

A physically based approach to modelling distributed snowmelt in a small alpine catchment

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ABSTRACT A distributed approach has been used to investigate the influence of topography on snowmelt processes for a small basin located in an open site, in the Eastern Italian Alps. Based on hourly records of shortwave radiation, the distributed fluxes of direct and diffused radiation are considered. Snow albedo variations in time and space are simulated, by using semi-empirical formulae. Atmospheric emissivity is estimated by means of Satterlund's equation; the turbulent exchange components are evaluated in the light of the classical mixing-length theory, and the computations have been carried out by processing hourly records of air temperature, relative humidity, and wind speed. A continuous simulation covering five years shows the contribution of net shortwave flux to be dominant, and the concomitant spatial variability of snowpack melt to be not negligible. Although remote sensing techniques were not used directly to provide data input for continuous simulation, the possibility has been investigated to simulate distributed snow albedo from radiance levels measured by the Landsat 5-MSS sensor. The spatially-distributed framework could be utilized to improve the performance of simpler degree-day models.

INTRODUCTION

Recent trends of hydrological research pointed out the capability of the distributed approach to account for the spatial variability of input (e.g. rainfall) and state (e.g. soil moisture) variables of a basin, and to describe the effects of such variability on the basin response (e.g. streamflow). In the case of snowmelt processes, such a distributed approach is seldom embedded in the mathematical models [World Meteorological Organization, 1987], except for altitude zones subdivision: this is due to the simplicity of the model structure, and data requirements. Nevertheless, many physical issues indicate that a distributed approach should be "a fortiori" required in snowmelt modelling, because the spatial variability of the snowpack, and related snowmelt can be pronounced, at least for some particular topography. In fact, at mid-latitudes shortwave fluxes play a basic role in the surface heat exchange balance during the snowmelt season, because of the high solar altitude angles. Moreover, alpine areas are subject to a pronounced shadowing effect due to relief and basin geomorphology. This can yield large fluctuations in the spatial distribution of net shortwave fluxes, which control the dynamics of snowpack water equivalent.

A simplified model of solar radiation is developed here for the purpose, and a quite large set of albedo records is analysed by using semi-empirical formulae. In addition, a 5 years record of meteorological variables, needed to solve the energy balance equation, is used. It is then possible to compare the relative importance of different components of the energy balance, showing the major role played, on average, by the net shortwave flux.

The mathematical model used to perform this analysis is a physically based model describing the distributed process involved in snowmelt and related runoff production at the basin scale, and will be referred to as PDSM in the following. Spatial variability of state and

input variables is simulated by means of raster grids, which easily allow for the linkage of the hydrological model with Digital Elevation Models (DEM) and remotely sensed data of land-use and other hydrological parameters. The possibility to use albedo estimates from radiance levels of snow surface, as measured from the Landsat 5-MSS sensor, has also been successfully tested.

The field area under exam is a small alpine creek, the major features of which are briefly reported in the following section. The third section of this paper deals with the effects of topography on solar radiation and it also briefly reports on the radiation model. Albedo variability in space and time is examined in the fourth section. Other heat-exchange components are briefly described in the fifth section; snowpack dynamics and melt-runoff transfer functions are only mentioned there.

FIELD AREA

The field area (see Fig. 1) is a small basin in the upper part of the Cordevole river, located in the Eastern Dolomites (North-Eastern Alps of Italy). The catchment area is 6.9 km². Elevation ranges from 1 800 m above sea level to 3 152 m, with an average altitude of 2 256 m, and the average basin slope is 23°. The main stream, the average slope of which is about 15°, is oriented from West to East. The latitude of the basin outlet, where a streamflow gage is located, is about 46° 29' N, and the longitude is about 11°51' E. The major rock types are carbonatic in the northern part of the basin, and conglomerates derived by eruptive rocks in the southern one. Vegetation cover is poor and mainly represented by shrubs, and pastures, while the area covered by forest is negligible. The discharge measurements at the basin outlet (Vizza station) are performed by a sharp-crested weir with continuous digital recording equipment. Two stations provide continuous measurements of meteorological variables, such as precipitation, air moisture and temperature, wind speed and direction, and global incoming shortwave radiation. These are located near Passo Pordoi and Arabba. The first gage is located in the middle of the basin at an altitude of 2 142 m above sea level; the second one (Arabba, 1 630 m in altitude) is close to Vizza station, at the basin outlet. A five years hourly data record is available for the period 1984-1988 and measurements taken at Arabba are used to complement data missing at Pordoi. The monitoring system is operated by CSVDI (Centro Sperimentale Valanghe e Difesa Idrogeologica), and data preprocessing, storage and retrieval is provided by CSIM (Centro Sperimentale per l'Idrologia e la Meteorologia). Two snow-gauge stations provide measurements of snow albedo, and snowpack height, density, temperature and grain size.

