Application of the Preissmann scheme on flood propagation in river systems in difficult terrain

K.W. CHAU
Department of Civil & Structural Engineering, Hong Kong Polytechnic, Hong Kong

ABSTRACT This paper presents the application of an accurate as well as efficient Preissmann type implicit finite difference solution to approximate flood propagation in a river network system of difficult terrain, tailored on the IBM/PC/XT or PC/AT or their compatible personal computers. With the use of this mathematical model, the hydrograph at any point on the river and hence the probable extent of flooding can be estimated in advance. The potential flood hazard in Shing Mun River and Sham Chun River in Hong Kong have been studied.

INTRODUCTION

A flood hydrograph changes as storm water flows from upstream to downstream of a river channel, including the magnitude and timing of the peak. It is imperative to estimate the hydrograph at any point on the river channel during a rainfall or flood event and to know in advance the probable extent of flooding.

Apart from the development of efficient, high-speed computer program, two alternatives are available in modelling free surface flow: (i) analytical solutions, and (ii) physical hydraulic models. Despite providing the most accurate results, analytical solutions are extremely rare and apply only in highly simplified situations. Physical models have the drawbacks of expensive, time-consuming and lack of flexibility for adaptation to different uses.

In this paper an accurate as well as efficient numerical solution for use on the IBM or compatible personal computers is implemented to simulate unsteady flood propagation in a river system of difficult terrain. The program is written in Fortran and is compiled by PC software Microsoft Fortran Version 4.0. The model developed is based on the 4-point operators Preissmann implicit finite difference scheme (Cunge, 1980 & Preissmann, 1960). As a yardstick against which the model performance can be compared, an explicit scheme has also been coded (Dronkers, 1969) which is conditionally stable. Real hydraulic features, including branched channels and tidal flats, are also simulated.

THE UNSTEADY FLOW EQUATIONS

The governing equations of mass and momentum transport, the de Saint-Venant Equations, are utilized to describe unsteady constant
density flow in an open channel which may be subject to tidal forcing and/or upstream freshwater inflows. The de Saint-Venant Equations can be derived from the general equations for fluid flow by integrating the governing equations over the cross section and making certain valid approximations (Abbott, 1980).

The governing equations of unsteady open channel flow are expressed as:

Continuity equation

\[ bs \frac{\partial z}{\partial t} + \frac{\partial Q}{\partial x} = q \]  \hspace{1cm} (1)

Momentum equation

\[ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} (uQ) + gA \frac{\partial z}{\partial x} + gA \frac{Q |Q|}{K^2} = 0 \]  \hspace{1cm} (2a)

or its linearized version

\[ \frac{\partial Q}{\partial t} + gA \frac{\partial z}{\partial x} + \tau Q = 0 \]  \hspace{1cm} (2b)

where \( Q \) = flow discharge, \( u \) = velocity, \( z \) = water stage above a datum, \( bs \) = storage width, \( q \) = lateral inflow, \( A \) = area of the channel, \( g \) = acceleration due to gravity, \( K \) = conveyance factor of the channel and \( \tau \) = constant linearized bottom friction coefficient.

The unknown variables involved are the discharge \( Q \) and the height of the free surface above a datum \( z \), or equivalently the water depth \( h(z) \) and the horizontal velocity \( u = Q/A(h) \), as shown in Fig. 1.

For an explicit finite difference scheme a stability analysis usually results in the Courant-Friedrichs-Lewy (CFL) stability criterion, equation 3, in choosing \( \Delta t \) for given \( \Delta x \).

\[ | C \frac{\Delta t}{\Delta x} | \leq \gamma, \quad C = \sqrt{gh} \]  \hspace{1cm} (3)

often \( \gamma = 1 \)
The stability limit \( \gamma \) is equal to 1 or a certain number below unity. The value \((C \Delta t / \Delta x)\) is called the Courant number, \( C_r \). One needs to use smaller \( \Delta t \) than that specified by the Courant and other analogous conditions to assure stability. The limitation on \( \Delta t \) for a selected \( \Delta x \) may require a lengthy computation, subsequently making the method impractical (Dronkers, 1969). As such, implicit finite difference scheme is chosen in the mathematical model.

THE PREISSMANN TYPE IMPLICIT FINITE DIFFERENCE SCHEME

Amongst the two implicit finite difference schemes most widely used in engineering practice, Preissmann scheme has advantages over Abbott-Ionescu scheme since it allows non-equidistant grids and computes discharge \( Q \) and elevation \( z \) at the same point. Thus the Preissmann type of implicit finite difference scheme was chosen in the mathematical model.

The salient features of the Preissmann scheme are as follows:

(a) The Preissmann scheme is unconditionally stable as long as \( \Theta \geq 0.5 \). Consequently, the time step is only a function of the required accuracy. The time step \( \Delta t \) used can be chosen freely to be comparable with the particular physical phenomena under consideration.

