A rainfall-runoff model for small ungauged watersheds in Poland

JANUSZ OSTROWSKI
Small Watersheds Project, Institute of Meteorology and Water Management, Podlesna 61, 01-673 Warsaw, Poland

Abstract Results are presented of research carried out in the Institute of Meteorology and Water Management (IMWM) on the peaks storm runoff evaluation and the rainstorm flood hydrographs simulation (for ungauged catchments up to 250 km²) by the Small Watershed Regional Model (SWRM).

Modèle pluies-débits pour de petits bassins versants non observés, en Pologne

Résumé On présente les résultats de recherches effectuées par l'Institut de la Météorologie et de la Gestion des Eaux sur l'évaluation du débit maximum et la simulation d'hydrogrammes de crues (dans les petits bassins versants de superficie jusqu'à 250 km²) par le modèle "Small Watershed Regional Model".

INTRODUCTION

The problem of estimation of flood rates from small ungauged basins has not yet been solved. This is very important purpose for research on engineering practice, planning, watershed development as well as for water management and environmental protection in rural catchments.

INPUT DATA

The rainfall hyetograph (1-h time step or multiple of hours) and 16 characteristics of the catchments are needed as input data to the Small Watershed Regional Model (SWRM) (Ostrowski, 1989). Values of the characteristics concerning basin area, river network, and Horton's ratios, as well as land use and vegetation cover are determined from hydrographical or topographical maps.

RAINFALL EXCESS AND GROUNDWATER CONTRIBUTION

The watershed initial conditions (WIC) (antecedent moisture) are represented by the total rainfall over the nine days preceding the flood event. The
accumulated rainfall excess \((SRE)\) up to time \(t\) during a given storm is evaluated by the modified Soil Conservation Service (SCS) Method based on the accumulated storm rainfall \((SRS)\):

\[
SRE(t) = \begin{cases} 
(SRS(t) - 0.2 * S)^2 & \text{for } SRS(t) > 0.2S \\
0 & \text{for } SRS(t) \leq 0.2S 
\end{cases}
\]

where \(RT\) is the duration of the rainfall.

The parameter \(S\) (maximum potential difference between storm rainfall and direct runoff) depends on the curve number \((CN)\), which is determined for a given storm as a function of total storm rainfall \((TRS)\) and watershed initial conditions:

\[
CN = a * TRS^b * WIC^c
\]

where \(a, b, c\) are regionalized coefficients (see Table 1). This means that the curve number is constant for a given storm, which differs to the original principle of the SCS Method which suggests it is constant for a basin. The rainfall excess hyetograph \((RE)\) is given by the relationship:

\[
RE(t) = SRE(t) - SRE(t - 1) \quad \text{for } t = 1, ..., RT
\]

The "soil profile feeding" \((SPF)\) is described as the difference between the total storm rainfall and the sum of rainfall excess, less an interception storage \((INT)\):

\[
SPF = TRS - SRE - INT
\]

Interception is assumed constant for a given basin and depends on land use and vegetation cover (similar to the Stanford Watershed Model IV). The groundwater flow hydrograph is composed of two elements: a dynamic part \((QGRD)\) and a time-invariant baseflow \((QBF)\):

\[
QGR(t) = QGRD(t) + QBF
\]

In essence, the "soil profile feeding" is divided into two parts. The first one causes the increment of soil moisture content \((SMC)\) and the other part adds to the groundwater zone. This is the "groundwater feeding" \((GRF)\) which generates the dynamic part of the underground flow:

\[
SPF(t) = GRF(t) + SMC(t)
\]

The "groundwater feeding" hyetograph is evaluated from:

\[
GRF(t) = ALFGR * SPF(t)
\]

where:
### Table 1: Results of regionalization of the Small Watershed Regional Model coefficients