Basin topography is simulated by means of a DEM, which has been obtained by processing IGM (Istituto Geografico Militare) maps with scale 1:25 000. The basin is described by means of 140 rectangular grids of size 210x230 m².

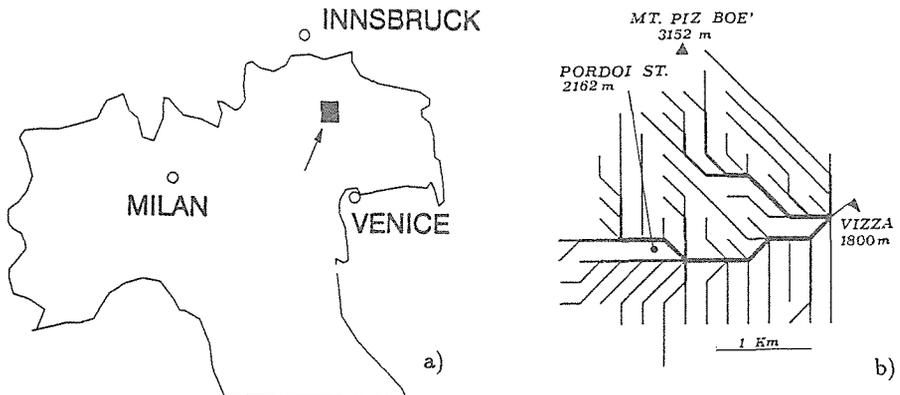


FIG. 1 Location of Cordevole basin (a) and the automatically derived stream network of Upper Cordevole basin (b).

SOLAR RADIATION

The global incident shortwave radiative flux, G , transmitted by the atmosphere, varies with latitude, season, time of the day, atmospheric conditions, and topography. Irradiance is given by a direct component, I , and a diffused component, D ; this latter accounts for 36 ÷ 51% of the total radiation in spring and for 24 ÷ 41% in summer, at latitudes between 40° and 60° [Obled and Harder, 1979]. The relative incidence of diffused and direct radiation is controlled by all the above mentioned factors; the present analysis addresses the role topography plays in determining incident direct radiation.

Sometimes "topographic correction factors" are introduced to improve the performance of degree-day methods used for snowmelt simulations. Ghilardi et al. (1988), for instance, introduced an empirical correction factor as an extra parameter to account for topographic effects when simulating snowmelt in three alpine watersheds by means of a modified version of the TANK model (Sugawara et al. , 1974) . However, the physical grounds of such corrections are not always evident. Charbonneau et al. (1981) applied a similar parametric correction to the Durance and HSP model they developed, and they finally argued that integrating over a varied topography, and over a sufficiently large time scale should produce a smoothing effect on the interactions between different radiation components. However, it is unclear which spatial and temporal scales of discretization are able to efficiently describe this effect.

In the following a distributed model of solar radiation is presented, which is designed to investigate the spatial variability of net shortwave radiative fluxes, and to introduce the physical grounds for possible further simplifications.

Solar radiation under clear-sky condition

An exhaustive description of radiative exchange under clear-sky condition is reported by Dozier (1980). Either the spectral model presented there, or the global radiation model further proposed by Munro and Young (1982), provide powerful tools for the purpose; however, these models require several parameters describing atmospheric attenuation to be estimated, and further correction factors to be fixed or calibrated. Therefore, a model formulation which is considerably simpler than these physical models has been adopted here in order to reduce the number of parameters and input data required.

