Quantitative geomorphology, stream networks and instantaneous unit hydrograph

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ABSTRACT One of the most important tasks of theoretical, as well as practical hydrology, is to find the laws of runoff processes in river basins. The difficulty of this task increases when there is a lack of hydrological data as is often the case in mountainous areas. Where no hydrological data are available at all, one of the possibilities is the determination of the runoff hydrograph by means of a relationship between characteristics of quantitative geomorphology of drainage basins and channel networks and parameters of an instantaneous unit hydrograph (IUH).

BASIC CHARACTERISTICS OF QUANTITATIVE GEOMORPHOLOGY

Quantitative studies of geomorphic processes of drainage basins and channel networks were first published by Horton (1945) and Strahler (1964). They proposed stream orders (stream segment orders) for the case where the smallest fingertip tributaries are designated order 1 (Fig. 1). This system is opposite to the one which has been used in Europe (Gravelius, 1914).

The law of stream numbers due to Horton (1945) has the form

\[ N_u = R_B^{k-u} \]  

where \( N_u \) is the number of stream segments of order \( u \), \( R_B \) is the bifurcation ratio (it tends to be constant for a given basin) and \( k \) is the order of the trunk segment for the given river basin.

The law of stream lengths (Horton, 1945) is

\[ L_u = L_1 R_L^{u-1} \]  

where \( L_u \) is the mean length of segments of order \( u \), \( L_1 \) is the mean length of segments of order 1, \( R_L \) is a length ratio which tends to be constant for a given basin.

Finally, the law of stream areas (Schumm, 1956) is

\[ A_u = A_1 R_A^{u-1} \]  

where \( A_u \) is the mean area of basins of order \( u \), \( A_1 \) is the mean area of basins of order 1 and \( R_A \) is an area ratio analogous to the length ratio \( R_L \) (Fig. 2).
GEOMORPHOLOGICAL INSTANTANEOUS UNIT HYDROGRAPH (GIUH)

The first relations between geomorphological characteristics and runoff involved maximum discharge and drainage basin area, drainage density, stream order (according various authors), stream frequency and other nonhydrological characteristics (e.g. Dickens, 1869; Patton and Baker, 1976; Blyth and Rodda, 1973).

The next step was the determination of the runoff hydrograph by means of effective rainfall and the unit hydrograph (UH). Estimation methods for the UH (or IUH) were also developed for such river basins where no measured hydrological data were available (relations between parameters of UH or IUH - lag time, unit hydrograph peak and time base - by means of quantitative characteristics such as drainage basin area, slope and length of the main stream, e.g. Snyder, 1938).

Nash (1960) expressed parameters n and K in his IUH model also by means of physical characteristics of a river basin. His equation is

\[ u(t) = \frac{1}{K} \left( \frac{t}{K} \right)^{n-1} \frac{1}{\Gamma(n)} n^{-t/K} \]  

(4)

where K is storage delay time, n is the number of conceptual reservoirs and \( \Gamma \) is the Gamma function.

When the parameters of UH (IUH) are expressed by means of the

Fig. 1 Designation of stream orders
above mentioned physical characteristics of the basin and the drainage network the relations (e.g. Snyder, 1938; Nash, 1960), are valid only for the conditions for which they were determined. Therefore, methods have been sought which could overcome these shortcomings and could be applied to any ungauged basin. One of such methods uses a relation between parameters of UH (IUH) and characteristics of quantitative geomorphology of basins and drainage networks (geomorphological instantaneous unit hydrograph—GIUH).

The idea of GIUH was published first by Rodriguez-Iturbe & Valdés (1979). The equation of IUH (GIUH) was expressed by means of basic geomorphological characteristics $R_B$, $R_L$, $R_A$, the length of the trunk segment $L_n$ (Strahler’s ordering system) and the mean runoff velocity from the basin. This theory was further developed by Gupta, et al. (1980), and Żelaziński (1986).

