WATER RESOURCES MANAGEMENT IN THE ZAMBEZI VALLEY: ANALYSIS OF THE KARIBA OPERATION

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ABSTRACT The authors applied multi-objective optimization techniques to analyze operation of the Lake Kariba hydropower scheme. The goal was to balance two conflicting objectives: first, maximization of hydropower production; and second, minimization of peak flood releases. A comparison of the test results with historical operating data shows that operating results can be significantly improved through adoption of optimization techniques and improved inflow forecasting. Further analysis can be done using the Interactive River System Simulation Program IRIS to assess the impact of the optimal policy on secondary objectives and verify robustness of developed policy.

INTRODUCTION

The literature dealing with various aspects of operation of reservoir systems is rich and extensive, but the majority of case studies analyzed is focused on well recognized and extensively used rivers flowing in the northern hemisphere: in Europe, the USA and Canada. Less recognized but no less important are rivers flowing in other parts of the world, for instance in Africa. One of the biggest African rivers is the Zambezi river (see Fig. 1) situated south of the Equator between 12° and 20°S latitude. It is the largest African river flowing into the Indian Ocean, about 2500 km from its source in the Central African Plateau (Balon & Coche; 1974, Balek, 1977). Its total catchment is about 1 300 000 km², and it is shared by eight countries, for some of which the river constitutes the main water resource. The total population within the river basin is about 20 million (UNEP, 1987).

Although the catchment possesses large development potential, the main water uses have been limited so far to the construction of three large hydroelectric schemes: Kariba and Cahora Bassa on the Zambezi itself, and Kafue Gorge - Itzhi-tezhi on the Kafue river, one of its main tributaries. Kariba and Kafue Gorge hydropower schemes provide more than 70 % of the electricity to the interconnected energy system of Zambia and Zimbabwe (Williams, 1984; ZESA, 1986) and therefore their efficient operation is very important for these two
countries. In recent years the Zambezi basin countries have taken political and organizational steps to improve development planning and management practices in the basin. The main initiative in this respect is the Zambezi River Action Plan (ZACPLAN) adopted in 1987 (see UNEP, 1987; Salewicz, 1990) and then followed by research undertaken by the International Institute for Applied Systems Analysis in Laxenburg, Austria (Salewicz & Loucks, 1988, Salewicz, Loucks & McDonald, 1989, 1990).

The research conducted at IIASA (see Salewicz, 1991, for an extensive overview) has addressed the broad scope of problems that confront the Zambezi basin countries, but the main interest of the study was the application of an Interactive River System Simulation Program called IRIS (IIASA 1990a, 1990b; Salewicz, Loucks & Gandolfi, 1990) to such a complex system of internationally shared water resources. IRIS could provide decision-makers in different countries responsible for the planning and management of water resources with a general and consistent framework for the analysis of proposed development plans and management strategies. A prototype of IRIS implementation is therefore being completed, containing a description of the main hydropower schemes in the region: Kariba; Kafue Gorge - Itezhi-tezhi, and Cahora Bassa.

The current focus is on the analysis of the operation of the first two schemes, Lake Kariba and Kafue Gorge - Itezhi-tezhi (Gandolfi & Salewicz, 1990). A multi-objective optimization analysis was carried out for each of them,
and a set of efficient operating rules was developed. Selected operating rules were then introduced into IRIS to allow for extensive simulations of the river system and for the analysis of the results from different points of view.

This paper reports on the analysis of the management of the Kariba hydropower scheme, the most important hydropower scheme in the region. Following a description of the Kariba hydropower scheme and its basic characteristics, formulations of the management problem are provided and followed by the analysis of optimization and simulation results.

LAKE KARIBA HYDROPOWER SCHEME

When Kariba filled to capacity in 1962, it was then the largest man-made lake in the world. Currently it is the fourth largest. At maximum retention level the lake covers an area of more than 5600 km\(^2\) and has an active storage exceeding 70 km\(^3\). Hydropower generating facilities consist of six 111 MW turbo generators located on the southern bank (Zimbabwe) and four 150 MW units on the northern bank (Zambia). The scheme is operated jointly by Zambia and Zimbabwe and generated electricity is equally shared by the two countries.