The equations describing the apparent motion of the sun around the earth are solved in order to derive daily values of the exoatmospheric parallel beam flux I_0 (also known as "solar

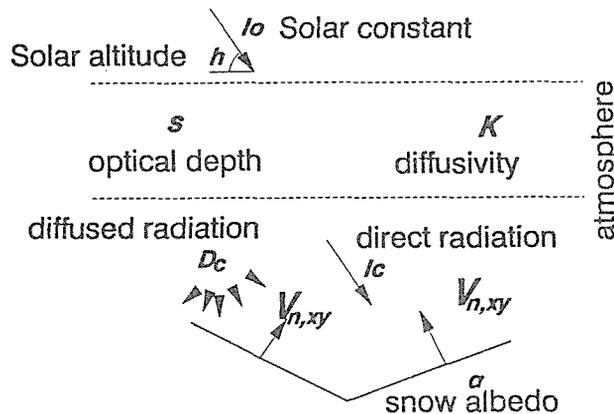


FIG. 2 Scheme of solar radiation model under clear-sky condition.

constant", which varies due to the elliptic motion of the earth around the sun) and hourly values of the solar altitude angle $h(t)$ and azimuth $\psi(t)$. Under clear-sky condition, direct radiation I_c at the Earth's surface on a horizontal plane (see Fig. 2) is computed by:

$$I_c = I_o \exp[-s/\sin(h)] \sin(h) \quad (1)$$

with s denoting the (dimensionless) optical depth of the atmosphere: its value, measured without optical filters at an altitude of 2300 m, is about 0.094.

Diffused radiation under clear-sky condition, D_c , is evaluated from:

$$D_c = K I_o \sin(h) \{ 1 - \exp[-s/\sin(h)] \} \quad (2)$$

where K , a dimensionless diffusion parameter, takes values in the range between 0.2 and 0.6. It represents the percentage of the radiation that is scattered out of the beam, toward the Earth, without being absorbed from the atmosphere's molecules: it depends on atmospheric turbidity.

Although some authors (see, e.g. Dubayah et al. 1990) proposed to introduce a 'configuration factor' in order to account for the radiation reflected by surrounding terrain, this has been neglected in the present analysis because of the expensive computer time needed for achieving a reliable simulation; under this simplification, the sensor located on a horizontal surface should theoretically measure a value of global shortwave radiation equal to:

$$R_c = I_c + D_c \quad (3)$$

Cloud cover

The influence of cloud cover on both shortwave and longwave radiation fluxes at the snow surface level has been widely studied. As related to the solar radiation flux, cloudiness yields both absorption and enhanced diffusion of incoming radiation, G , and an increase of snow albedo α , because of the augmented relative incidence of the diffused component of G . In addition, net longwave radiation also increases with increasing cloudiness. An index of cloudiness should therefore be evaluated or measured. Although the values of average cloudiness conditions are available for some basins (as in the present case study), these are not used as an input to the PDSM, at least at the present stage of development; we preferred to estimate a cloudiness index, d_i , from the hourly records of measured incoming shortwave radiation, R_m , and from the theoretical value of R_c given by (3). This procedure assumes a reference value of 0.22 for t_{clo} , the transmissivity coefficient for 8/8 stratocumulus cloud cover (Male & Granger, 1981); when the measured radiation, R_m , is less than the lower threshold corresponding to the minimum radiation needed to be transmitted by clouds, $t_{clo} R_c$, the cloudiness index takes its maximum value of $d_i=1$ and radiation is thus completely diffused. Otherwise a linear relationship is inferred in the form:

$$d_i = (R_c - R_m) / [(1-t_{clo}) R_c] \quad (4)$$

where a minimum value of $d_i=0$ is assumed. Accordingly, cloudiness c_i is evaluated from:

$$c_i = \text{MIN} \{ 1, \text{MAX}[d_i, 0] \} \quad (5)$$

and the resulting hourly values of d_i are taken as an index of diffused radiation. The relative incidence of the diffused component R_d , and the direct beam component, R_b , of the measured solar radiation, can be then computed by means of a linear weighting of D_c and R_m , that is:

$$R_d = (1-c_i) D_c + c_i R_m \quad (6)$$

$$R_b = R_m - R_d \quad (7)$$

Using a DEM to evaluate spatial variability of direct radiation

For operational purposes it is reasonable to assume the diffused component R_d to be isotropic, and uniformly distributed over the basin (Obled & Harder, 1979) ; on the contrary, slope and aspect affect substantially the net incident flux of direct beam radiation. Dubayah et al. (1990) recently derived the mean \bar{F}_s , and the variance σ_F^2 of the direct flux over slopes with a fixed angle β , and azimuths distributed uniformly for any direction, under clear sky conditions. These are given by:

$$\bar{F}_s = I_0 \exp[-s/\sin(h)] \cos(\beta) \sin(h) \quad (8)$$

$$\sigma_F^2 = \frac{1}{2} I_0^2 \exp[-2s/\sin(h)] \sin^2(\beta) \cos^2(h) \quad (9)$$

and they further showed that the variance has a maximum for the value of h which satisfies:

$$\frac{\sin^3(h)}{\cos^2(h)} - s = 0 \quad (10)$$

A similar approach has been followed here, further extending the simulations also to partial cloud-cover conditions. For any grid point (x,y) of the basin we computed the local normal vector to the terrain surface, $\vec{V}_{n,xy}$, by using Sobel's algorithm (see Gonzalez & Ritz, 1987); because solar altitude $h(t)$ and azimuth $\psi(t)$ at time t are already known, the sunbeam vector \vec{V}_{R_b} can be derived straightforwardly from:

$$\vec{V}_{R_b} = [R_b/\sin(h)] \{v_x, v_y, v_z\} = [R_b/\sin(h)] \{\cos(h)\sin(\psi-\pi/2), \cos(h)\cos(\psi-\pi/2), \sin(h)\} \quad (11)$$

and its norm results to be $R_b/\sin(h)$. For each grid point, it is then verified whether the sun is obscured by shadowing from the slope itself or from adjacent ridges: in this case radiation is assumed diffused and equal to R_d . Any unobscured hillslope in the catchment then receives a global shortwave flux, $R_{g,xy}$, which varies in space; this is computed as:

$$R_{g,xy} = R_d + \vec{V}_{n,xy} \bullet \vec{V}_{R_b} \quad (12)$$

with \bullet denoting the scalar product .

TEMPORAL AND SPATIAL VARIABILITY OF ALBEDO

Albedo is a fundamental parameter in the analysis of the earth's energy balance, and it needs to be carefully estimated, especially when shortwave fluxes are dominant; a minor error (e.g. $\pm 10\%$) affecting its estimation can significantly be amplified (i.e. $\mp 40\%$) when net shortwave flux is computed. Albedo is strongly influenced by the physical properties of incoming radiation (wavelength, angle of incidence, presence of diffused radiation), by snow crystals metamorphism, and morphology, and by bulk properties of the snowpack as a whole. Physically based models of spectral reflectivity of snow were proposed (see e.g. Choudhury, 1979); these models generally require a deep knowledge of physics of incident radiation, and snow (e.g. density, grain size, and shape), which is seldom available. The model of albedo proposed here is based on a large set of albedo measurements recorded from two automatic stations, located not far from the area under investigation; unfortunately these measurements were not continuous in time, and they could not be used as input to the snowmelt model, apart for some testing periods. Some of the albedo records have been therefore used to calibrate three semi-empirical formulas, and the performance of these formulas has been then tested against the remaining data.

First it has been observed that the albedo of freshly fallen snow under clear-sky conditions

and high solar elevation angles, A_{cs} , decreased from winter to spring. This decay is related to increasing snow humidity, and it results in the following formula fitted to the available data:

$$A_{cs}(t_{aws})=A_{sp}-A_0 \arctan[A_1(t_{aws}-t_{sp})] \tag{13}$$

where t_{aws} denotes the number of days after the winter solstice; A_{sp} , A_0 , A_1 in (13) are parameters accounting for the possible range of albedo variability, and t_{sp} is a time location parameter.