(b) The space intervals \( \Delta x \) may be variable. This enables a more flexible schematization of the river, especially in the case of strongly varying cross sections.

(c) Both unknown flow variables are computed at the same computational grid points. Stage/discharge rating curves and similar relationships may be introduced at the same locations with no particular difficulty.

\[
\begin{align*}
Q, z_t & = Q, z_{t-1} + \Delta t \left[ \sum_{i=1}^{n} \left( \frac{Q, z_i}{\Delta x} \right) - \frac{1}{2} \frac{Q, z_{t-1}}{\Delta x} \right] \\
\end{align*}
\]

\[
\begin{align*}
(1-\Theta) \frac{Q, z_{t-1}}{\Delta x} & \rightarrow X
\end{align*}
\]

Fig. 2 Data layout of the Preissmann type implicit scheme.

Fig. 2 and equations 4 show the actual discretization of dependent derivatives according to Preissmann (Cunge et al., 1980):
where \( f \) is the flow variable, discharge \( Q \) or water elevation \( z \), and \( \Theta \) is a time weighting coefficient, \( 0 \leq \Theta \leq 1 \), introduced in the spatial derivatives to aid in the numerical solutions.

For single channels with one boundary condition given at each end, the coefficient matrix contains elements only in the band along the main diagonal. A computational point \( j \) is not linked directly to all other points, but only to adjacent points \( j-1 \) and \( j+1 \). The discretized form of continuity and momentum equations, written for each adjacent pair of the \( N \) computational points in the model, together with the two boundary conditions constitute the following penta-diagonal system of equations in matrix form:

\[
\begin{align*}
\begin{bmatrix}
-E & 1 &  &  & \\
-C & -D & H & B & 0 & 0 & 0 \\
-C' & -D' & H' & B' & 0 & 0 & 0 \\
-C_2 & -D_2 & H_2 & B_2 & & & \\
-C'_2 & -D'_2 & H'_2 & B'_2 & & & \\
 & & & & & & \\
0 & . & . & . & & & \\
0 & 0 & -C_{N-1} & -D_{N-1} & H_{N-1} & B_{N-1} & \\
0 & 0 & -C'_{N-1} & -D'_{N-1} & H'_{N-1} & B'_{N-1} & -E_N & 1
\end{bmatrix}
\begin{bmatrix}
\Delta z_1 \\
\Delta Q_1 \\
\Delta z_2 \\
\Delta Q_2 \\
\Delta z_3 \\
. \\
. \\
. \\
. \\
\end{bmatrix}
= 
\begin{bmatrix}
F_1 \\
G_1 \\
G'_1 \\
. \\
. \\
. \\
. \\
. \\
. \\
\end{bmatrix}
\end{align*}
\]

Thus this system may be solved by the conventional double-sweep method for any time step \( \Delta t \). The amount of core storage is minimized by taking advantage of the banded nature of the coefficient matrix, which is not destroyed by the branch channel algorithm.

RIGOROUS ANALYTICAL TESTS ON RECTANGULAR CHANNEL

All numerical models need to be checked and tested to ensure that the physical boundary problem is being solved numerically with sufficient accuracy to provide useful and worthwhile engineering results.

The numerical model is tested against several carefully chosen representative analytical solutions: i) a standing wave in a short rectangular channel with constant depth and in the absence of bottom friction, ii) a co-oscillating tide in a rectangular channel with quadratic bottom bathymetry and linearized bottom friction (Lynch,
TABLE 1 Comparison of water elevations at \( t/T = 0.25 \) computed by implicit scheme with analytical solution for frictionless flow in rectangular channel with constant depth.

<table>
<thead>
<tr>
<th>( x/l )</th>
<th>Exact Solution</th>
<th>( \Theta = 0.6 ) Cr=1.990</th>
<th>( \Theta = 0.6 ) Cr=8.957</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.1011E-0</td>
<td>0.1011E-0</td>
<td>0.1011E-0</td>
</tr>
<tr>
<td>0.4000</td>
<td>0.1009E-0</td>
<td>0.1009E-0</td>
<td>0.1009E-0</td>
</tr>
<tr>
<td>0.8000</td>
<td>0.1004E-0</td>
<td>0.1004E-0</td>
<td>0.1004E-0</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.1000E-0</td>
<td>0.1000E-0</td>
<td>0.1000E-0</td>
</tr>
</tbody>
</table>

FIG. 3 Comparison of water elevations along the channel computed by implicit scheme with analytical solution for frictionless flow in rectangular channel with constant depth.
These tests also give guidelines on mesh size and length of time step in prototype applications. Table 1 and Fig. 3 show the comparison of elevations along a rectangular channel computed by the scheme and the analytical solution for frictionless flow with constant depth. It can be observed that excellent agreement are obtained for the linearized analytical cases.

