Peakflow estimation using an antecedent precipitation index (API) model in tropical environments

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ABSTRACT An antecedent precipitation index (API) model is presented which utilizes a hydrograph recession coefficient in conjunction with precipitation amounts and timing to simulate streamflow during large storm events. Application of the methodology is illustrated for estimating peakflows on a 865 ha watershed in Hawaii, USA, and simulating stream levels of the Wainganga River in India.

INTRODUCTION

The prediction of peakflows associated with tropical catchments represents an important problem in applied hydrology. Historical rainfall and runoff data often forms the basis for undertaking empirical frequency analyses or to develop and calibrate hydrologic models which may be used for flow simulations and predictions. In most countries, precipitation data is generally more commonly available (both the number of stations and length of record) than is streamflow data. Thus, there is a need for methodologies that utilize existing precipitation-runoff information as a basis for estimating peakflows and associated return periods.

A variety of hydrologic models exist for simulating catchment hydrology and streamflow. These models range from complex physically-based process models to simple regression models that require little hydrologic understanding of processes. Three general categories of rainfall-runoff models are often identified (Wood & O'Connell, 1985): (1) distributed physically-based models which attempt to simulate the vast array of hydrologic process and physical laws that govern runoff on natural and disturbed watersheds (e.g., Beven, 1985), (2) lumped parameter models which are quasi-physical in nature and offer a simplified conceptual representation of the various hydrologic processes (e.g., Blackie & Eeles, 1985), and (3) input-output or "black-box" models which focus on relationships between rainfall and runoff without necessarily identifying any of the internal mechanisms whereby this transformation takes place.

A linear regression of total storm runoff as a function of rainfall amount would represent a black-box model in perhaps its simplest form. Incorporating hydrologic concepts into an input-output model might
allow one to characterize such a formulation as a "grey-box" model. For example, a unit hydrograph approach which postulates a linear relationship between effective rainfall and storm runoff fits this category. A rainfall-runoff model that utilizes antecedent precipitation to adjust runoff responses could similarly be categorized as a grey-box model. In comparison to other modeling strategies for rainfall-runoff simulations, input-output models are of simple construction and tend to have minimal data and computational requirements.

The purpose of this paper is to present an input-output model for peakflow simulation that is based on antecedent precipitation concepts. Application of the methodology in the coastal mountains of the Pacific Northwest, USA, indicated the methodology provided reasonable estimates of peakflows (Fedora & Beschta, In press). Similarly, Ziemer & Albright (1987) found that an API approach was useful for evaluating pipe-flow hydrology in steep mountainous terrain of the Pacific Northwest, USA.

THE ANTECEDENT PRECIPITATION INDEX METHODOLOGY

High flows at the mouth of a catchment are primarily dependent upon the occurrence of large amounts of rainfall over a relatively short period of time. The API model presented in this paper can be used to simulate storm runoff and requires essentially three steps: (1) recession analysis of storm hydrographs, (2) calculation of API values, and (3) correlation of API values with stream discharge.

Recession analysis

An underlying assumption of an API modeling approach is that antecedent precipitation influences the runoff efficiency from precipitation occurring at time t. Precipitation that occurs several days prior to time t has less effect on rainfall-runoff relationships than precipitation that has occurred more recently. Thus, the capability of antecedent precipitation to influence rainfall-runoff relationships decreases or decays with time.

For the API model presented herein, the temporal decay of antecedent precipitation amounts is indexed by a storm hydrograph recession coefficient C. The recession coefficient integrates various effects of a catchment's soils, geology, topography, vegetation, etc. In general, catchments that are relatively small, and which have steep topography and shallow soils, tend to have larger recession coefficients than catchments which are large or have gentle terrain and deep soils.
Rainfall hyetographs and storm hydrographs are required to undertake recession analysis. After peak discharge occurs during a given storm, and flows continue to recede, the recession coefficient is determined. This coefficient is obtained by deriving the slope of the line formed from plotting stream discharge at time t against the discharge at time t-Δt (Figure 1). Information regarding rainfall patterns is also needed because the recession analysis is only undertaken for those periods during which no rainfall was occurring. The recession coefficient can also be approximated by the slope of the recession hydrograph when plotted on semilogarithmic paper (Linsley et al., 1982).