An exponential decay describing snow metamorphism, as related to an index-variable, was already proposed in "Snow Hydrology", the pioneering study by the U.S. Army Corps of Engineers (1956). A re-formulation of that approach has been used here to determine albedo under clear-sky condition, A_{mel} , as a function of a temperature-index T_{md} ; this is taken as the sum of maximum daily temperatures above -10°C , after the last snowfall. These temperatures are rescaled with respect to -10°C and are computed for each grid point of the DEM, in order to account for variability of albedo with altitude. Two additional parameters, A_{M0} and C_{AM} , are also required to describe the ranges of possible values taken by A_{mel} , which is given by:

$$A_{mel}(T_{md}) = (A_{cs}-A_{M0}) \exp(-C_{AM} T_{md}) + A_{M0} \tag{14}$$

Cloud cover, and low angle of incidence of radiation result in higher values of albedo; both these factors are related to the major role played by diffused radiation under these conditions. It is therefore reasonable to assume the index of diffused radiation (and cloudiness), d_i , as a meaningful variable to explain the albedo changes from that occurring under clear-sky condition. Petzold (1977) suggested to use a cloud-amount index, N , in order to explain the increase of albedo with cloudiness. We found that Petzold's formula could be generalized, accounting for diffused radiation due to both cloudiness, shadows and low angle of incidence of solar radiation. The formula was slightly modified, assuming that the index d_i , and not only a cloud amount one, could be successfully (see Fig. 3) applied in it, resulting in the third albedo-equation:

$$A_{dif} = \text{MIN}[1, A_{mel} (1.00449+0.097 d_i^3)] \tag{15}$$

The equations presented here provide the framework of net shortwave radiation modelling.

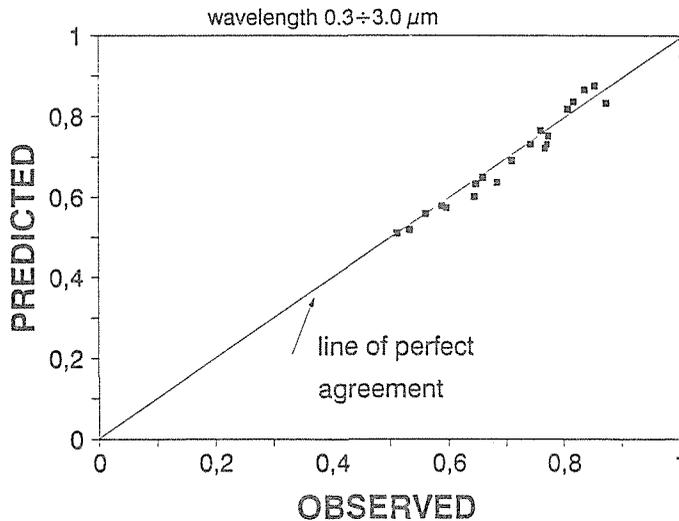


FIG.3 Influence of diffused radiation over albedo: observed vs. predicted values using the modified Petzold formula (15) (from CSVDI data).

HEAT EXCHANGE BALANCE EQUATION

The previous results are significant but incomplete. In fact the importance of systematic errors introduced due to neglecting the spatial distribution of shortwave fluxes also depends on the relative importance of these fluxes as compared to the other components of the heat exchange. Obviously, when the radiant fluxes are less important than the turbulent ones, the complexity required by the distributed approach to radiation may not be justified. The energy balance for the snowpack at a point is given by:

$$H_m + H_s = S_{io} + L_{io} + H_c + H_l + H_p + H_g \quad (16)$$

H_m = Energy available for snowmelt
 H_s = Change in the internal energy of the snowpack
 S_{io} = Net Shortwaves radiation
 L_{io} = Net Longwaves radiation
 H_c = Sensible heat exchange
 H_l = Latent heat exchange
 H_p = Heat content of precipitation
 H_g = Heat exchange at snow-ground interface

and the role of different components must be investigated. Because we are focusing our attention on the effect of spatial distribution of shortwave fluxes, we just mention that L_{io} is computed by means of Satterlund's (1979) equation for atmospheric emissivity ϵ_a , that is:

$$\epsilon_a = 1.08 \{ 1 - \exp[-e_a^{(273.14 + T_a)/2016}] \} \quad (17)$$

where e_a is screen vapor pressure in millibars and T_a is screen air temperature in °C. This formulation was chosen because it yields satisfactory results at low temperatures. Snow is assumed to have a value of 0.99 for emissivity in the range of longwaves; the snow temperature is computed by solving equation (16) with an explicit finite difference scheme.

