A distributed model describing the interaction between flood hydrographs and basin parameters

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Abstract The contribution deals with the development of a detailed model, which offers the possibility to determine the connection between a given flood hydrograph and basin parameters. First the determination of a \(T_r\)-years flood hydrograph using a bivariate regionalization formula is introduced. A description is then given of how the drainage basin is structured into square grids and how the two parameters, the relative runoff coefficient and the concentration time value are calculated for each square. The main part of the paper deals with the description of the detailed model, which is based mainly on the application of a linear routing model in order to calculate the relation between a normalized mass-transport diagram and the standardized flood hydrograph.

INTRODUCTION

The solution of water management problems often requires synthetic flood hydrographs. For the determination of such hydrographs, various methods are used in applied hydrology. Some methods compute the volume of the flood...
runoff by taking the precipitation height of the respective rain event and use an appropriate standard flood hydrograph for the modelling of the distribution in time. Another possibility is the application of a flood formula to calculate the peak flood discharge and to determine the shape of the flood hydrograph in connection with a standard hydrograph. Most methods produce flood hydrographs which reflect more or less the properties of the drainage basin upstream. However it is almost never possible, to estimate those dependencies exactly enough, in order to be able to identify the part of the discharge of a subcatchment within the flood hydrograph of the whole area.

In many areas we are confronted with drastic changes of the runoff situation, caused by flood protection measures like the regulation of water courses or the erection of flood retention basins as well as other manipulations to the landscape like deforestation, die-back of woodland, a change in soil use or urbanization of rural areas. Such changes call for answers to the question about possible consequences on the runoff regime of the drainage basin. In particular the estimation of the change of design floods of a certain return period $T_r$ requires a method which allows the estimation of the runoff of subcatchments in such a way that the superposition of all calculated partial hydrographs give the flood hydrograph of the whole basin. Solving this problem means finding an answer to the question about the connection between a statistically determined flood hydrograph, described by certain characteristics, and the most significant parameters of the respective drainage basin. Therefore, this problem cannot be solved by the direct way of using a rainfall-runoff model. Moreover, a detailed model with appropriate accuracy must be developed, a model which combines the properties of the drainage basin as well as the given characteristics of the flood hydrograph in such a way that the influence of every sub-basin can be identified. Through this approach a reasonable estimation of the effect of changes in the basin on the flood runoff is made possible.

The paper deals with the description of such a detailed model, which presumes that a hydrograph, obtained from a bivariate statistical analysis of the significant characteristics of a $T_r$-year flood event, is given at the outlet of the drainage basin. Furthermore it is assumed that those regional basin characteristics, which have most influence on the runoff regime can be taken from geodata-banks or from topographic maps.

$T_r$-YEAR FLOOD HYDROGRAPHS – REGIONALIZATION METHOD

The model requires flood hydrographs of a certain return period at the outlet of the drainage area. As there are usually no discharge measurements at the respective river profile and therefore a direct statistical analysis is not possible, a brief description of a regionalization model is given for the determination of the $T_r$-year flood hydrographs. The method is based on an investigation of the correlation between flood- and basin characteristics, such as geometry, topography, river network characteristic or vegetation, and a bivariate statistical description of flood peaks and volumes.
Bivariate statistical analysis of flood peaks and volumes

The two characteristics that characterize best a flood event, are the peak $Q_D$ and the volume $V_D$ of direct runoff. If a sufficient number of observed flood hydrographs of a sufficient long observation time is available, the pairs of values can be analysed directly. By a characteristic regional hydrograph shape (standard hydrograph) and a mean initial base flow, a spectrum of hydrographs can be determined for every certain return period. The method developed for this purpose is described by Sackl (1987) and is based on the "transformed binormal distribution". To describe the hydrograph shape either 2-parameter analytical functions (Sackl, 1987) or the method described later on are used. At ungauged sites first the $T_r$-year peaks and volumes of direct flood runoff have to be estimated, which are points on the $x$- and $y$-axes of a coordinate system. These points can be connected by lines of equal "upper right probability"

$$P_{ur} = \int_{V_D1}^{\infty} \int_{Q_D1}^{\infty} g(V_D, Q_D) \, dVdQ = \text{const.} \quad (1)$$

These lines are also called design curves of certain return periods. In connection with flood analysis they are approximative ellipses (Fig. 1).