If the time interval for precipitation observations and the time interval used to derive the recession coefficient are not they same, the recession coefficient needs to be adjusted to a time interval that is consistent with that of the precipitation data. The coefficient derived from recession analysis can be easily adjusted to the time interval of precipitation observations by the following relation:

\[ C = C' \left( \frac{\Delta t}{\Delta t'} \right) \]  

(1)

where \( C \) = recession coefficient for time interval \( \Delta t \) (0≤C≤1), \( C' \) = recession coefficient derived from time interval \( \Delta t' \) (0≤C'≤1), \( \Delta t \) = time interval of precipitation observations (in hours), and \( \Delta t' \) = time interval used to derive recession coefficient \( C' \) (in hours).

Calculation of the antecedent precipitation index (API)

The API model is mathematically formulated as follows:

\[ API_t = (API_{t-\Delta t} \times C) + P_t \]  

(2)

where \( API_t \) = antecedent precipitation index at time \( t \) (mm), \( \Delta t \) = time interval of precipitation observations (h), \( C \) = storm hydrograph recession coefficient (dimensionless), and \( P_t \) = precipitation that occurs from \( t-\Delta t \) to \( t \) (mm). Although calculated values of \( API_t \) are theoretically dependent upon all precipitation occurring prior to time \( t \), precipitation that occurs during the most recent time interval has a greater effect on \( API_t \) than an equivalent amount of rainfall that fell during any previous period. Precipitation during the time interval immediately prior to time \( t \) contributes fully to \( API_t \), while the effect of previously fallen precipitation (i.e., prior to \( t-\Delta t \)) is decayed through time. A simple computer program can be used to calculate \( API_t \) values; spreadsheet programs can also be used.

For a particular gaging station, several of the largest runoff events of record are selected.
Corresponding precipitation data for the catchment are also needed for each period of high flow and for several days prior to each event. Hopefully, the precipitation records will provide reasonably accurate indications of catchment amounts and temporal distributions. Runoff events used for analysis can be defined to begin and end using the baseflow separation technique described by Hewlett & Hibbert (1967). Precipitation amounts associated with these runoff events are then used in conjunction with Equation (2) to calculate $\text{API}_t$ values during the period of high flow.

Even though peakflow may be of primary interest, simulations need to begin several days prior to the occurrence of peak discharge. This is because the effects of antecedent precipitation amounts upon $\text{API}_t$ decay through time and generally become insignificant after a period of several days. For example, the effect of precipitation which occurred 4 days prior to the time $t$ upon $\text{API}_t$ will have decreased by 99%, assuming an hourly recession coefficient of 0.95. In this manner, the cumulative effectiveness of previous precipitation
amounts for influencing $API_t$ is decayed through time. Determining the length of time prior to the occurrence of peak discharge that $API_t$ calculations should be undertaken is somewhat arbitrary, but it should be sufficiently long so that at least 90% of the relative effectiveness of the earliest precipitation amount has "decayed".

**Correlation of API with stream discharge**

Once API values have been calculated for each storm, corresponding values of the antecedent precipitation index $API_t$ and stream discharge $Q_t$ are then correlated. The coefficient of determination (i.e., $r^2$) for this line will provide an initial indication of the goodness-of-fit between the two variables. It may be desirable to transform either variable to obtain a straight-line relationship. For example, in western Oregon, USA, Fedora & Beschta (In press) found that the square root of discharge provided a linear relationship with $API_t$ (Figure 2).

The slope $S$ of the regression line in Figure 2 represents the rate of change in discharge with a unit change in precipitation. The y-axis intercept $I$ represents the average base flow immediately before and after high flow events.