The catchment area upstream of Kariba Gorge is approximately 664 000 km\(^2\) (Santa Clara, 1988). Rainfall over the catchment is strongly seasonal. Normally the rainy season extends from November to March. The Zambezi has been gauged at Livingstone, near Victoria Falls, approximately 400 km upstream of Kariba Gorge, as early as of 1905. The river's peak discharge reaches Victoria Falls at the beginning of May. The discharge at the Falls returns to base flow in October or November. In January the flows begin to raise. Additional inflow to Lake Kariba comes from the Lower catchment located between Victoria Falls and Kariba. Almost 60% of the 9 . \(10^9\) m\(^3\) of average annual flow from the Lower catchment is concentrated between January and March (Santa Clara, 1988). The Kariba scheme is the main source of energy for the Zambian-Zimbabwean interconnected electricity supply system. Since the completion of the generating facilities in 1977, the scheme has supplied a monthly average of about 600 GWh, with an almost constant distribution throughout the year.

The central management problem of the Kariba operation is to balance two conflicting objectives: first, the need to maximize the hydropower output; and second, the need to have a safe capacity at the beginning of the rainy season to store peak flows and avoid peak discharges through the floodgates. Opening of the floodgates is extremely inconvenient for three reasons: first, from the power generating point of view, the rise of the tailwater level associated with the opening of one floodgate reduces the net head by about 5 m, and thereafter by 3 m for every additional gate opened; second, from the dam safety point of view, the vibrations caused by very high discharges through the floodgates should be avoided; and third, extremely large releases may endanger the population living downstream and create operational problems at the Cahora Bassa dam. The main objective of Lake Kariba management is to maintain the maximal fixed level of energy production. The second objective is to avoid very high discharges through the floodgates. There are other objectives, related to human activities and to environmental protection of wildlife areas downstream of the reservoir. They
seem, however, to be less important for the current management. Therefore they were not considered in the formulation of the optimization problem, although IRIS can analyze the effects of the system operation on such objectives.

FIRST FORMULATION OF THE MANAGEMENT PROBLEM

Given the characteristics of the Lake Kariba management problem described in the previous paragraph, a first analysis was conducted based on the following hypothesis:

(a) present firm energy target of 600 GWh/month is a rigid constraint that should not be violated;
(b) the operation of the reservoir is conducted according to simple operating rule shown in Fig. 2, that specifies the value of release \( r_t \) as a function of

\[
y_t = s_t + \hat{i}_t - \hat{v}_t
\]  

\( \text{(1)} \)

where \( s_t \) is a current amount of water stored in the reservoir and \( \hat{i}_t \) and \( \hat{v}_t \) denote forecasts of the inflow and evaporation losses respectively.

The operating rule is such, that until a certain value \( Y_t \) is reached, the release decision \( r_t \) is equal to the amount \( r_E \) necessary to generate a given value of monthly energy production target \( E \). When \( Y_t \) is exceeded, \( r_t \) increases steadily as a function of water available in the reservoir in order to avoid excessive reservoir filling, and consequently, to maintain a flood reserve needed to store peak flows and avoid floodgate discharges. The value of \( Y_t \) varies according to the risk of occurrence of flood inflows from the catchment, as indicated in Fig. 3, which was obtained based on an analysis of inflows to the reservoir and
historical operating policy (Gandolfi & Salewicz, 1990). Thus analyzed the operating rule is identified by three parameters $\alpha$, $Y_L$, $Y_U$ and is given as:

$$r_t = r(t, y_t, \alpha, Y_L, Y_U)$$

(2)

In order to formulate the management problem, let us consider now the reservoir mass balance equation

$$s_{t+1} = s_t + i_t - v_t - u_t$$

(3)

where:
- $i_t$ is the volume of inflow in month $t$;
- $v_t$ is the volume of evaporation in month $t$ and;
- $u_t$ is the volume of release in month $t$.

