Analysis of Shanghai Land Subsidence

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ABSTRACT A synthetic analysis is carried out by means of incorporating computations and laboratory testing with field measurements. The field observations can be regarded as a large-scale in-situ test. Laboratory testing is used to study the mechanism of subsidence and the soil model as well as to obtain the soil parameters. In the computational analysis the actual boundary water level is resolved into several components and then the one-dimensional consolidation equation is solved in each case to obtain appropriate expressions for excess pore-water pressure and deformation. The computational results are comparable with the in-situ data. The results of analysis provide a scientific basis for determining the effective control measures and indicate the directions of future work.

NOTATION

A, B - Fourier coefficients
Cv - average coefficient of consolidation during cyclic loading
Cvc - coefficient of consolidation
Cvs - coefficient of rebound
e0 - initial void ratio
G - specific gravity
H - thickness of soil layer
H1, H2 - depths of upper & lower boundaries of the compressible layer
h2 - boundary water-level drawdown
K - hydraulic conductivity
mv - coefficient of volume change
mvc - coefficient of volume compression
mvs - coefficient of volume rebound
(mvc)f - coefficient of compression from field data
(mvs)f - coefficient of volume rebound from field data
N - number of cycles
n - integral number
OCR - overconsolidation ratio
P - amplitude of cyclic water pressure
S - deformation of soil layer
T - time factor
t - time
t0 - equivalent time
INTRODUCTION

Land subsidence in Shanghai started in 1921. A dish-like depression has been forming continuously with increase of pumpage of underground water. The maximum accumulated subsidence had reached 2.63 m during 1921-1965, and the maximum subsiding rate - 200 mm per year. Since 1963, when pumping water had been reduced, the subsiding rate has decreased, and the average subsidence in 1965 was 23 mm. In 1966 the control measures like artificial water recharging into the aquifers was adopted, since then some rebound of the ground surface took place during 1966-1971. However, since 1972, in spite of the fact that the water recharging exceeds out-pumping, minor subsidence has been observed again. The minor subsidence can especially be demonstrated by the cumulative deformation curves of the compressible layers, which are shown in Fig. 1.

In order to study the land subsidence, a large amount of stratification benchmarks, observation wells and piezometers were set up, and many geotechnical boreholes were drilled by the Shanghai Geological Department (Su, 1979). Data from field observations serve as the basis for this analysis. Furthermore, analysis of the laboratory tests under the stress state simulating the field conditions provides the soil model and parameters for computations. Finally, through the computational analysis the theoretical and observed data of pore pressure and deformation are compared, and the validity of the computation method is warranted. Thus, the analysis provides a scientific basis for predicting subsidence and determining effective control measures.
ANALYSIS OF FIELD OBSERVATION DATA

Aquifers and aquitards

In Shanghai overlying the bedrock are sediments of Quaternary system with thickness of 300 m, within which one phreatic and five artesian aquifers are involved. The phreatic water surface is at +2m. Before the control measures were taken, underground water was pumped mainly from the II and III aquifers. The hydraulic gradient of the water level depression is very small, generally about 1/1000, and never exceeds 1/160. A typical curve of water-level changes in aquifer II is shown in Fig. 2.

![Figure 2: Water level fluctuation in aquifer II.](image)

The stratification benchmarks show that subsidence takes place mainly in three compressible layers above aquifer II, with a thickness of 70 m. The vertical profile of the soil layers is shown in Fig. 3. The geotechnical indexes of the three compressible layers are given in Table 1. Thicknesses of each compressible layer are less than 20 or 30 m, which is less than 2/1000 of the width of the overall pumping area. Therefore, seepage flow and deformation may be assumed to occur mainly in a vertical direction.

The distribution of soil layers in the whole urban area is not uniform. According to the different combination of soil layers, four engineering geological zones may be designated (see Fig. 4). Within each zone soil distribution and water-level changes in the aquifers are relatively uniform. But subsidences within zones III and IV are somewhat larger than that in zones I and II, and minor subsidence has been continuing in III and IV even after adopting the control measures. Thus, analysis of the zones III and IV is emphasized herein.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Geotechnical indexes of three compressible layers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer No.</td>
<td>W (%)</td>
</tr>
<tr>
<td>I</td>
<td>45-58</td>
</tr>
<tr>
<td>II</td>
<td>32-42</td>
</tr>
<tr>
<td>III</td>
<td>28-35</td>
</tr>
</tbody>
</table>
Piezometric data

The mono-tubing piezometers of open type are used to adopt the long-term measurement and the prevailing negative pore-water pressures. Pore-pressure fluctuation curves at various depths are recorded. The fact that pore-pressure gradients in the compressible layers caused by the water-level fluctuations in the aquifers continue to exist implies that primary consolidation is continuing to the present.