Turbulent fluxes H_c and H_l are computed by means of the classical formulas based on the mixing-length theory; the available measurements provide hourly records of air temperature and relative humidity, taken at a level l_0 from the ground, and wind speed u_1 , taken at a level l_1 from the ground. A convective exchange coefficient D_w is then computed as:

$$D_w = \frac{k^2 u_1}{\{\ln[(l_1-d)/z_0]\}^2} \frac{(l_1-d)}{(l_0-d)}$$

where k is von Karman's constant, z_0 is the snow roughness height, and d is the depth of the snowpack. Sensible heat H_c , and latent heat H_l fluxes result respectively as:

$$H_c = D_w (K_h/K_m) C_p \rho_a (T_a - T_s) \quad (18)$$

and

$$H_l = 0.622 L_v D_w (K_e/K_m) \rho_a/p (e_a - e_s) \quad (19)$$

where K_h , K_m and K_e denote the turbulent transfer coefficients for heat, momentum and water vapor, with ratios K_h/K_m and K_e/K_m taken equal to 1 as generally assumed in the hydrological practice, for atmospheric stable condition. C_p is the specific heat of air at constant pressure, L_v is the latent heat of vaporization of water, ρ_a and p denote air density and pressure. Finally T_s and e_s denote snow surface temperature and saturated vapor pressure at the temperature of snow. It was then possible to compute in detail the relative contribution of radiative exchange of net shortwaves S_{io} , net longwaves L_{io} and turbulent fluxes of sensible

TABLE 1 Global heat exchange balance and relative contribution (percentages are reported in brackets) of radiative and turbulent fluxes. Values are integrated over each of five snowmelt seasons . Data reported are point values, relative to the central cell of the basin, where meteorological data are recorded.

Heat flux [MJ m ⁻²]	Simulation period				
	1984 12 Apr-15 Jun	1985	1986	1987 1 Apr-15 Jun	1988
H _m +H _s	309	190	361	315	210
S _{io}	430(139%)	291(153%)	282(78%)	465(147%)	300(143%)
L _{io}	-159(-51%)	-170(-89%)	-108(-30%)	-186(-59%)	-107(-51%)
H _c	153(40%)	143(75%)	245(68%)	188(60%)	73(35%)
H _l	-125(-40%)	-125(-66%)	-96(-27%)	-186(-59%)	-65(-31%)