VERIFICATION ON REAL PROTOTYPE CASES

In the development of a numerical model, apart from the analytical test cases, it is worthwhile to investigate its effectiveness in real prototype cases (Chau, 1990). As such, the mathematical model has also been verified on tidal hydraulics of two prototype estuaries, namely, the Delaware Estuary and the Chincoteague Bay in the United States, where enormous information was available (Harleman & Lee, 1969 & Ippen, 1966) and where irregular geometries occur.

Table 2 gives the comparison of elevations at t/T = 0 computed by implicit scheme with explicit scheme for Delaware Estuary while Fig. 4 shows the results of velocities over the periodic cycle at different locations of the channel computed by implicit scheme in Chincoteague Bay. It can be noted that the finite difference solutions are in good agreement with historical data of tidal range, mean water level and high/low water times.

TABLE 2 Comparison of elevations at t/T = 0 computed by implicit scheme with explicit scheme for Delaware Estuary (Cr = 0.5).

<table>
<thead>
<tr>
<th>x/ℓ</th>
<th>Explicit Scheme</th>
<th>Implicit Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17</td>
<td>116</td>
</tr>
<tr>
<td>width</td>
<td>grid</td>
<td>grid</td>
</tr>
<tr>
<td>0.0303</td>
<td>433</td>
<td>-0.0060</td>
</tr>
<tr>
<td>0.0909</td>
<td>543</td>
<td>-0.0516</td>
</tr>
<tr>
<td>0.1515</td>
<td>681</td>
<td>-0.1325</td>
</tr>
<tr>
<td>0.2121</td>
<td>854</td>
<td>-0.2519</td>
</tr>
<tr>
<td>0.2727</td>
<td>1070</td>
<td>-0.3681</td>
</tr>
<tr>
<td>0.3333</td>
<td>1342</td>
<td>-0.4657</td>
</tr>
<tr>
<td>0.3939</td>
<td>1683</td>
<td>-0.5338</td>
</tr>
<tr>
<td>0.4545</td>
<td>2111</td>
<td>-0.6029</td>
</tr>
<tr>
<td>0.5152</td>
<td>2648</td>
<td>-0.6645</td>
</tr>
<tr>
<td>0.5758</td>
<td>3320</td>
<td>-0.7027</td>
</tr>
<tr>
<td>0.6364</td>
<td>4164</td>
<td>-0.6927</td>
</tr>
<tr>
<td>0.6970</td>
<td>5222</td>
<td>-0.6095</td>
</tr>
<tr>
<td>0.7576</td>
<td>6549</td>
<td>-0.4703</td>
</tr>
<tr>
<td>0.8182</td>
<td>8213</td>
<td>-0.3450</td>
</tr>
<tr>
<td>0.8788</td>
<td>10300</td>
<td>-0.2304</td>
</tr>
<tr>
<td>0.9394</td>
<td>12918</td>
<td>-0.1225</td>
</tr>
<tr>
<td>1.0000</td>
<td>16200</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
FIG. 4 Variation of tidal velocities over the periodic cycle at different locations of the channel computed by implicit scheme in Chincoteague Bay.

APPLICATION ON FLOOD PROPAGATION

The model is then applied to study the potential flood hazards in Shing Mun River together with its three tributary branches — Tin Sam Nullah, Fo Tan Nullah and Siu Lek Yuen Nullah (Maunsell, 1979). The effect of a proposed river training scheme in River Indus on the flood propagation in the future Sham Chun River network along the Hong Kong/China border is also evaluated (Maunsell, 1988). Both channels are of trapezoidal shape.

Fig. 5 depicts the computed water velocities along the Shing Mun River due to the effect of flood hydrographs applied at the upstream end of the river channel with tidal forcing of amplitude 1.05 m at the ocean end. It is found that the areas adjacent to the two river channels are protected against flooding if a severe storm surge does not occur. However, flooding is possible in the event the following combination occurs, i.e. 50-year Rainstorm + High Tide + Storm Surge.

CONCLUSIONS

It is useful to predict, accurately and efficiently, water stages and
flow quantities in river channels, particularly during heavy rainstorm, since the prediction of the time and magnitude of flood peaks can give alarm signals to the people at the downstream end. Appropriate measures to alleviate the effect of flooding can hence be made accordingly.

The model is able to describe the physical processes correctly and, for comparable accuracy, is more convenient and reliable than the explicit scheme. Solution features attributable to the nonlinearity of the equation can be noted. The accuracy of the implicit model is good at the Courant Number up to 10. The rigorous verification for the analytical test cases and the successful applications of the model to various prototype cases with different characteristics provide confidence for its capability as predictive tools in situations involving complex bathymetry and/or nonlinear bottom frictional effects, which pure analytical models is not able to incorporate.

The PC based mathematical model for simulation of unsteady flood propagation in an open channel network system is useful for predictive purposes and, as well, for design of proper urban stormwater drainage schemes to cope with different rainstorm runoffs or to carry off the surplus water.
REFERENCES


Maunsell Consultants Asia (1979) Tolo Harbour Rivers, Pollution & Sedimentation Study Part 1 — Shing Mun River.

