A routing model for continental-scale hydrology

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Abstract A simple linear convective-diffusion approach to the routing of flow through a river network is explored in the context of large UK catchments. Network width functions derived from different scale maps are compared and network response functions for different time scales are given. The methods are applied to two different catchment subdivisions - one based on grid square elements in the Severn and one based on geological variations in the Thames. These examples highlight the utility of the method and the importance of channel routing in modelling daily flows.

INTRODUCTION

In developing models for continental-scale hydrology, there are essentially two requirements - a runoff-generation component and a routing component. While considerable attention has been devoted to the runoff-generating component (Famiglietti & Wood, 1991) and the development of aggregation schemes for soil-vegetation-atmosphere transfers (Avissar & Pielke, 1989; Entekhabi & Eagleson, 1989), relatively little attention has been paid to the routing of the generated flows. However, this is an important component of any model for the derivation of time series of flows into the oceans and for studies of the impact of climate/land use change on water resources. The work of Vörösmarty et al. (1989) in South America and Vörösmarty & Moore (1991) in the Zambezi are perhaps the exceptions. These studies use a monthly time step and are at a space scale of 0.5° latitude and longitude. Flood plain inundation is important and modelled; otherwise, flow routing is simply from cell to cell using a single transfer coefficient and a single dominant flow direction defined using the major rivers and other topographic information. However, routing of flow takes place both within as well as between grid cells, and this becomes particularly important when shorter time steps are required.

This paper focuses on a method for routing flows through the river network which is relatively simple and lends itself to applications at all scales. The method is based on the network width function and a linear convective-diffusion approach to routing (Naden, 1992). The applications discussed relate to the most downstream gauging stations on the rivers Thames and Severn. While not continental in scale, these catchments are of the order of 10 000 km² and represent two of the largest catchments in the UK. The principles demonstrated in these examples are not limited to this scale of application.

METHODOLOGY

The methodology for the routing is that presented in Naden (1992) in the context of a
semi-distributed unit hydrograph. In the present paper, however, the network response function is convolved with the generated input into the stream network. The network response function itself is derived by weighting a routing function by the standardized network width function.

The network width function (Kirkby, 1976; Mesa & Mifflin, 1986) is simply a distribution of the number of channels at successive distances, as measured along the network, away from the basin outlet. This provides a picture of the spatial pattern of the water courses within a catchment. It can readily be derived from maps or, more appropriately in the case of large catchments, from digital data sources. It is standardized by the total number of channels featuring in the distribution.

The routing function used is a linear convective-diffusion solution to the equations of continuity and momentum balance (Dooge, 1973; van de Nés, 1973). The network is thought of as a series of individual channel links of length $\Delta s$, each receiving a lateral inflow, and being connected to the basin outlet at a distance $s$. The solution can be derived for any time period $\Delta t$ and the full derivation of the $\Delta t$-period response over unit distance $\Delta s$ is given in Naden (1992). The function has two parameters - a wave velocity and a diffusion or attenuation coefficient which expresses the rate at which the wave spreads out. These parameters are considered to be constant throughout the network. Under certain assumptions, the wave velocity is proportional to the water velocity and the attenuation parameter is related to the water velocity, channel slope, and Froude number.

This routing method, being a linear approximation, has the disadvantage that the model parameters cannot vary with the flow. However, linear approximations to the St Venant equations are acceptable under a wide range of conditions, as reported by Price (1982) and Beven & Wood (1993). Furthermore, the great advantage of a linear routing method is that the catchment can be subdivided in any appropriate manner and the generated flows from each subdivision independently routed and summed to give the flow at the catchment outlet. For large-scale modelling in which the water balance and the general pattern of flow into the oceans are the predominant foci of interest, this advantage appears to outweigh the disadvantages. If necessary, linear reservoirs could be added to provide a better representation of any major flood plain storage elements which are crucial to the water balance. Examples are given below of how the method works on both a grid square subdivision and a geological subdivision of a catchment.

SPACE AND TIME SCALES

Both space and time scales are dealt with explicitly in the derivation of the network response function and so the method can be applied at any scale. In terms of spatial scales, perhaps the most interesting question is how far the description of the channel network is dependent on map scale.

Figure 1(a) shows a map of the digitized river network for the Thames basin as taken from the 1:50 000 Ordnance Survey first series map sheets. The network width function derived from these map sheets on a spatial scale of 1 km is given in Fig. 2(a). Any spatial step could be used but 1 km is close to the mean link length (i.e. mean distance between successive tributaries) of 1.3 km. Much smaller space steps would not, therefore, add further information and the plot would eventually take on more of a stepped character. A spatial scale of 1 km also ties in with the size of Representative
Fig. 1 (a) River network digitized from 1:50 000 Ordnance Survey first series map sheets (© Crown Copyright). Thick line indicates catchment boundary of the Thames to Teddington. (b) Subdivision of Thames catchment based on Winter Rain Acceptance Potential soil classification (see text for details); ▲ small catchments used in scaling-up procedure (Table 3).
Elementary Areas introduced by Wood et al. (1988). Space steps larger than 1 km simply provide a sampling of the verticals in Fig. 2(a).