<table>
<thead>
<tr>
<th>Ord No.</th>
<th>Model</th>
<th>Parameter</th>
<th>Eq No.</th>
<th>Regional independent Coeff.</th>
<th>Value</th>
<th>Eq No.</th>
<th>R2 Value</th>
<th>SEE Value</th>
<th>Number of Basins</th>
<th>Floods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CH</td>
<td>a</td>
<td>X20.X25.X27</td>
<td>L</td>
<td>16</td>
<td>192</td>
<td>.778</td>
<td>15.1350</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>b</td>
<td>X20.X25.X27</td>
<td>L</td>
<td>16</td>
<td>192</td>
<td>.706</td>
<td>0.0539</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>c</td>
<td>X13.X27.X28</td>
<td>NL</td>
<td>16</td>
<td>192</td>
<td>.741</td>
<td>0.0181</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>ALFGR</td>
<td>d</td>
<td>X17.X23.X2</td>
<td>L</td>
<td>16</td>
<td>189</td>
<td>.841</td>
<td>0.0227</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>e</td>
<td>X2.X17.X20</td>
<td>L</td>
<td>16</td>
<td>189</td>
<td>.912</td>
<td>0.0016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>N</td>
<td>-</td>
<td>-</td>
<td>RB, RL, RA</td>
<td>NL</td>
<td>51</td>
<td>-</td>
<td>.994</td>
<td>.0166</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>K</td>
<td>K2</td>
<td>-</td>
<td>RB, RL, RA</td>
<td>NL</td>
<td>51</td>
<td>-</td>
<td>.995</td>
<td>.0047</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>LT</td>
<td>-</td>
<td>X1.LRE.SRE</td>
<td>L</td>
<td>14</td>
<td>136</td>
<td>.843</td>
<td>2.0780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>OBF</td>
<td>f</td>
<td>-</td>
<td>RB, X17.X26</td>
<td>NL</td>
<td>16</td>
<td>192</td>
<td>.870</td>
<td>.4851</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>g</td>
<td>X18.X21.X40</td>
<td>L</td>
<td>16</td>
<td>192</td>
<td>.944</td>
<td>.0034</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Designation of the independent variable:**

<table>
<thead>
<tr>
<th>Characteristics of area</th>
<th>Characteristics of land use and vegetation cover</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 X1</td>
<td>basin area</td>
</tr>
<tr>
<td>2 X13</td>
<td>average hills length</td>
</tr>
<tr>
<td>3 X40</td>
<td>soil index</td>
</tr>
<tr>
<td>Characteristics of river network</td>
<td>Horton’s ratios</td>
</tr>
<tr>
<td>4 X17</td>
<td>main river length</td>
</tr>
<tr>
<td>5 X18</td>
<td>tributaries length</td>
</tr>
<tr>
<td>6 X20</td>
<td>river network density</td>
</tr>
<tr>
<td>7 X21</td>
<td>stream number</td>
</tr>
<tr>
<td>8 X23</td>
<td>river sources altitude</td>
</tr>
<tr>
<td>9 X25</td>
<td>main river slope</td>
</tr>
</tbody>
</table>

L - linear function  
NL - nonlinear function
\[ ALFGR = d \ast WIC^e \tag{8} \]

where \( d \) and \( e \) are regionalized coefficients.

**THE FLOOD HYDROGRAPH**

The flood hydrograph (\( Q \)) consists of two components: direct runoff (\( QDR \)) and groundwater flow (\( QGR \)):

\[ Q(t) = QDR(t) + QGR(t) \tag{9} \]

where \( QT \) is the duration of the flood.

The direct runoff and the dynamic groundwater flow are simulated by means of two Nash instantaneous unit hydrographs (NIUH). Generally these can be written:

\[ Qf(t) = \int_{0}^{t} RE(\tau) \ast NIUH(t - \tau) \, d\tau \tag{10} \]

for \( \tau = 1, ..., t; \ t = 1, ..., QT \) and \( j = 1, 2 \),

and

\[ NIUH(t,N,K_j) = [K_j \ast ((N)\ast 1) \ast (t/K_j)^{N-1} \ast \exp(-t/K_j)] \tag{11} \]

for \( j = 1 \quad Q_1 = QDRS(t) \quad \text{and} \quad K_1 = K \)

for \( j = 2 \quad Q_2 = QGRDS(t) \quad \text{and} \quad K_2 = KGR \)

Therefore the simulated flood hydrograph for an ungauged watershed (\( QSYM \)) can be calculated from:

\[ QSYM(t) = QDRS(t) + QGRDS(t) + QBFS \quad \text{for} \ t = 1, ..., QT \tag{12} \]

The parameters of the Nash cascades are identified by equations based on the reservoir version of the geomorphological instantaneous unit hydrograph theory (Rodriguez-Iturbe & Valdes, 1979; Zelasiński, 1986; and Ostrowski, 1984/1985, 1987/1988).