Rosso (1984) found the following empirical relations between the Nash model (eq. 4) and the GIUH.

$$n = a = 3.29 \left( \frac{R_B}{R_A} \right)^{0.78} R_L^{0.07}$$  (5)

$$K = k = 0.70 \left[ \frac{R_A}{(R_B R_L)} \right]^{0.48} v^{-1} L$$  (6)

where $L$ is the length of the highest order stream and $v$ is the average cross-sectional streamflow velocity at the outlet during peak discharge for the given flood wave. Note that $L$, $k$ and $v$ must be measured in coherent units.

Of all the published methods, the method of Rosso (1984) is the simplest and quickest. The fact is, however, that none of the published methods has solved the problem of the important "velocity parameter" (runoff velocity from the basin) unambiguously and satisfactorily.

RESULTS OF RESEARCH WORK IN UPPER HRON RIVER BASIN

The upper Hron River basin was analysed, namely the Hron River down to Svermovo, the Rohožná River down to Michalová, the Čierny Hron River down to Čierny Balog and the Vydrovo River down to Čierny Balog (no basin exceeds 60 km²). The basic geomorphological characteristics ($R_B$, $R_L$, $R_A$ and $L$) for the Hron River are presented in Fig. 2. The first named basin is of order 4., while the three other ones are of order 5.

The parameters of the Nash model were determined from the above mentioned characteristics (eqs. 5 and 6) and also by the method of moments. Then the following IUHs for each basin were calculated:

a) average empirical IUH

b) IUH by the Nash model (method of moments)

c) GIUH by the method of Rosso, 1984.

Because the Rosso model is very sensitive to average streamflow velocity (Fig. 3), it was important to try to find some method for determining this velocity. Various values of parameter $K$ were substituted in eq. (6) and a GIUH was chosen whose maximum ordinate
Fig. 2 Law of stream numbers, lengths and areas, Hron drainage basin of order 4.
corresponded with the maximum ordinate of the average empirical IUH. In this way the values of the average streamflow velocity in the outlet cross-section during the "average" peak discharge of the given flood waves were determined for each watershed. These average velocities were compared with those from the relation between the average velocity and the elevation of water level (measured values). It was found that, for the given GIUH, it is reliable to take those average cross-sectional streamflow velocities at the outlet which correspond to the bankfull discharge.

![Graph showing the influence of change of the average stream velocity on the shape of GIUH, Čierny Balog-Čierny Hron](image)

Fig. 3 Influence of change of the average stream velocity on the shape of GIUH, Čierny Balog-Čierny Hron

Parameters of the Nash model determined by the two above mentioned methods were also compared with the aim to improve the correspondence of the derived GIUH for each basin and the average IUH by Nash model (method of moments). The result is shown in Table 1.

<table>
<thead>
<tr>
<th>Basin</th>
<th>GIUH</th>
<th>Nash (method of moments)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>K</td>
</tr>
<tr>
<td>Hron/Švermovo</td>
<td>3.92</td>
<td>0.37</td>
</tr>
<tr>
<td>Rohožná/Michalová</td>
<td>3.08</td>
<td>0.48</td>
</tr>
<tr>
<td>Č. Hron/Č. Balog</td>
<td>3.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Vydrovo/Č. Balog</td>
<td>2.76</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 1 Comparison of parameters for GIUH and Nash models
Finally, the flood hydrographs were reconstructed using all the three above mentioned methods for determining IUH and they were compared with the observed ones (example for one basin is shown in Fig. 4). From these results we can say that for the physical conditions of Czechoslovakia the GIUH model provides sufficiently accurate estimates of the reconstructed flood waves.

Fig. 4 Observed and reconstructed flood waves; a) observed, b) from empirical IUH, c) by the Nash model (method of moments), d) by the GIUH

CONCLUSIONS

Applicability of the IUH (GIUH) model for small basins in Czechoslovakia was demonstrated. Application of this model requires evaluation of the parameters $R_B$, $R_L$, $R_A$, $L$ (from maps) and a "velocity parameter" corresponding to average cross-sectional velocity at the outlet for bankfull discharge for each basin. The reconstructed flood waves have been known to be sufficiently accurate. It would be desirable to verify this model for other types of river basins in Czechoslovakia, in particular for larger ones where flood forecasting is considered important.
REFERENCES

Dickens, A. (1869) Professional papers on Indian engineering. 133, No. 2