Soil parameters from field data

Since the total overburden pressure of the soil-water system remains constant, if negative excess pore-pressure increases, the effective stress in the soil skeleton increases, in a like manner that tends to result in compression of the soil layer and vice versa.

The relationship between the pore pressure change and soil deformation is:

\[ S = m_v \int_{H_1}^{H_2} \Delta udz \]  

(1)

By differentiating the changing direction of the pore pressure \( m_{vc} \) and \( m_{vs} \) can be obtained. The average values of \((m_{vc})_f\) and \((m_{vs})_f\) are listed in Table 2.

ANALYSIS OF THE LABORATORY TESTS

Consolidation tests under pumping

To study the consolidation process under different loading conditions, consolidation tests with pumping at the lower boundary and the conventional consolidation tests with double drainage are compared (Gu, 1965). It is shown
that the theoretical expressions for degree of consolidation are the same for both cases:

\[ U = 1 - \sum_{n=0}^{\infty} \frac{8\exp[-(n+1/2)^2\pi^2T]}{(2n+1)^2\pi^2} \]  

(2)

Similar results of the primary consolidation are also obtained from the experiments. It is concluded therefore that in the land subsidence studies soil parameters may be obtained by following the routine consolidation test procedures.

To determine method of analyzing complicated water-level fluctuating conditions, tests with different water heads at top and bottom of a sample are conducted so as to set up a trapezoidal distribution of effective stress increments within the sample (Tsien, 1968). The deformation-time curves obtained are comparable to the ones obtained by superposing a rectangular and a triangular effective stress increment pattern, indicating the validity of the superposition theorem in this analysis.

### Cyclic consolidation tests

The yearly water-level fluctuation data as demonstrated in Fig. 2 stress the importance of cyclic consolidation tests with low frequencies (Fu & Lou, 1968; Gu et al, 1988). Since load increment ratio has long been known to have an obvious effect on the consolidation behavior, the loading and unloading values are selected in conformity with the actual conditions.

Lowering of boundary water-level implies loading, so coefficient of consolidation \( C_{vc} \) is substituted in place of \( C_v \) in the one-dimensional consolidation equation:

\[ \frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} + \frac{\partial \sigma^*}{\partial t} \]  

(3)

Likewise, rising of boundary water-level means unloading, and \( C_{vs} \) is substituted instead. Fig. 5 shows the cyclic loading test results. On the abscissa \( N \), odd and even numbers represent loading and unloading respectively. \( C_{vs} \) is larger than \( C_{vc} \) when \( N \) is not too large, but then they become close to each other. In both cases of unloading or during repeated loading after a certain number of cycles the consolidation process completes rapidly and no secondary consolidation is observed. Fig. 5 also shows that \( m_{vc} \) is always larger than \( m_{vs} \) in one cycle. With increase of \( N \) \( m_{vc} \) decreases, but \( m_{vs} \) changes moderately. Thus \( (m_{vc} - m_{vs}) \) declines exponentially. Therefore,
for the condition of no appreciable secondary consolidation, the soil may be regarded as a nonlinear elasto-plastic medium which tends to behave elastically as the number of cycles increases.

COMPUTATION ANALYSIS

Computation method

As above mentioned, land subsidence caused by withdrawal of ground water may be considered as a one-dimensional consolidation problem as described by equation (3). Prior to pumping, excess pore pressure equals zero, \( u(z,0) = 0 \). The boundary conditions are: i) \( u(0,t) = 0 \), i.e. the water level at the upper boundary remains unchanged; ii) \( u(H,t) = g(t) \) where \( g(t) \) represents the water-level change of aquifer II. By using the superposition theorem it is possible to resolve the actual boundary water-level fluctuation into several simple components, namely: fixed drawdown, linear rising, and cyclic change (Tsien & Gu, 1981), in each of which only linear behavior is considered. By solving equation (3) separately, excess pore pressure for different boundary components are obtained:

**Fixed drawdown:**

\[
\begin{align*}
\dot{u}_1 (z,t) &= -\gamma w h_2 \left[ \frac{2}{H} \sum_{n=1}^{\infty} (\cos(n\pi/H) - 1)^n \sin(n\pi H) \exp\left[-\left(1-\left(\frac{n\pi}{H}\right)^2\right)^{2C_v H^2/T^2}\right] \right] \\
&= \alpha \left[ \frac{z}{H} - \frac{z}{H} \left[ 1 - \left(\frac{z}{H}\right)^2 \right] \right] + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \sin \left( n\pi z / H \right) \times \exp\left[-\left( n\pi / H \right)^2 \cdot C_v / H^2 \right] \\
&\quad \times \frac{(n\pi)^3 C_v / H^2}{(n\pi)^3 C_v / H^2} \\
&= \alpha \left[ \frac{z}{H} - \frac{z}{H} \left[ 1 - \left(\frac{z}{H}\right)^2 \right] \right] + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \sin \left( n\pi z / H \right) \times \exp\left[-\left( n\pi / H \right)^2 \cdot C_v / H^2 \right] \\
\end{align*}
\]

**Linear rising:**

\[
\begin{align*}
\dot{u}_2 (z,t) &= u_1 (z,t) \\
&= \alpha \left[ \frac{z}{H} - \frac{z}{H} \left[ 1 - \left(\frac{z}{H}\right)^2 \right] \right] + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \sin \left( n\pi z / H \right) \times \exp\left[-\left( n\pi / H \right)^2 \cdot C_v / H^2 \right] \\
&\quad \times \frac{(n\pi)^3 C_v / H^2}{(n\pi)^3 C_v / H^2} \\
&= \alpha \left[ \frac{z}{H} - \frac{z}{H} \left[ 1 - \left(\frac{z}{H}\right)^2 \right] \right] + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \sin \left( n\pi z / H \right) \times \exp\left[-\left( n\pi / H \right)^2 \cdot C_v / H^2 \right] \\
\end{align*}
\]

**Cyclic change:**

\[
\begin{align*}
\dot{u}_3 (z,t) &= P \exp \left[ - \sqrt{\frac{\omega}{2C_v}} (H-z) \right] \sin \left[ \omega t - \sqrt{\frac{\omega}{2C_v}} (H-z) \right] \\
&= P \exp \left[ - \sqrt{\frac{\omega}{2C_v}} (H-z) \right] \sin \left[ \omega t - \sqrt{\frac{\omega}{2C_v}} (H-z) \right] \\
\end{align*}
\]
Since many cycles have elapsed, only steady part of the analytical solution is considered here. An average value $C_V$ is used. For the actual water-level fluctuation Fourier transformation must be carried out, then equation (6) becomes:

$$u_3(z,t) = \sum_{n=0}^{\infty} A(n) \exp\left[-\sqrt{\frac{n\omega}{2C_V}}(H-z)\right] \sin[n\omega t - \sqrt{\frac{n\omega}{2C_V}}(H-z)] +$$

$$+ \sum_{n=0}^{\infty} B(n) \exp\left[-\sqrt{\frac{n\omega}{2C_V}}(H-z)\right] \cos[n\omega t - \sqrt{\frac{n\omega}{2C_V}}(H-z)]$$

Finally, total excess pore-water pressure $u = u_1 + u_2 + u_3$.

It should be noted that the water-level records started in 1964, which is far after the year consolidation process began in the soil strata. It may be considered that soil consolidation had been underway in an equivalent time $t_0$ (Tsien, 1965). Therefore, $t_0$ should be calculated beforehand according to the field data, then $t + t_0$ is used instead of $t$ in equation (4).

By substituting equations (4) and (5) into (1), expressions (8) and (9) for deformation under different boundary conditions are obtained as follows:

**Fixed drawdown component:**

$$S_1 = \frac{1}{2} m_v c \gamma w h_2 H_2 \left[1 - \left(\frac{H_1}{H_2}\right)^2\right] - 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{(n\pi)^2} \left[\cos n\pi - \cos\left(\frac{H_1}{H_2}\right)n\pi\right] \times \exp[-(n\pi)^2 C_v t/H_2^2]$$

**Linear rising component:**

$$S_2 = m_v s \alpha H_2 \left[\frac{1 - \left(\frac{H_2}{H_2}\right)^2}{2}\right] t - \left[1 - \left(\frac{H_1}{H_2}\right)^2\right]^2 + 2 \sum_{n=1}^{\infty} (-1)^n \times \frac{C_v \gamma w h_2}{\left[\cos n\pi - \cos\left(\frac{H_1}{H_2}\right)n\pi\right] C_v (n\pi)^4/H_2^2} \exp[-(n\pi)^2 C_v t/H_2^2]$$