The shape parameter \( N \) (i.e. reservoir number) is constant for a watershed (and the same for both cascades) and can be determined from the equation:

\[ N = 3.329 \ast (RB/RA)^{0.744} \ast RL^{0.072} \tag{13} \]

However the scale parameter \( K \) (reservoir storage time) is estimated from the following equation:

\[ K = KZ \ast LR \ast V^{-1} \tag{14} \]
where:

\[ KZ = 0.695 \times (RB/RA)^{0.485} \times RR^{-0.476} \]  \hspace{1cm} (15)

and

\[ K = 0.695 \times (RB/RA)^{0.485} \times RL^{-0.476} \times LR \times V^{-1} \]  \hspace{1cm} (16)

where \( RB, RL, RA \) are bifurcation, length and area ratios (Horton's ratios) respectively, \( LR \) is the main stream length (scale variable), \( V \) is the mean streamflow velocity in the flood period.

From (13) and (16) the following conclusions have been drawn:
- parameter \( N \) is constant for the catchment,
- parameter \( K \) is time invariant within a flood in a given catchment.

Computer experiments (trial and error method) and other studies have ascertained that the groundwater travel time (i.e. underground reservoir storage time) varies from four to five times longer than reservoir storage time for direct runoff. Hence, the groundwater scale parameter \( (KGR) \) is assumed to be \( 4.5 \times K \). However the reservoir numbers of Nash cascades \( (N) \) are equal for both surface and underground responses.

For a basin of Strahler order \( \Omega \), Rodriguez-Iturbe & Valdes (1979) have formulated the geomorphological theory of the instantaneous unit hydrograph (GIUH) in which the time to peak \( (t_p) \) is given by:

\[ t_p = 1.584 \times (RB/RA)^{0.55} \times RL^{-0.38} \times LR \times V^{-1} \]  \hspace{1cm} (17)

In classical theory a very common and useful analytical form for the instantaneous unit hydrograph (IUH) is the two-parameter gamma density function (11). In this case, the time to peak is the mode of the gamma function shape:

\[ t_p = K \times (N - 1) \]  \hspace{1cm} (18)

and the scale parameter is:

\[ K = LT/N \]  \hspace{1cm} (19)

where \( LT \) is the lag time estimated as the difference between the rainfall excess hyetograph and the flood hydrograph centres of gravity.

The assumption of equality of \( t_p \) allows the estimation mean streamflow velocity by equating the left-hand sides of (17) and (18) as well as taking into consideration (19):

\[ V = 1.584 \times (RB/RA)^{0.55} \times RL^{-0.38} \times \frac{LR \times N}{(N - 1) \times LT} \]  \hspace{1cm} (20)

The lag time is an unknown value of the above-mentioned equation and can be evaluated from the empirical relationship:
where $XI$ is basin area, $LRE$ is rainfall excess duration time and $SRE$ is sum of rainfall excess.

The baseflow (constant for the flood) is computed from the watershed initial conditions:

$$QBF = f + g \times WIC$$  \hspace{1cm} (22)

where $f$, $g$ are regionalized coefficients.

MODEL STRUCTURE

The Small Watershed Regional Model is stationary for floods because the time scale parameter depends on the lag time as well as on streamflow velocity, which both vary for every flood event. The watershed is treated as a lumped parameter system.