By a typical regional standard hydrograph, the pairs of values $(V_D, Q_D)$ on a design curve can be converted into hydrographs of direct runoff. After adding the initial baseflow $Q_{BO}$ as a multiple of the mean runoff $MQ$, the $T_r$-year flood hydrographs are achieved (Fig. 2).

**Return period of flood peaks**

At the streamflow gauges of the specific region, flood peaks are statistically
analysed and a regional flood formula, depending on basin characteristics, is calibrated by multiple linear regression. As characteristics, the catchment area $A_E$ in km$^2$, the length of the main river $L$ in km, a circularity index $C_b$, an index for the location of the basin centre $L_{C/L}$ (the ratio of the flow path to the basin centre to $L$) and the mean slope of the main channel $I$ (dimensionless), the stream density $S_D$ in km/km$^2$ and the proportion of forest $F$ (dimensionless) are used. In regions with a great inhomogeneity concerning soil and geology or meteorological characteristics, these have to be considered too. The following formula for the estimation of flood peaks (return period $T_r = 100$ years) shows as an example the result of a regionalization for East and West Styria, Austria (Sackl, 1988).

$$Q_{100} = 6.71 \cdot A_E^{0.542} \cdot C_b^{0.219} \cdot (L_{C/L})^{-0.33} \cdot I^{0.016} \cdot S_D^{0.236} \cdot F^{-0.169} \text{ [m}^3 \text{ s}^{-1}] \quad (2)$$

**Return period of flood volumes of direct runoff**

The following formula is used for the determination of flood volumes of direct runoff with certain return periods:

$$V_{D,r} = A_E \cdot h_{r,t} \cdot \psi_s$$

The height of precipitation $h_P$ is taken from regional "rainfall duration curves" (return period $T_r$), assuming the relevant rainfall duration $t$ as 10 times the time of concentration after Kirpich (1940). The "statistical" runoff coefficient $\psi_s$ is "recomputed" by regional statistical analysis of flood volumes at streamflow gauges. As investigations have shown, $\psi_s$ can be assumed as approximately constant for a catchment – independent of the return period – and within a range from 0.25 to 0.35.
Modelling the flood hydrograph/basin parameters interaction

Time to peak $t_p$ - flood hydrographs

The time to peak $t_p$ of an event results from

$$t_p = t_m/s \ [h]$$

with the "peak runoff time"

$$t_m = \frac{V_D}{Q_D} [h]$$

and the standard volume (the characteristic) of the used standard hydrograph $s \equiv 1.2$ to 2. These values are determined by flood data analysis (Sackl, 1988). So, for a certain return period flood hydrographs are achieved for different "peak runoff times" (Fig. 2).

MODELLING OF THE DRAINAGE BASIN

Systematic division of the basin

One aim of this detailed model is to achieve knowledge on the effects on the runoff regime, caused by different changes in the catchment area. In order to consider all these possible changes (areal, linear and point changes) and to learn more about the effects of local changes on the whole system, a detailed description of the basin is necessary. Usually the borders of the regions where changes of the runoff parameters take place are not identical with the borders of the hydrological basin. That is why a systematic division into squares of equal size $A_j$ is proposed. Using those incremental areas, it is possible, to define the sub-basins which are necessary for the investigations with sufficient accuracy (Fig. 3).

The systematic division of the catchment area offers as well a good basis for the mathematical formulation of the model. The choice of the size of the area element $\Delta A$ depends on the one hand on the problem but on the other hand has to consider the size of the whole drainage basin, the variability of the basin parameters and finally the computation time of the model. Besides, it has to be kept in mind, that various geodata banks have been installed already so these data can be utilized so long as the same grid is used.

Choice and estimation of relevant basin parameters

When setting up the model, it was determined that only such parameters should be used which have a significant influence on the runoff, and can be clearly estimated. Based on these considerations, the following two parameters were finally chosen for describing the basin properties:

(a) the relative runoff coefficient (mass-percentage factor) $\psi_j$ which is calculated for each square depending on the proportion of the wooded part. This relative coefficient has to be calibrated by linear distortion;
In this model, the runoff hydrograph for the observed profile is calculated in two steps (Bergmann & Richtig, 1988). First, an "unretended" hydrograph is estimated by superposing the discharges of the individual squares, considering their times of concentration. In the second step the retention effect of the drainage area is simulated by using a linear routing formula.

