Probability Analysis of Shanghai Shallow Layer Subsidence

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ABSTRACT This paper endeavours to provide a suitable approach for the solution of the calculation about Shanghai shallow layer subsidence. An analytical study of deformation model concerning clay creep has been made by the optimization method, along with the observed data of the pore water pressure and deformation of Shanghai shallow layer under the pumping and recharging conditions. A viscoelastic Solution of Shanghai land subsidence derives from the selected model and the elastic solution of Shanghai land Subsidence. The statistical rule of the rheological parameters can be likewise obtained by the extended Lee's analogy method. The probability distribution of the deformation of shallow layer at a typical location in Shanghai is calculated with Monte-carlo simulation method. This new approach is intended for the prediction of land subsidence in Shanghai in the future ten years, which suggests a method for the reduction of the subsidence on Shallow layer.

INTRODUCTION

Shanghai land subsidence is chiefly attributed to groundwater pumping. This grave occurrence, since the beginning of 1960's has been kept under control by means of the limitation of the use of the groundwater in parallel with the application of the artificial recharge of the groundwater. However starting from 1972, there reoccurred minor subsidence, which, as shown by field observation, was concentrated in the upper part of Overburden (from the surface to the depth of 40m, hereinafter called shallow layer). And the laboratory tests further indicated that the creep of clay in shallow layer was an obvious cause of the minor subsidence (Su, 1979). So the adoption of proper measures both for the prediction of deformation and for the stabilization of the subsidence of shallow layer has become a key factor to further control of land subsidence in Shanghai.

GEOLOGICAL CONDITION AND DEFORMATION REGULARITY OF SHALLOW LAYER

In the area of Shanghai, there is a deposit of two compressible layers (Known as the 1st and 2nd compressible layers) in the upper part of
the Overburden, from the depth of 40m upward. The typical columnar section of the soil layers is shown in Fig. 1.

The 1st and 2nd Compressible layers are composed of muddy clay with low permeability and high moisture content, which possess a higher compression and a long time lag during the process of groundwater pumping. This is considered to be an internal factor, which will lead to the high compression in the shallow layer. The cumulative deformation and water table fluctuation of the shallow layer in a typical site are both shown in Fig. 2, which states that the cumulative deformation of this layer has a tendency of continuous compression as well as cyclic swelling and compression with periodic fluctuation of groundwater level. Furthermore the peak valley value of deformation has a short lag of groundwater level. This phenomenon indicates the creep behaviour of the soft clay layer.

FIG. 1. Typical geological columnar section of shallow layer.

FIG. 2. Curves showing the cumulative deformation of shallow layer and water level fluctuation in the typical site.
And the laboratory rheological test further reveals that the shallow soil has a creep behaviour. As shown in Fig. 3, the

![Graph showing deformation (D) by odometer test versus time (t).](image)

The deformation of soil shows no tendency to approach stabilization during the 90 days testing and the ratio between the secondary consolidation and the principal consolidation is 3:1 (Su, 1979).

In view of the above mentioned cyclical variation of the groundwater level and of the deformation, in our computation, we divide one year's groundwater and deformation variation into a rising period and a lowering period. It is supposed that the variations of groundwater level both in the rising period and the falling period are with linear pattern. By the annual field observation data of the groundwater level and of the deformation, we can reverse the rheological model parameters individually for the compression period and the rising period. Only in this way, can the compression and the swelling deformation of the shallow layer be computed.

OPTIMUM ANALYSIS FOR RHEOLOGICAL MODEL AND THEIR PARAMETERS

Kelvin's model which connects with elastic elements and viscous elements is usually used to explain strain-stress. Thus, three-elements model shown in Fig. 4 can be selected.

![Schematic drawing of model structure.](image)
Based on the principle of the creep mechanics, strain about
generalized non-relax model, inclusive of three-elements, can be
expressed as follows:

\[
\varepsilon(t) = \sigma(t) \cdot J(o) + \int_0^t \sigma(\tau) \frac{dJ(t-\tau)}{d(t-\tau)} \cdot d\tau
\]  

(1)

where \( \varepsilon(t) \) = strain;
\( \sigma(t) \) = stress;
\( t \) = time.

\[
J(t) = \frac{1}{E_o} + \frac{1}{K_o} \sum_{i=1}^{N} \frac{1}{E_i} \left[1 - e^{-\left(E_i/K_i\right) t}\right]
\]

is the strain acted by unit stress.