Evaporation from the reservoir can not be neglected in African climatic conditions and consequently it was assumed that evaporation is proportional to the reservoir surface $A$ which, within the operational range of storage values, can be approximated by a linear function of storage. The average value $A_t$ of the reservoir surface in month $t$ is thus given by

$$A_t = a - \frac{s_{t+1} + s_t}{2} + b$$

(4)

while the evaporation loss volume $v_t$ is given by

$$v_t = k_t \cdot A_t$$

(5)

The coefficients $k_t$ characterizing evaporation intensity are periodical with respect to $t$ and were calculated based on the available time series of evaporation from the reservoir (CAPC, 1978). Substituting eq. (5) and (4) for equation (3) we obtain the reservoir equation in the form:
\[ s_{t+1} = c_t \cdot s_t + c_t^2 \cdot (i_t - u_t) + c_t^3 \]  \hspace{1cm} (6)

where \(c_t^1\), \(c_t^2\), and \(c_t^3\) can be easily obtained for known \(a\), \(b\), and \(k\). A distinction must also be made between the release \(u_t\) in eq. (6) and the release \(r_t\) given by the operating rule (2). In fact, since the decision \(r_t\) is taken when the values of \(i_t\) and \(v_t\) are unknown, it can not be \textit{a priori} guaranteed that \(r_t\) is physically feasible. In order to secure physical feasibility of the release, its monthly volume \(u_t\) is given as

\[ u_t = \min \left( \max \left[ r_t; R(S_{\text{MAX}}, s_t, i_t); R(S_{\text{MIN}}, s_t, i_t) \right] \right) \]  \hspace{1cm} (7)

where \(S_{\text{MAX}}\) and \(S_{\text{MIN}}\) denote maximum and minimum value of the storage respectively, and

\[ R(X, s_t, i_t) = \frac{c_t^1}{c_t^3} \cdot s_t - \frac{1}{c_t^3} X + \frac{c_t^2}{c_t^3} + \frac{c_t^3}{c_t^3} \]  \hspace{1cm} (8)

gives the volume of the release that has to be made to obtain at the moment \(t+1\) storage \(s_{t+1} = X\), starting from the storage \(s_t\) and for given inflow \(i_t\). In the management problem analysis three cases of information pattern about future inflows to the reservoir were considered (see equation (1)). The reservoir operating policy was therefore given as:

1) policy \(r_t^1\) relating release to current storage \(s_t\) only:

\[ y_t = s_t \]  \hspace{1cm} (9)

2) policy \(r_t^2\) relating release to the estimated amount of water available \(y_t^2\) in period \([t, t+1]\) and given as the sum of storage \(s_t\), inflow \(i_t\) during this period of time minus estimated evaporation losses \(\hat{v}_t\) obtained from formulae (4) and (5) by assuming \(s_{t+1} = s_t\):

\[ y_t = s_t + i_t - \hat{v}_t \]  \hspace{1cm} (10)

3) Policy \(r_t^3\) in which release depends on the sum of storage \(s_t\) and inflow in the previous time interval \(i_{t-1}\) (inertial forecast) minus estimated evaporation losses \(\hat{v}_t\):

\[ y_t = s_t + i_{t-1} - \hat{v}_t \]  \hspace{1cm} (11)

In order to determine values of the parameters \(\alpha\), \(Y_L\), \(Y_U\), the following mathematical programming problem was then solved for each of the distinguished information patterns (9), (10), (11):

\[ \min \; \left[ F \right] \hspace{1cm} \text{(12)} \]

\[ (\alpha, Y_L, Y_U) \]

where:
\[ \max \left[ f_i \right] \quad 0 \leq t \leq T \]  

while \( T \) is the length of the optimization horizon and \( f_i \) is defined as:

\[ f_i = \begin{cases} r_i - r_E & \text{if } r_i > r_E \\ 0 & \text{if } r_i \leq r_E \end{cases} \]

Formulation of problem (12) is completed by the following constraints:
- the continuity equation (6);
- the operating rule (2);
- feasibility condition (7);
- the rule determining the division of the implemented release \( u_i \) between turbines and floodgates discharge (\( u_i^t \) and \( u_i^f \) respectively):

\[ u_i^q = \min \left[ u_i; M_i \right] \]

\[ u_i^f = \max \left[ 0; (u_i - M_i) \right] \]

where \( M_i \) is the volume discharged in month \( t \) if all the turbines are fully operating for 90% of the time;
- the equation relating the net head on the turbines to the storage and release;
- the constraint on the energy output:

\[ e_i \geq E \]

where \( E \) is the energy production target.

Solution of problem (12), if feasible, gives the operating rule that produces the lowest value of the peak floodgates discharge, maintaining in any time period an energy production of at least 600 GWh/month.