The procedure uses the following features for flood events with a return period of \( T_r \) years:

- peak of direct runoff \( Q_D \) \( [m^3 \cdot s^{-1}] \)
- volume of direct runoff \( V_D \) \( [m^3] \)
- time to peak \( t_a \) \( [s] \)

First, a standard hydrograph based on the information of a topographical map of the drainage basin is computed.

The most important parameter of the procedure is the value called "characteristic of the standard hydrograph"

\[
s = \frac{V_D}{Q_D \cdot t_a}
\]  

For the normalization of the standard hydrograph the following relations are employed:
relative time \( T = t/t^a \)
relative time to peak \( T = 1 \)
discharge ordinate \( u = Q/Q_D \)
peak-flow ordinate \( u_{\text{max}} = 1 \)

Since there is an infinite number of possible configurations of inputs ("precipitation"), the standard hydrograph is basically calculated from a uniform precipitation.

The runoff-contribution of every single square as explained above compared to the whole discharge volume is

\[ V_j = \phi_j \cdot V_D \]

where the mass-percentage factor is:

\[ \phi_j = \psi_j / \sum \psi_j \]

Using equation (6) it can be found that:

\[ V_j = \phi_j \cdot s \cdot Q_D \cdot t_a \]

from which the characteristic of the hydrograph for each square:

\[ s_j = V_j / Q_D \cdot t_a = s \cdot \phi_j \]

can be calculated, under the condition that \( \sum s_j = \sum s \cdot \phi_j = s \cdot \Sigma \phi_j = s \) is true.

Translation model

Relative time of concentration Using the topographic information, the time of concentration for each square is calculated and normalized with the maximum time of concentration \( t_{\text{max}} \) which is divided into a number of \( K \) intervals of time steps:

\[ \Delta t = t_{\text{max}} / K \]

For the times of transportation:

\[ t_k = \sum_{0}^{K} \Delta t \]

the relative times of concentration are calculated:

\[ \kappa = t_k / t_{\text{max}} \cdot K \]

and finally the relevant time of transportation is found:

\[ t_k = \kappa \cdot \Delta t \]
Diagram of transported mass At first a diagram for the mass transport (discharge volume) is computed with a method similar to the time-area method. Since the normalized "transported mass" of one single square element is $s_j$ according to equation (11), for the relative time step $k$ the sum of the masses of all squares with the same concentration time is:

$$s_k = \sum_k s_j = \sum_k s \cdot \phi_{jk} = s \sum_k \phi_{jk} = s \cdot a_k$$  \hspace{1cm} (16)

from which the complete transported mass of one time-step can be calculated according to equation (10):

$$V_k = s_k \cdot Q_D \cdot t_a = s \cdot a_k \cdot Q_D \cdot t_a$$  \hspace{1cm} (17)

With $a_k = s_k/s$ (equation (16)) the mass-transport diagram which is shown in Fig. 4 can be obtained.

Routing model

It is assumed that it is possible to calculate the standard hydrograph by applying a linear storage element to the mass-transport diagram. That means, that the maximum of the hydrograph $a$ has to lie somewhere on the diagram. Since there is nothing said yet about the time relations, the diagram can be transformed in a way, so that the areas of the diagram rectangles remain constant. For an arbitrary diagram area with the real time step $\Delta t$ and the transport-mass $V_p$ a mean discharge

$$Q_p = \frac{V_p}{\Delta t}$$  \hspace{1cm} (18)

can be calculated, where $\Delta t$ can be found using equation (12), but also using the time to peak $t_p$ and the number of time steps to the peak $p$: 

![Mass-transport diagram](image_url)
\[ \Delta t = \frac{t_p}{p} \]  

(19)

Equation (19) together with equation (18) results in:

\[ Q_p = \frac{V_p}{t_a} \cdot p \]  

(20)

After division with the normalization value \( Q_D \) (equation (7)):

\[ u_p = \frac{Q_p}{Q_D} = \frac{V_p}{Q_D} \cdot t_a \cdot p \]  