By substituting the pore water pressure observed in the field
into Eq. 1, the layer strain at any time can be calculated. Optimum
method is used to select rheological model and determine their
parameters and target function is formed:

\[
F(E_0,E_1,K_1,\ldots) = \sum_{i=1}^{N} \left(\varepsilon(t_i) - \varepsilon_i\right)^2
\]

(2)

where \( \varepsilon \) = observed strain in certain period;
\( \varepsilon_i \) = calculated strain in the same period;
\( N \) = period numbers;
\( E_0,E_1,K_1,\ldots \) = parameters of model.

So, value of target function \( F(E_0,E_1,K_1,\ldots) \) becomes a
measurement of coinciding precision. Three-elements model is accepted
as a calculating model for shallow layer because it is relatively
simple.

By putting the data of observed pore water pressure and
deformation in rising and lowering period of water level individually
into the target function, one set of parameters both in rising period
and lowering period can be calculated.

**DEFORMATION FORMULA OF SHALLOW LAYER**

By assuming that soil is elastic and basing on the Terzaghi's
one-dimensional consolidation theory, Qian, S.Y & Gu, X.Y have
derived the solution of the deformation of layer (Fig. 5) under the

![FIG.5. Schematic diagram of layer structure.](image-url)
Probability analysis of Shanghai shallow layer subsidence

case linear variation of central water level line. i.e.:

\[ S(t) = a \frac{M_v}{\sqrt{2}} \left\{ \frac{1}{(H_2^2 - H_1^2)} t - \frac{1}{(2H_2^2 - 2H_1^2 - H_2^2 + H_1^2)} \right\} \]

\[ + \sum_{n=1}^{\infty} \frac{H^3 \cos n\pi (H_2/H) - \cos n\pi (H_1/H)}{(nx)^4 C_V} e^{-(nx)^n C_V t/H^2} \]

where \( a \) = slope of variation of water level;
\( C_V \) = coefficient of consolidation;
\( M_v \) = coefficient of volume compressibility;
\( K \) = coefficient of permeability.

By extended Lee's analogy method, viscoelastic solution can be derived through following procedures:

(a) Selecting a three-elements viscoelastic model and write out it's physical equation

\[ E_0 K_1 \dot{\epsilon} + E_0 E_1 \epsilon = \sigma (E_0 + E_1) + K_1 \dot{\sigma} \]

applying Laplace transformation to Eq.4

\[ E_0 K_1 S \ddot{\epsilon} + E_0 E_1 \ddot{\epsilon} = \ddot{\sigma} (E_0 + E_1) + K_1 S \ddot{\sigma} \]

equivalent elastic modulus

\[ E = \frac{\ddot{\sigma}}{\ddot{\epsilon}} = \frac{E_0 K_1 S + E_0 E_1}{K_1 S + E_0 + E_1} \]

(b) Replacing \( C_V \) and \( M_v \) with elastic modulus

If one-dimensional compression, \( M_v = 1 / E, C_V = K E / r_w \),
assuming \( D = (nx)^2 K / (H^2 r_w) \), \( N = D E \), hence Eq. 3 becomes:

\[ S(t) = a \frac{M_v}{\sqrt{2}} \left\{ \frac{1}{(H_2^2 - H_1^2)} t - \frac{1}{(2H_2^2 - 2H_1^2 - H_2^2 + H_1^2)} \right\} \]

\[ + \sum_{n=1}^{\infty} \frac{H^3 r_w \cos n\pi (H_2/H) - \cos n\pi (H_1/H)}{(nx)^4 K E^2} e^{-Nt} \]

where \( E = \) elastic modulus;
\( r_w = \) unit weight of water.

(c) Apply Laplace transformation to Eq. 7
\[ S(S) = a \left\{ \frac{1}{2H} \left( \frac{H_2^2-H_1^2}{E^2} \right) - \frac{1}{24 KH} \left( \frac{2H^2H_2^2-2H^2H_1^2-H_2^4+H_1^4}{E^2S} \right) \right\} \]

\[ \sum_{n=1}^{\infty} \frac{H^3r_w[\cos nx (H_2/H) - \cos nx (H_1/H)]}{(n \pi)4 K} L^{-1}(\frac{1}{E^2(S+N)}) \]  

