as those involving piezometers close to the external boundary.

The grid had, therefore, to be modified by adding new elements along this external limit in order to extend the model in space and thus to allow interpretations for longer times of pumping. The prescribed head boundary previously situated at 590 m distance (59 elements) from the "pumping well" element was thus shifted to twice that distance.

The period of time within which the interpretation of the test is allowed without being disturbed by the influence of the limit has been determined by tracking the evolution of the flow rate through this limit and by graphical visualization with the semi-log Jacob type plotting.

The pumping tests were then interpreted with both the Jacob and the Theis methods. The results are similar but the use of Jacob's method allows a better visualization of the different phenomena occurring during the pumping test. The semi-log plot of drawdown vs. time for the same two lognormal distributions used for the steady state experiments are presented for the pumping well and two nearby piezometers (Fig. 4) on Fig. 6.

It appears clearly that whatever the position of the piezometer used for the interpretation, the value of the transmissivity that is determined varies with time during the test as the slope of the straight lines observed varies. Each of these slopes can be considered to be representative of different portions of the aquifer where the integration over a very large area is
Fig. 6 Transient state pumping tests for the realizations which respectively overestimate and underestimate the most effective computed transmissivity.
obtained for the later lines. It may then, however, be difficult to distinguish them from boundary effects.

An emphasis can therefore be placed on the following main results:

- As observed during the steady-state tests, the earliest straight line that can be distinguished at the different observation wells furnishes a better evaluation of the global transmissivity of the aquifer when observed at a piezometer located far from the pumping well. However, because of the presence of the external boundary, the interpretation is very difficult or even impossible when not performed in the neighbourhood of the pumping well. At the production well, the interpretation of the earlier straight line gives an average transmissivity which is relatively close (although not identical) to the geometric mean in the area of this well. At the observation wells, the validity of Jacob's approximation appears for later times i.e. when the areal influence of the drawdown cone is much greater. Thus the better results obtained for the evaluation of the global transmissivity can be explained.

- For very long pumping times, the transmissivity obtained from the interpretation of the last straight line observed is very close to the actual geometric mean of the entire tested heterogeneous aquifer.

As a conclusion, it could be said that the transmissivity which is obtained from a long pumping test is representative of an average over an area which is a function of the duration of the test. Thus, when performing very long time pumping tests, the transmissivity obtained is very close to the geometric mean of the aquifer transmissivity and thus from the real effective transmissivity.

Note that Toth (1966, 1973) mentioned that in Alberta (Canada), very long pumping tests performed in the same highly heterogeneous formation ended up giving very similar values of the transmissivity if the interpretation was made using the final portion of the plot and not the earlier one which gave values very different one from the other.

CONCLUSION

This paper has tried to show that the determination of aquifer parameters in the field should be made with a clear understanding of the type of average that the instruments, method or device generate automatically (and almost secretly); this is, of course, relevant for aquifer systems which are heterogeneous, while homogeneous systems can be studied classically. Making predictions of the behaviour of aquifers often requires another type of average than the one which is produced by the method or instrument. For that purpose, geostatistics can be of a great help to estimate both these averages as well as local values at any scale of interest (e.g. the scale of the mesh of a model). It has been seen that it is important to apply geostatistics to the correct variable, e.g. $K$, $1/K$ or $\ln K$.

Among the methods which have been reviewed, it seems that the slug test or injection test over a small section of a well or piezometer when
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performed at numerous locations is suitable to determine both the type of distribution and the variogram of the local permeability values.

Long-term pumping tests can be used as a means to validate the averages which could be obtained from local measurements. It has been shown that the mean transmissivity value established from such a test is identical to the one derived by averaging results from local tests. Thus both methods seem to lead to similar results. Long-term pumping tests are indeed very costly, and do not provide sufficient information on the variability of the system unless a large number of piezometers is available.

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