From a computational point of view the solution can be found by simulating the system operation over a sufficiently long period with a large number of different operating rules (i.e. triples \( \alpha, Y_L, Y_U \)). In the case at hand the simulations were carried out using the inflow data covering period from 1950 until 1984. A random search algorithm (Karnopp, 1963) in the space of \( (\alpha, Y_L, Y_U) \) was used to limit a region around the optimum, and then a deterministic method (Himmelblau, 1972) was applied for refinement of the results.

Analysis of Results

The management problem formulated in terms of the optimization task presented in the previous section has been solved for three types of information pattern (see eq. (9), (10) and (11) respectively). The maximum values of the reservoir releases and respective values of the optimal parameters (\( \alpha, Y_L, Y_U \)) are given as follows:

<table>
<thead>
<tr>
<th>Information Pattern</th>
<th>Max. release</th>
<th>( \alpha )</th>
<th>( Y_L )</th>
<th>( Y_U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No inflow forecast</td>
<td>13640</td>
<td>0.3</td>
<td>34070</td>
<td>47325</td>
</tr>
<tr>
<td>Perfect forecast</td>
<td>9480</td>
<td>0.1918</td>
<td>36738</td>
<td>47675</td>
</tr>
<tr>
<td>Inertial forecast</td>
<td>13320</td>
<td>0.1888</td>
<td>36340</td>
<td>48757</td>
</tr>
</tbody>
</table>
It should be noted that in all three cases of information pattern the problem could be solved, which means that a firm energy production of 600 GWh/month could be always guaranteed. The use of the inertial inflow forecast (information pattern (11)) did not improve performance of the reservoir. There is, however, a significant margin for improvement of the performance by using more precise information on future inflow, as demonstrated in a case of perfect inflow forecast. A 30% decrease of the floodgate discharge $F$ can in fact be gained given a perfect knowledge of the inflow one month ahead. Though this utopic situation will never apply in practice, the results demonstrate that it might be useful to implement an inflow forecast system for reservoir management purposes.

The number of months in which the floodgates had to be open between 1977 and 1984 was: 30, 32 and 33 respectively for information patterns (9), (10), (11). These numbers are bigger than those obtained in historical management over the same period of time, when floodgates were opened 22 times. However, the theoretical operating policy produced very significant reductions of the maximum floodgates discharges to: 5386, 4882 and 5300 $10^6$ m$^3$/month respectively for the three information patterns, versus 12 780 $10^6$ m$^3$/month of the peak floodgates discharge registered in reality. The operating rule discussed here was verified for its robustness and the reservoir operation was simulated using historical sequences of inflows to the reservoir in the period 1924-49. Again in this case the firm energy output of 600 GWh/month could be maintained, and the maximum monthly floodgate discharges computed for three information patterns were 6081, 5051 and 6602 $10^6$ m$^3$/month respectively.

SECOND FORMULATION OF THE OPTIMIZATION PROBLEM

Operating rules obtained from the solution of the optimization problem presented in the previous section were introduced into IRIS and then system operation was simulated over the period 1924-84. Results of these simulations demonstrated that, apart from a very few critical years, the reservoir storage remains well

![Fig. 4. Modified version of the operating rule.](image-url)
above its minimum value and huge amounts of water are lost through floodgates discharge, suggesting that a higher energy output could be achieved if a more flexible operating policy was adopted. In order to check this possibility the following approach was followed:

(a) A modified operating rule was proposed, as shown in Fig. 4. This rule differs from the first one in that the range of values between 0 and $Y_t$ is divided into two zones: in the lower one the release is fixed and equivalent to the release $r_E$ necessary to generate $E = 600 \text{ GWh/month}$, while in the upper zone an increased energy target $E' = 700 \text{ GWh/month}$ was set and the release $r_E$ varied accordingly. Assuming that (see Fig. 5):

$$Y'_t = \beta_t \cdot Y_L$$

$$Y_t = \beta_t \cdot Y_U,$$

i.e. that the limits of the zones vary according to the same simple law, it turns out that the new operating rule is identified by four parameters: $\alpha$, $\beta_L$, $Y_L$ and $Y_U$:

$$r_t = (t, y_t, \alpha, \beta_L, Y_L, Y_U)$$

(b) A second objective was introduced to express the desire to achieve the highest possible energy output in the form:

$$\min \left[ N \right]$$

$$(\alpha, \beta_L, Y_L, Y_U)$$

where:

$$N = \sum_{t=1}^{T} n_t$$

\[ (17) \]

\[ (18) \]

\[ (19) \]

\[ (20) \]

\[ (21) \]
when $T$ is the length of optimization horizon and $n_i$ is defined as:

$$n_i = \begin{cases} 
1 & \text{if } e_i \leq \frac{E_1 + E_2}{2} \\
0 & \text{if } e_i > \frac{E_1 + E_2}{2}
\end{cases} \quad (22)$$