(21)

where \( u_p \) equals the ordinate of the standard hydrograph, equation (21) together with equation (11) can be transformed into:

\[ u_p = s_p \cdot p = s \cdot a_p \cdot p \]  

(22)

Since \( u_{p_{\text{max}}} \) has to be unity according to equation (7), the relation for the position of the maximum of the hydrograph can be found:

\[ a_p = 1/s \cdot p \]  

(23)

For the "unit value" of the hydrograph characteristic \( s = 1 \) follows:

\[ a_{p1} = 1/p \]  

(24)

As a result of equation (23), the graph in Fig. 5 shows a family of hyperbolas with the parameter \( s \). The maximum of the standard hydrograph with the characteristic \( s \) is situated in the point of intersection of the curve of

Fig. 5 Transformation of the normalized mass-transport diagram to the standardized mass-transport hydrograph.
equation (23) with the falling branch of the mass-transport diagram:

\[ a_i > a_p > a_{i+1} \]  

(25)

Besides the value \( p \), it is also possible now, to calculate the time interval of the hydrograph \( \Delta t \) using equation (19) as well as the normalized time interval:

\[ \Delta t = \Delta t |_a = 1/p = s \cdot a_p \]  

(26)

of the standard hydrograph.

**Routing factor** Using the linear equation:

\[ Q_{i+1} = C \cdot Q_i + (1 - C) \cdot Q_k \]  

(27)

where \( Q_i \) is the discharge value at the beginning and \( Q_{i+1} \) at the end of a time interval and where \( Q_k \) is the mean discharge value during this time interval, it is possible to estimate the routing constant \( C \) after the normalization of equation (27) with \( Q_D \):

\[ Q_{i+1}/Q_D = C \cdot Q_i/Q_D + (1 - C) \cdot Q_k/Q_D \]  

(28)

respectively together with equation (7) the general equation:

\[ u_{i+1} = C \cdot u_i + (1 - C) \cdot \bar{u}_k \]  

(29)

is achieved, where \( \bar{Q}_k \) can be found analogous to equation (17):

\[ \bar{Q}_k = s_k \cdot Q_D \cdot t_a/\Delta t \]  

(30)

or with equation (19):

\[ \bar{Q}_k = s \cdot a_k \cdot p \cdot Q_D \]  

(31)

By normalizing, equation (31) with \( Q_D \) follows in conformity with equation

\[ \bar{u}_k = s \cdot a_k \cdot p = a_k/\bar{a}_p \]  

(32)

This equation becomes equation (23) again when \( u_k = 1 \). Applying equation (32) it is now possible to transform the normalized mass-transport diagram to the standardized mass-transport hydrograph (Fig. 5).

Furthermore the general equation (29) can be used to find the routing constant \( C \) iteratively by solving equation (33) under the condition that:

\[ u_p = (1 - C) \sum_{k=1}^{p} C^{(p-k)} \bar{u}_k = 1 \]  

(33)

Once the routing constant \( C \) is computed, the ordinates \( u \) of the standard-hydrograph \( u = u(t) \) can be calculated using again the general
equation (29) for all relative points of time τ, which are obtained with equation (26) by summing up all time steps Δτ.

**EXAMPLE OF APPLICATION**

In connection with the planning of flood retention basins a frequent question concerns the effects of such structures on the hydrographs of flood events with a certain return period at a critical point downstream. Using the above described model this question can be answered as follows:

First, the whole drainage basin behind the investigated point of interest is split up into equal sized squares. Then all those squares are defined, which are influenced by the flood retention project (Fig. 3) and the respective parts of the mass-transport histogram are determined. Using the linear routing model with the routing constant which has been calculated for the whole basin, the part of the flood hydrograph for the area which will be affected by retention measures can be calculated separately. The flood hydrograph of the remaining part of the basin can easily be found by subtracting the above calculated hydrograph from the given hydrograph for the whole basin. Now the separated hydrograph of the influenced sub-basin can be taken as the input for any kind of hydraulic retention calculations. To get the changed flood hydrograph for the whole drainage basin as the desired result, it is finally necessary to superimpose the two partial hydrographs, the one of the area which has not and the other one of the area which has been affected by retention measures (Fig. 6).

![Fig. 6 Example for T_r-flood hydrographs without and with a retention basin.](image)

**REFERENCES**