**Cyclic change component:**

The stress-deformation relationship of the soil during one cycle is shown in Fig. 6. For an entire soil layer, the net deformation after one cycle $S_3'$ is (Tsien & Gu, 1981):

$$S_3' = -2P(2C_v/\omega)^{1/2}(m_v c - m_v s)$$

in which $(m_v c - m_v s)$ declines exponentially by a factor $\beta$. After taking the Fourier transformation, deformation due to cyclic changes can be written as:
\[ S_3 = -2 \sum_{n=0}^{\infty} \left[ A(n) + B(n) \right] \left( C_v/n\pi \right)^{1/2} (m_{vc}-m_{vs}) \beta \] (11)

For the case of using \((m_{vc})_f\) and \((m_{vs})_f\), which are the average parameters calculated from the field data in a specific time interval, \(\beta\) needs not be introduced. The expression for \(S_3\) becomes:

\[ S_3 = -2 \sum_{n=0}^{\infty} \left[ A(n) + B(n) \right] \left( C_v/n\pi \right)^{1/2} [(m_{vc})_f - (m_{vs})_f] \] (12)

**Selection of soil parameters**

After determining the computation method, the selection of soil parameters is the next important step to ensure correct results.

As described above, \(m_v\) and \(C_v\) values for different components are different. \(C_{vc}\) in equation (4) and \(m_{vc}\) in equation (8) and (11) are obtained from the conventional consolidation tests with load increment ratio = 1. The parameters in equations (5), (6), (9) and (11) are from the cyclic consolidation tests. \((m_{vc})_f\) and \((m_{vs})_f\) in equation (12) are from the field data.

Field observations can be regarded as large-scale in-situ tests, and laboratory testing can be used to study the mechanism of subsidence, the soil model as well as the relationships between various soil parameters. Therefore, the equivalent soil parameters obtained through the combination of laboratory testing and field observations can reflect the full range of soil layer behavior and the actual stress conditions (Tsien, 1981). The soil parameters obtained from the in-situ and laboratory testing together with the ones used in the computations are listed in Tables 2 to 4.

**Computation results**

Figs. 7 and 8 show the excess pore-water pressure and cumulative deformation curves at Bus Station No. 2 from zone III and at Laodong Park from zone IV respectively. It shows that the computation and the observed data are consistent. It indicated that the computation method based on primary consolidation is acceptable. Fig. 8 also shows the predictions of deformation to the year 2000 under the current water level condition.
Analysis of Shanghai land subsidence

FIG. 7 Computed and observed pore pressure curves.

FIG. 8 Computed and observed curves of accumulated deformation.

<table>
<thead>
<tr>
<th>Site</th>
<th>Compr. layer</th>
<th>$C_v \text{ (m}^2\text{/year)}$</th>
<th>$C_{vc}$</th>
<th>$C_{vs}$</th>
<th>$C_v$</th>
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<tbody>
<tr>
<td>Bus st. No. 2</td>
<td>I</td>
<td>3-8</td>
<td>4.7</td>
<td>30</td>
<td>22</td>
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<tr>
<td></td>
<td>II</td>
<td>16-100</td>
<td>60</td>
<td>148</td>
<td>150</td>
</tr>
<tr>
<td>Laodong Park</td>
<td>I</td>
<td>5-7</td>
<td>7</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>6-32</td>
<td>30</td>
<td>32-158</td>
<td>158</td>
</tr>
</tbody>
</table>
TABLE 4  Coefficients of volume change  (KPa⁻¹ x 10⁻⁵).

<table>
<thead>
<tr>
<th>Site</th>
<th>Compr. layer</th>
<th>Lab</th>
<th>Comput.</th>
<th>Components</th>
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<tr>
<td></td>
<td>mᵥc</td>
<td>mᵥs</td>
<td>S₁</td>
<td>S₂</td>
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<td>1</td>
</tr>
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<td>II</td>
<td>14</td>
<td>0.7</td>
<td>14</td>
</tr>
<tr>
<td>Laodong Park</td>
<td>I</td>
<td>35</td>
<td>3</td>
<td>18</td>
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<tr>
<td></td>
<td>III</td>
<td>7</td>
<td>0.8</td>
<td>4</td>
</tr>
</tbody>
</table>

FUTURE WORK

Secondary consolidation due to soil viscosity and readjustment of its skeleton structure under constant effective stress and the development of excess pore-water pressure and deformation characteristics under cyclic loading are an ongoing part of the authors' future investigations.

Trends of continuing small rates of subsidence reveal that recharging the aquifers is not a cure-all remedy. Determination of the most effective recharging program, including implementation time, depth, frequency and intensity, deserves careful consideration.

REFERENCES