(d) Substitute \( E \) in Eq. 8 with expression of \( E \) in Eq. 6 and apply Laplace inverse transformation to Eq. 8, then viscoelastic solution can be obtained as follows:

\[ S(t) = a \left\{ \frac{1}{2H} \left( \frac{H_2^2-H_1^2}{E^2} \right) L^{-1}(\frac{1}{E^2S}) \right\} \]

\[ \frac{r_w}{24 KH} - \frac{1}{24 KH} \left( \frac{2H^2H_2^2-2H^2H_1^2-H_2^4+H_1^4}{E^2S} \right) L^{-1}(\frac{1}{E^2S}) \]

\[ \sum_{n=1}^{\infty} \frac{H^3r_w[\cos nx (H_2/H) - \cos nx (H_1/H)]}{(n \pi)4 K} L^{-1}(\frac{1}{E^2(S+N)}) \]  

where \( L^{-1}(\frac{1}{E^2S}), L^{-1}(\frac{1}{E^2S}), L^{-1}(\frac{1}{E^2(S+N)}) \) are expression of Laplace inverse transformation.

**PROBABILITY ANALYSIS OF SHALLOW LAYER DEFORMATION**

By optimization method, model parameters from 1966 to 1986 can be calculated and their mean value and standard deviation are shown in table 1.

Using Kolmogorov-Smirnov method to examine the assumed distribution of parameters, the results indicate that distribution of the model parameters can be assumed as normal distribution.

Putting the values of model parameters, depth of layer and coefficient of permeability into deformation Eq. 8, every year’s value of compression in lowering period and value of swelling in rising period can be calculated, and the net cumulative values of deformation can also be evaluated. Fig. 6 shows the cumulative observed deformation values and cumulative calculated deformation values to be well coincided.

The deformation of shallow layer is affected by the variation of water level in underlying aquifer and also affected by meteorological phenomena, man’s activity, measurement errors and other random factors. The parameters reversed from observed data also reflect the influence
TABLE 1. Statistic Value of rheological parameters.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Parameters</th>
<th>Rising period</th>
<th>Compression period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>mean value</td>
<td>standard deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st</td>
<td>$E_0$ (MPa)</td>
<td>34480</td>
<td>16100</td>
</tr>
<tr>
<td>Layer</td>
<td>$E_1$ (MPa)</td>
<td>267</td>
<td>28.0</td>
</tr>
<tr>
<td></td>
<td>$K_1$ (MPa)</td>
<td>11410</td>
<td>1342</td>
</tr>
<tr>
<td>2nd</td>
<td>$E_0$ (MPa)</td>
<td>38020</td>
<td>16590</td>
</tr>
<tr>
<td>Layer</td>
<td>$E_1$ (MPa)</td>
<td>416</td>
<td>137</td>
</tr>
<tr>
<td></td>
<td>$K_1$ (MPa d)</td>
<td>12330</td>
<td>2830</td>
</tr>
</tbody>
</table>

of all these random factors. So, a group of model parameters should be considered as random variables. Monte-Carlo simulation method is used to correlate sampling and calculating the deformation distribution regularity.

Monte-Carlo method is known as statistical examination method. It is a method of using random variables, which can be used to calculate deformation distribution regularity.

After 1000 times sampling and calculating based on the above mentioned method, the ten year's deformation distribution regularity for the typical site is calculated and shown in table 2.

![FIG.6 Curves showing the variation of deformation and water level in typical site.](image)

<table>
<thead>
<tr>
<th>Depth of layer (m)</th>
<th>Deformation distribution</th>
<th>Deformation ranges when confidence interval is 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 - 19.50</td>
<td>N [-8.00, 5.38]</td>
<td>[-10.97, -5.03]</td>
</tr>
<tr>
<td>19.50 - 42.30</td>
<td>N [ 4.5, 2.50]</td>
<td>[ 2.58, 6.50]</td>
</tr>
</tbody>
</table>

CONCLUSION

The three-elements model can be used to simulate the deformation regularity concerning about clay creep of shallow layer in Shanghai and optimization method can be employed to select model and inversely calculate parameters by the data observed in the field for pore water pressure and deformation.

Under the present pumping and recharging conditions, the shallow layer will continuously subside with a rate of 0.5-1.1 mm per year. Under the constant amount of pumping and recharging conditions, to prolong the period of recharging and shorten the period of pumping will be greatly useful to reduce the subsidence of shallow layer.

REFERENCES