For large catchments, precipitation that falls on a distant portion of the catchment may require a significant period of time for it to be routed to the catchment outlet, even during high flow conditions. The API methodology, as currently formulated, does not specifically adjust for time-of-travel. Thus, rising limbs of storm hydrographs tend to be over-predicted by the model and recession limbs under-predicted. Although visually disconcerting, this effect may not greatly influence peakflow estimates. A relatively simple approach for overcoming this problem is to undertake cross-correlation analyses of $API_t$ and $Q_t$ values to determine an appropriate timing offset for precipitation amounts to account for time-of-travel effects.

**Regionalization of coefficients**

To use the API methodology for a specific catchment, three coefficients ($C$, $S$, and $I$) need to be established. Because storm hydrographs are generally less "flashy" as catchment size increases, recession coefficients tend to increase with catchment size. Thus, it may be possible to develop a relation between $C$ and catchment area (e.g., Fedora & Beschta, In press). Similarly, $S$ and $I$ may be associated with watershed characteristics such as soil depth, geologic rock type or depth of weathering, terrain steepness, drainage density, etc. If regional estimates of recession coefficients can be developed (or predicted
from regression analysis with other factors), the API methodology could be used for predicting peakflows from ungauged watersheds where precipitation records are available. Furthermore, if regionalized characterizations of large rainfall events (i.e., storm amounts and temporal distributions) were developed from historical precipitation records, peakflows could be estimated using an API model for ungauged catchments. Although initial results in western Oregon indicate that the API coefficients can be regionalized (Fedora & Beschta, In press), the API methodology has not been widely applied or tested.

APPLICATION AND DISCUSSION

Peak flow estimation in tropical areas

A preliminary analysis of rainfall-runoff data was undertaken for an 865 ha catchment in the Moanalua Valley, on the Hawaiian island of Oahu, to evaluate the potential applicability of the API procedure for tropical catchments which experience high intensity rainfall events. The four largest flow events within an eight-year period of record were analyzed. API values were calculated at five-minute intervals for each of three precipitation gages on the catchment. Based on hydrograph analysis, an hourly recession coefficient of 0.24 (i.e., $C' = 0.888$ for five-minute intervals) was used for all API calculations. To account for time-of-travel, the calculated values of API for each precipitation gage were delayed in relation to distance.
from the streamgage (e.g., a 30-minute delay was used for the farthestmost precipitation gage) and an aerially weighted \( \text{API}_t \) was calculated at each time \( t \). A relationship between the \( Q_{0.5} \) and \( \text{API}_t \) was then established by regression. The resulting \( C, S, \) and \( I \) coefficients were then used as a basis to simulate peakflow hydrographs.

The average absolute error of the four largest storms was 14 percent (Table 1). In a more comprehensive analysis of rainfall-runoff patterns on this catchment, Shade (1984) obtained an average absolute error of 15.4% for these same five events using the distributed routing rainfall-runoff model of Dawdy, Schaake, and Alley (DSA).

Flood hazard forecasting in real time

The potential usefulness of API as a flood-stage forecasting methodology is illustrated with stage data for the Wainganga River in India. Data for three periods of flooding (Chander et al., 1981) provide the basis for this example. Recession curve analysis of stage hydrographs indicated a hourly recession coefficient of approximately 0.87. Regression analysis of stage versus \( \text{API}_t \) was then undertaken to find the line of best fit. The plotting of stage vs. \( \text{API}_t \) values indicated a pronounced hysteresis effect whereby \( \text{API}_t \) greatly overpredicted stage on the rising limb of the flood-stage hydrograph and similarly underpredicted stage on the recession limb of the flood-stage hydrograph. Cross-correlation analysis between stage and \( \text{API}_t \) values indicated that "delaying" the occurrence of precipitation by 9 hours would tend to minimize this effect. Thus, precipitation amounts were lagged by 9 hours and \( \text{API}_t \) values calculated for the floods illustrated by Chander.