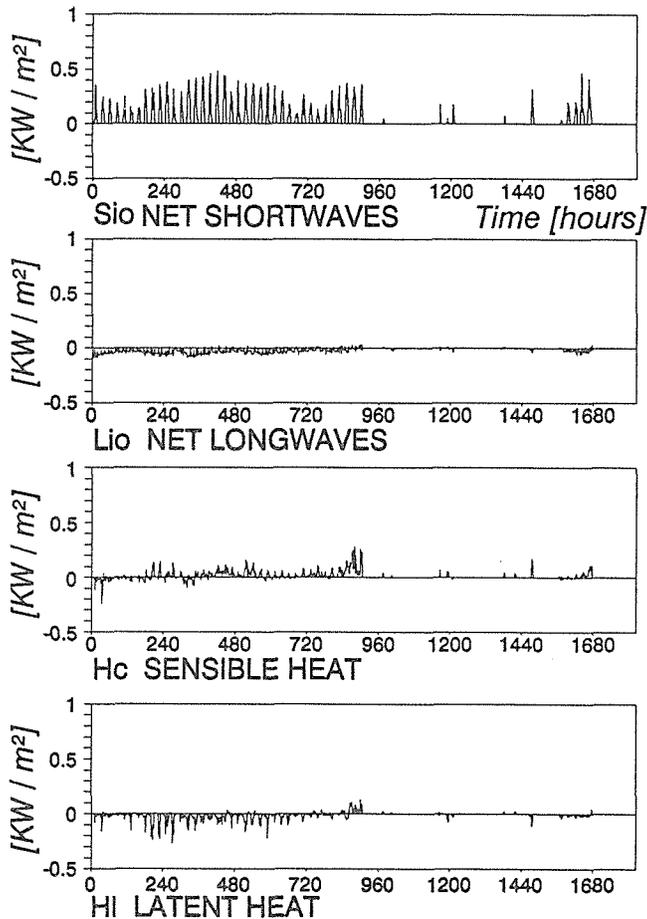


FIG. 4 Point and hourly values of radiative and turbulent heat fluxes during the 1988 snowmelt season, simulated for the central cell of Cordevole basin.

heat H_c and latent heat H_l . Simulations were performed over the five snowmelt seasons and integrated values are reported in Table 1. The simulation for the year 1988 is reported in Fig. 4. These data show the net shortwave fluxes to play the major role in the global energy budget of the snowpack covering the small alpine basin under analysis. Because low wind speed ($1 \div 2 \text{ ms}^{-1}$) generally characterizes the study area during the snowmelt season, the turbulent exchange components play a minor role. To compute these fluxes, a constant lapse-rate of air temperature has been assumed and a value of $0.0065 \text{ }^\circ\text{C m}^{-1}$ has been estimated by means of available data (Ca' Zorzi et al., 1984). Therefore, this result can be regarded as a first approximation only to the analysis of turbulent exchange. However, the above simplification was dictated by the insufficient information about the spatial distribution of air temperatures, and of large scale air mass properties, which may influence the role of advection in the energy balance (Olyphant & Isard, 1988). H_p and H_g were negligible on average.

The energy balance equation gives the input to the snowpack model, where the snowmelt flux within the snowpack is computed through a two-layer model with celerity estimated from the results of Colbeck's (1982) studies. Heat transfer and phase changes are obviously taken into account. Distributed values of snow cover temperature, density and water equivalent within the two layers are simulated, and the resulting snowmelt is computed. The distributed snowmelt provides the input to a conceptual snowmelt-runoff model where surface runoff volumes are computed by assuming that soil water storage capacity at a point is a stochastic variable with exponential probability density function (Clarke & Moore, 1981). A linear-exponential store-discharge relationship is used to model the transfer function (Ranzi, 1990) in order to account for nonlinearity in surface runoff, as expected for a small mountain creek (Caroni et al., 1986).

CONCLUSION AND FUTURE WORK

A model describing the topographic distribution of net solar radiation under both clear-sky, and cloud cover conditions, has been presented, where a "fine" resolution in time is used (i.e. using hourly records of incoming shortwave radiation). Net shortwave radiation has been shown to be the dominant component of the energy balance for the alpine creek under investigation. The optimal grid spacing of a DEM to be chosen for the reliable simulation of spatially integrated heat fluxes depends on basin ruggedness and on the spatial and temporal scales of the hydrological processes involved. The major geomorphological constraint involved in this operation is given by the requirement of preserving the effective features of the catchment stream system, which control runoff generation from hillslopes and the structure of basin hydrographs. Some preliminary attempts to analyze these problems have been performed for the basin under examination, showing a not negligible degree of sensitivity of the simulated snowmelt runoff to the grid spacing used for modelling. Additional research should be devoted to investigating this problem.

It must be pointed out that strong limitations affect the development of the physically-based distributed approach for operational purposes. These are mainly due to the huge experimental information required to approach real world applications. However, the conceptual models based on air temperature could be improved when applied to those environments where shortwave fluxes are dominant. Moreover, the stochastic models using solar radiation and degree-day index as the only exogenous variables (Bengtsson, 1986) could also be improved by introducing some parametrization of topography, in terms of global geomorphological features, such as statistical descriptors of slope and aspect. The model proposed here provides a useful framework for investigating possible improvements of operational snowmelt models.

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