Thus the new formulation of the management problem was given as:

$$\min_{(\alpha, \beta, Y_L, X_0)} [F, N] \quad (23)$$

subject to operating rule (19) and to the same set of constraints as in problem (12). The solution of problem (23) is not unique. The set of efficient (in the Pareto sense) solutions was determined using the method of weights and the same optimization algorithm as in case (12). Respective formulation of the optimization problem was solved for the three cases of information pattern (see (9), (10) and (11)).

**Analysis of the Results**

Efficient solutions shown in the space of two objectives $F$ and $N$ are shown in Fig. 6. The curves representing the cases with no information about inflows (A) and inertial inflow forecast (C), (see (9) and (11) respectively) almost overlap each other: thus it confirms that the inertial inflow forecast does not add any valuable information to the management problem. Curve A in Fig. 6 shows, however, that very interesting results can be obtained for the case representing no inflow forecast. Considering for example point $P_1$ ($\alpha = 1.158; \beta_L = 0.654$;
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\[ Y_L = 6640; \quad Y_U = 69251 \] it can be seen that energy output of 700 GWh/month or more could be achieved for 81.2 % of the time over the period of 35 years; that energy output lower than 650 GWh/month occurred only 8 times ( 1.9 % of the time!); and that the minimum monthly energy production did not fall below 600 GWh/month. At the same time the maximum release obtained during the period 1977-84 was lower (7906 \times 10^6 \, m^3) than the maximum one registered historically. The floodgates had to be opened 23 times in the same period of time versus 30 in the previously analyzed case. Curve B in Fig. 6 shows efficient solutions obtained assuming perfect information about inflow. Simulation results demonstrate that the improvement obtained in the utopian conditions assuming perfect knowledge about future inflow can be significant. Point \( P_2 ( \alpha = 0.443; \beta = 0.694; \gamma_L = 10484.1; \gamma_U = 70630.3 ) \) for example, gives a 58 % reduction of the indicator N and a 23 % reduction of the maximum flood release maintaining energy production at 700 GWh/month or more 81.2 % of the time over 35 years. At the same time energy production lower than 650 GWh/month during the period 1977-1984 occurred 3 times ( 0.7 % of the time !); the maximum floodgates discharge was 6109 \times 10^6 \, m^3/month and floodgates were open 24 times.

DISCUSSION AND CONCLUSIONS

As Dávid (1987) pointed out, the problem of finding solutions to the conflicts generated in the river basins by water projects and water related activities must be considered at a basinwide scale, especially in the case of international water systems. The assessment of the extent to which the search for such solutions can be supported by computer based systems was the strategic objective of a research conducted at IIASA (see Salewicz, 1991). The Zambezi system was selected as one of the case studies. In fact, the river still offers great potential for development that asks for effective development planning and management practices. This paper presented the results of the analysis of the management of the Kariba scheme that constitutes an essential element of the present and future configuration of the Zambezi water system, and was therefore one of the main topics of the IIASA study. Two basic conclusions can be drawn:

(a) a highly reliable firm energy output can be maintained and at the same time relatively low values of peak discharges through the floodgates can be obtained. The analysis shows possible tradeoffs between the two objectives, that is lower energy output versus peak discharge reduction;

(b) a significant improvement in the system performance can be obtained if hydrological knowledge is improved; based on the results obtained, a careful evaluation of the technical and economical feasibility of implementing an inflow forecasting system for management purposes is highly desirable.

The results obtained for Lake Kariba scheme and other facilities in the region are being introduced into the IRIS package to allow for further extensive simulations of the system operation under different hydrological scenarios in order to validate the results of the studies and the effectiveness of whole approach.
Finally, we would like to stress importance of bringing the outcomes of investigations like the ones presented in this paper, to the attention of those who are responsible for water resources management in shared international rivers in order to improve efficiency of the management. Unfortunately such interactions are rare and future investigations dealing with problems of international rivers management should be more closely associated with managers and decision makers.

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