While the detail shown in Fig. 1(a) is appealing, it is important, in the context of continental-scale modelling, to have regard to how the models might be applied in areas with less good spatial data than in the UK. To test whether the map scale is fundamental, a network width function for the Thames at Teddington was derived, by hand, from the 1:250 000 map sheets. These maps show much fewer channels and Table 1 compares some of the river network statistics for the two map scales which highlight the difference in detail. The network width function derived from the 1:250 000 scale map is shown in Fig. 2(b), using the same axes as Fig. 2(a) for
Table 1  Network characteristics – Thames at Teddington.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>From 1:50 000 map sheets</th>
<th>From 1:250 000 map sheets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sources</td>
<td>3058</td>
<td>427</td>
</tr>
<tr>
<td>Total stream length (km)</td>
<td>7852</td>
<td>3657</td>
</tr>
<tr>
<td>Maximum stream length (km)</td>
<td>257</td>
<td>248</td>
</tr>
<tr>
<td>Mean link length (km)</td>
<td>1.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Maximum link length (km)</td>
<td>4.4</td>
<td>28.8</td>
</tr>
</tbody>
</table>

comparison. It clearly shows the fewer channels but is remarkably similar in overall shape to that derived from the 1:50 000 maps.

A direct comparison of the two network width functions in their routing role is provided in Fig. 3 in which the 1-h network response function is shown under the assumption of a wave velocity of 0.6 m s\(^{-1}\) and an attenuation parameter of 1.0 m\(^2\) s\(^{-1}\). The two network response functions are very similar, with just some additional detail and a slightly longer tail shown in the one derived from the 1:50 000 maps. The reason for the similarity is that, despite the lack of detail on the 1:250 000 maps, the structure of the river network is preserved and so the network response functions derived from the two different scale maps are very similar. How far this finding holds true as map scale is decreased still further, as for instance in the use of the Digital Chart of the World (Denko, 1992), is yet to be identified.

In terms of the slightly longer tail shown in Fig. 3, it is worth considering whether it might be possible to apply a simple correction for map scale to the derived river lengths. Under the assumption that rivers are fractal in nature, Hjelmfelt (1988) and

![Network response function for Thames at Teddington with wave velocity 0.6 m s\(^{-1}\) and attenuation parameter 1.0 m\(^2\) s\(^{-1}\): solid line from 1:50 000 maps; dashed line from 1:250 000 maps.](image)
La Barbera & Rosso (1989) suggest that river lengths are related by a factor \((d_1/d_2)^{-D}\) where \(d_1\) and \(d_2\) are the two map scales and \(D\) is the fractal dimension – usually quoted as between 1.1 and 1.2 for a single stream. This suggests a correction factor of between 1.17 and 1.38 when moving from the 1:250 000 scale to the 1:50 000 scale whereas the ratio of the two maximum stream lengths given in Table 1 is 1.04. It must, however, be borne in mind that the two data sets were derived by different procedures and that differences in geology cause distinct inhomogeneity in the network. Consequently, perhaps a more thorough examination of this question is warranted in the future.

As time scales are explicit in the routing procedure developed above, the method can be applied to any time sequence of flows. This might range from calculations based

![Fig. 4](image-url)  
**Fig. 4** (a) Network width function for Severn at Haw Bridge; (b) Network response function for Severn at Haw Bridge; solid line 1-h response function; dashed line 24-h response function.
on 5 minute rainfall data from weather radar or 15 minute output derived from a General Circulation Model to the daily runoff used in the examples below. The effect of changing time scale is shown in Fig. 4. The network width function for the Severn at Haw Bridge, derived at a spatial scale of 1 km from 1:50 000 Ordnance Survey first series map sheets, is given in Fig. 4(a). Under the assumption of a wave velocity of 0.6 m s\(^{-1}\) and an attenuation parameter of 1.0 m\(^2\) s\(^{-1}\), Fig. 4(b) shows the 1-h network response function (solid line) and the 24-h network response function depicted as a stepped function (dashed line). As shown, the method provides a sound averaging procedure.

**EXAMPLES OF CATCHMENT SUBDIVISION**

The main advantage of the linear routing method is that a catchment can be subdivided in whatever way is appropriate for the best representation of runoff generation. Naden (1992) looked at the effect of weighting the network response function by the rainfall variation for the purpose of flood estimation in individual events. Here, the focus is on subdivisions based on grid squares in the case of the Severn and on geological differences in the case of the Thames.