Six model parameters ($CN$, $ALFGR$, $N$, $K$, $KGR$, $QBF$) are needed for each flood hydrograph simulation and they are identified on the basis of the above-mentioned four group of characteristics, by empirical equations with regionalized coefficients (see Table 1). All parameters have physical meaning or their range is determined ($CN$, $ALFGR$, $N$).

The human impact on the catchment (i.e. result of river training activities or land use changes) can be reflected in the model.

MEASUREMENT NETWORK

Field studies for the Small Watersheds Project of IMWM (which is the basis for this study) have been carried out on a network of 46 small Polish basins (areas from 2.8 to 231.6 km$^2$). The data from 192 floods in a five-year period (1981–1985) from 16 catchments were used in the regionalization, processed by the Statigraphics procedures.

FINAL NOTES

Model verification is based on the following statistics (Ostrowski, 1986, 1987/1988):

- relative mean error of the simulated flood peak ($RME$),
- determination coefficient ($R^2$) between observed and simulated flood hydrograph.

A classification of model quality is presented in Table 2.

Preliminary results of model verification on quasi-independent and independent data sets are highly promising for future development of the model.

Results of flood hydrograph simulation for the Wilga River (see Table 3) for the 19–20 October 1989 event are showed on Fig. 1.
Table 2  Classification of model quality

<table>
<thead>
<tr>
<th>Statistic criteria</th>
<th>Range</th>
<th>Model quality estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0.99 \leq R^2 &lt; 1.00$</td>
<td>Excellent</td>
</tr>
<tr>
<td></td>
<td>$0.95 \leq R^2 &lt; 0.99$</td>
<td>Very good</td>
</tr>
<tr>
<td>Determination</td>
<td>$0.90 \leq R^2 &lt; 0.95$</td>
<td>Good</td>
</tr>
<tr>
<td>coefficient</td>
<td>$0.85 \leq R^2 &lt; 0.90$</td>
<td>Fair</td>
</tr>
<tr>
<td>$(R^2)$</td>
<td>$0.80 \leq R^2 &lt; 0.85$</td>
<td>Sufficient</td>
</tr>
<tr>
<td></td>
<td>$0.70 \leq R^2 &lt; 0.80$</td>
<td>Poor</td>
</tr>
<tr>
<td></td>
<td>$0.00 \leq R^2 &lt; 0.70$</td>
<td>Unsatisfactory</td>
</tr>
</tbody>
</table>

Table 3  Details of the Wilga catchment

<table>
<thead>
<tr>
<th>Catchment area</th>
<th>231.6 km²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swamp area</td>
<td>0.0%</td>
</tr>
<tr>
<td>Forest area</td>
<td>15.3%</td>
</tr>
<tr>
<td>Ploughed area</td>
<td>66.8%</td>
</tr>
<tr>
<td>Meadow area</td>
<td>11.7%</td>
</tr>
<tr>
<td>Other</td>
<td>6.3%</td>
</tr>
<tr>
<td>Period</td>
<td>1978-1989</td>
</tr>
<tr>
<td>Number of events (V-X)</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 1 Result of flood simulation for the Wilga River at Oziemkowka (19–20 October 1980; $R^2 = 0.807$, RME = 8.0%).
Soon the SWRM model will be tested on independent data sets for basins from different morphological, hydrological and climatological conditions.

Comparability of observed and simulated hydrographs largely depends on simulating accurately the rainfall excess hyetograph, watershed initial conditions and mean streamflow velocity for particular floods.

Future research will be directed to the development of the model structure as well as to more detailed descriptions of the main components of the SWRM model.

Acknowledgements The author would like to acknowledge the financial support of the Institute of Geophysics (Polish Academy of Science) from the Government Fundamental Research Programme 03.09. "Water Resources Analysis and Development Methods" (CPBP-03.09). The author is grateful to Professor M. Ozga-Zielinska, Director of the Institute of Environment Engineering (Warsaw Technical University), research coordinator of the Programme. The author wishes to thank research technicians (from the Small Watersheds Project of IMWM) who assisted in all phases of this research.

REFERENCES


Part II: Hydrograph separation. vol. X, no. 2-3.
Part V: Results of the rainfall-runoff model testing (in press).