<table>
<thead>
<tr>
<th>Storm No.</th>
<th>Date of storm</th>
<th>Observed peak (( \text{m}^3 \text{s}^{-1} ))</th>
<th>Simulated peak (( \text{m}^3 \text{s}^{-1} ))</th>
<th>Error ( % )</th>
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<td>-9</td>
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<td>4/5-6/71</td>
<td>51.5</td>
<td>56.6</td>
<td>10</td>
</tr>
</tbody>
</table>

\( \text{Error} = \left( \frac{\text{Simulated} - \text{Observed}}{\text{Observed}} \right) \times 100\% \)
et al. (1981). The following relationship between stage (m) and $\text{API}_t$ was then obtained by regression ($r^2 = 0.83$):

$$\text{Stage}_t = 2.04 + (0.219 \times \text{API}_t)$$

In this example, river stage is linearly related to $\text{API}_t$ so that each additional five millimeters of rainfall causes, on average, a one-meter increase in the stage of the Wainganga River.

Hydrographs of the observed and synthesized stages for three storms are shown in Figure 3. Even though precipitation was lagged 9 hours for these examples, the predicted stage overpredicts the rising limb stages for the 1st flood. This overprediction could be the result of significant storm precipitation going into retention storage. For the remaining floods, the API methodology replicates the general shape and magnitude of the flood-stage hydrographs reasonably well. Absolute errors in peak discharges for the three floods illustrated in Figure 3 averaged 0.4 meters. With each incremental amount of precipitation, predicted flood-stage hydrographs can be simulated up to 9 hours in advance. Thus, API provides a relatively simple technique for flood prediction that can be easily used in real time.

General comments

The API methodology implicitly assumes that abstractions from rainfall amounts, such as increased soil moisture storage, and evaporation or transpiration, are relatively insignificant for large rainfall events. Perhaps the inclusion of a long-term or seasonal antecedent factor might be useful in areas where seasonal changes in soil moisture levels have an important effect on storm discharges.

The API model is based on the concept that the relative efficiency of precipitation for generating storm runoff depends on both the amount and the time distribution of storm precipitation. Because the model attempts to account for antecedent precipitation effects on rainfall-runoff relationships, the method does not require a priori assumptions about the temporal distribution of storm precipitation. The utilization of a storm hydrograph recession coefficient provides the basis for "decaying" the hydrologic significance of antecedent precipitation amounts through time. This feature of the model provides a mechanism for linking the model against a single, integrative catchment characteristic that is easily and systematically derived.

The widely used SCS runoff curve model (e.g., US Department of Agriculture, 1972) assumes a systematic increase in runoff efficiency as a storm progresses. In contrast, the basic premise of the API model and the results of API simulations indicate that the relative
efficiency of a catchment to produce streamflow from a given unit of precipitation continuously varies. Hence, the API method appears to have potential application for geographical areas where the temporal distribution of storm precipitation amounts is highly variable.

![Comparison of observed and simulated flood-stage hydrographs for the Wainganga River in India: (a) 30 July to 6 August 1969, (b) 28 August 1972, and (c) 28 August 1973.](image)

Although the slope $S$ and intercept $I$ of the relation between $Q_t$ and $\text{API}_t$ may have a hydrologic interpretation, little is known about how these parameters vary with different catchments, or how they are affected by topography or catchment characteristics.

Because of its relative simplicity and reasonably accurate simulations, the API methodology may have widespread application in tropical regions for simulating storm discharges from large rainfall events. Once calculated $\text{API}_t$ values have been developed from existing rainfall-runoff records, theoretically storms of any temporal distribution can then be used to synthesize storm discharge. However, additional simulations over a wide range of hydrologic conditions in tropical catchments are needed to further evaluate the potential applicability and accuracy of the API method.
REFERENCES