**Grid square subdivision**

The Severn basin has a steep rainfall gradient, which to some degree is reflected in a grid square subdivision, and the runoff process is dominated by a quickflow response. Jolley & Wheater (1992) are developing a model based on the Meteorological Office Rainfall and Evaporation Calculation System (MORECS; Meteorological Office, 1981) which uses 40 by 40 km grid squares. The grid squares on the Severn are shown in Fig. 5. Network width functions for each of the MORECS grid squares on the Severn to the downstream gauge of Haw Bridge have been calculated. In order to demonstrate the effect of the routing alone, typical small catchments representing each of the grid squares have been identified and measured mean daily flows obtained. The locations of these gauges are also shown in Fig. 5. Table 2 gives, for each grid square, the gauge number and basin area of the chosen small catchment along with the area of the grid square which contributes to the Severn at Haw Bridge. Suitable small catchments have not always been available (maximum basin area used is 178 km\(^2\)), and it will be noted that some small catchments have had to serve as the response for more than one grid square.

Taking the year 1981, for which daily flows are available for all the catchments selected, the total annual observed flow at Haw Bridge was compared with the summation of the scaled-up flows (i.e. observed small catchment flows multiplied by area of grid square contributing divided by the area of the small catchment) for each grid square. The cumulated scaled-up flows came to 3681 \(\times\) 10\(^6\) m\(^3\) compared to the observed flow of 3673 \(\times\) 10\(^6\) m\(^3\) - a 0.2% error over the year. However, comparison on a monthly basis was less good and, looking at the daily flows (Fig. 6(a)), then it is clear that a simple summation of the scaled-up runoff is not appropriate with the summed peaks occurring too early and being too large, while the baseflow is underestimated.
Fig. 5 MORECS squares in the Severn basin; ▲ small catchments used in scaling-up procedure (see Table 2).

The scaled-up catchment flows were, therefore, routed through the river network using the appropriate network response function for each of the grid squares. Typical parameter values of 0.6 m s$^{-1}$ and 1.0 m$^2$ s$^{-1}$ were used. The value of 0.6 m s$^{-1}$ for the wave velocity compares with 1.0 m s$^{-1}$ used by Mesa & Mifflin (1986) on a catchment of 1.24 km$^2$, and 0.5 m s$^{-1}$ used by Moore & Bell (in press) on a catchment of 275 km$^2$. The diffusion parameters (using the definition in Naden, 1992) employed by Mesa & Mifflin ranged between 2.0 and 9.0 m$^2$ s$^{-1}$. In practice, over large time steps, the results are relatively insensitive to the diffusion parameter. The results of routing are shown in Fig. 6(b). This shows a marked improvement in the fit, with an increase in efficiency (given by $1 - \left[ \Sigma (O_i - M_i)^2 / \Sigma (O_i - \bar{O})^2 \right]$ where $O_i$ is the observed mean daily flow, $M_i$ is the modelled mean daily flow; Nash & Sutcliffe, 1970) from 0.12 to 0.76. This increase in efficiency indicates the important role which routing plays in the Severn basin. The example illustrates the main advantage of the routing method which lies in the ability to subdivide the catchment into grid squares, which to some degree reflect the rainfall gradient of the Severn. It will also be noted, however, that the high flood peaks, still overestimated after routing, would be overbank at Haw Bridge, which suggests that at individual gauging stations consideration has to be given to flood plain storage if flood estimates are required.
Table 2  MORECS grid squares on the Severn.

<table>
<thead>
<tr>
<th>Grid square (Fig. 5)</th>
<th>Area of square within Severn (km²)</th>
<th>Typical small catchment</th>
<th>Area of small catchment (km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>65.2</td>
<td>54022 Severn at Plynlimon Flume</td>
<td>8.7</td>
</tr>
<tr>
<td>113</td>
<td>540.4</td>
<td>54066 Platt Brook at Platt</td>
<td>15.7</td>
</tr>
<tr>
<td>114</td>
<td>690.9</td>
<td>54062 Stoke Brook at Stoke</td>
<td>13.7</td>
</tr>
<tr>
<td>115</td>
<td>7.7</td>
<td>54062 Stoke Brook at Stoke</td>
<td>13.7</td>
</tr>
<tr>
<td>122</td>
<td>408.9</td>
<td>54022 Severn at Plynlimon Flume</td>
<td>8.7</td>
</tr>
<tr>
<td>123</td>
<td>1560.6</td>
<td>54018 Rea Brook at Hookagate</td>
<td>178.0</td>
</tr>
<tr>
<td>124</td>
<td>1600.0</td>
<td>54046 Worfe at Cosford</td>
<td>54.9</td>
</tr>
<tr>
<td>125</td>
<td>368.2</td>
<td>54046 Worfe at Cosford</td>
<td>54.9</td>
</tr>
<tr>
<td>126</td>
<td>227.5</td>
<td>54048 Dene at Wellesbourne</td>
<td>102.0</td>
</tr>
<tr>
<td>127</td>
<td>44.7</td>
<td>54048 Dene at Wellesbourne</td>
<td>102.0</td>
</tr>
<tr>
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</tr>
<tr>
<td>134</td>
<td>218.0</td>
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<tr>
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<tr>
<td>136</td>
<td>1460.0</td>
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<td>138</td>
<td>50.0</td>
<td>54048 Dene at Wellesbourne</td>
<td>102.0</td>
</tr>
<tr>
<td>147</td>
<td>28.1</td>
<td>54026 Chelt at Slate Mill</td>
<td>34.5</td>
</tr>
<tr>
<td>148</td>
<td>454.8</td>
<td>54026 Chelt at Slate Mill</td>
<td>34.5</td>
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<tr>
<td>149</td>
<td>125.9</td>
<td>54048 Dene at Wellesbourne</td>
<td>102.0</td>
</tr>
</tbody>
</table>

Geological subdivision

While the rainfall gradient is the dominant feature of the hydrology of the Severn, the dominant feature of the Thames basin is the combination of surface runoff with a significant groundwater component. This suggests that the most important subdivision in the case of the Thames is on the lines of geological boundaries. Indeed, the geological variation within the Thames basin is seen quite clearly in Fig. 1(a), with the chalk Downs and Chilterns extending from southwest to northeast across the centre of the map and the Jurassic limestone of the Cotswolds standing out in the northwest of the catchment as another area with lower drainage density. The routing method provided here can equally be applied to this geological subdivision.

The catchment has been subdivided into four main areas (Fig. 1(b)) on the basis of geology, using the Winter Rain Acceptance Potential (WRAP) soil classification.
from the Flood Studies Report (NERC, 1975) as a guide. There are 5 WRAP classes and a WRAP class of 1 is taken to be indicative of the chalk and limestone areas. The WRAP map is digitally held as scan lines with a spatial resolution of 100 m. The four geologically distinct areas (Fig. 1(b)) identified for the Thames catchment to Teddington comprise the Jurassic limestone; the Oxford and Kimmeridge clays extending from Swindon through Oxfordshire; the chalk; and the London clay, Reading beds and Bagshot beds located from the west of Reading to London. The areas of each subdivision are given in Table 3, along with the small catchments (Fig. 1(b)) chosen to represent them. Mean daily flows were obtained and, again, both observed total flow and cumulated scaled-up flows were derived. In this case, the correspondence between
the observed annual water total \((2340 \times 10^6 \text{ m}^2)\) and the scaled-up total \((2504 \times 10^6 \text{ m}^2)\) is not as good as on the Severn, with an error of 7%. The reasons for this may be the coarser subdivision of the catchment, abstractions for water use – particularly in the lower Thames, differences in surface and groundwater catchment areas, and the difficulty of representing groundwater responses by typical small catchments.

Figure 7(a) shows the cumulated scaled-up daily flows and the observed mean daily flow for the year 1981 for the Thames at Teddington, while Fig. 7(b) shows the effect of routing the scaled-up flows, using the typical parameter values of 0.6 m s\(^{-1}\) and 1.0 m\(^2\) s\(^{-1}\), with no attempt to optimize. The efficiency (see above for definition) increases from 0.07 to 0.78. This represents about the same order of improvement as on the Severn – a catchment of similar size. The main source of errors remaining in the Thames example accrues from the lower flows observed during the summer compared to those derived from the scaled-up flows. This discrepancy arises largely from the scaled-up contribution from the groundwater-dominated areas (Fig. 7(c)) and the example highlights the more complex picture of catchments dominated by groundwater and human influence, thereby pointing to some of the additional considerations which have to be accommodated in continental-scale modelling.

**CONCLUSION**

This paper has focused on a simple convective-diffusion routing model for large catchments which is based on the network width function derived from the observable river network. It identifies space and time scales explicitly and, at least for the move from 1:50 000 to 1:250 000, the scale of map from which the network width function is derived has relatively little impact. Work on estimating the parameters of the routing model from measurable catchment characteristics, such as river slopes, is proceeding.

The examples quoted illustrate the use of the routing method with catchment subdivisions based on either grid squares, as on the Severn, or on geological boundaries, as on the Thames. Routing is seen to be highly important in both examples. The method has the advantage over other grid routing procedures in that it builds on the full representation of the river network and considers within-square as well as between-square routing.
Fig. 7 Flows for Thames at Teddington: (a) observed (solid line) and cumulated scaled-up flows (dashed line); (b) observed (solid line) and routed scaled-up flows (dashed line); (c) routed scaled-up contributions from areas dominated by groundwater (solid line) and surface-water (dashed line).
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