Numerical investigation of contaminant transport in shallow water bodies

L. A. KRUHKIER & G. V. MURATOVA
Computing Centre of Rostov State University, 200/1 Stachki av., Rostov-on-Don, Russia 344104

Abstract A numerical investigation of the processes of hydrodynamic and contaminant transport in shallow water bodies is presented. The calculations were made for suspended sediments distribution in the Don River estuary. The Don River enters the Azov Sea located in Russia and the Ukraine. To check the adequacy of the developed mathematical model, the calculated results were compared with field observations. A series of numerical experiments were conducted to estimate the influence of the designed dam in the neck of Taganrog Bay. It was supposed that the dam could reduce water salinity in the Bay. The results of the calculations showed that narrowing the neck stimulated the cycling current, which in turn, promoted sediment accumulation near the dam.

INTRODUCTION

Environmental impact assessment of any technological and socio-economic projects becomes vitally important. Mathematical modelling based on numerical methods is one of the effective ways of conducting such investigations.

The models that describe the physical behaviour of the receiving medium, i.e., the hydrodynamic models of the water body under different external influences (Nikolaev et al., 1992), are the most difficult for the numerical realization. This paper includes the model of shallow water body hydrodynamics based on the solution of modified equations of Navier-Stokes (Surkov et al., 1989) and advection diffusion equation (Volsinger & Pyaskovsky, 1977).

The Azov Sea, which is very shallow, was chosen for investigation (Fig. 1). It is located between 45°6′ and 47°17′N latitude and 33° and 36°E longitude, united with the Black Sea by Kerch-strait. The area of the Azov Sea is 37 800 km². It is less than 14 m deep and less than 100 km wide. Two large rivers, the Don and the Kuban, are the main tributaries to the Azov Sea.

During the last 30-40 years, the ecological situation has seriously deteriorated in the Azov Sea. The river's fresh water flow has been reduced, but the total pollutant input to the sea has increased. As a result, concentrations of pollutants and salinity of the sea has increased and the area of fish habitation and biodiversity has been dramatically reduced. To restore the Azov Sea ecosystem and to increase its fish productivity, the project of narrowing the mouth of the Taganrog Bay of the Azov Sea was suggested. It was assumed that this project could reduce the water exchange between fresh water Taganrog Bay and the more saline Azov Sea.
Realization of this project, however, can change the circulation pattern of transport contaminant in the Sea and the Bay. To solve these problems the simulations were made on a hydrodynamic mathematical model of a shallow water body.

MODEL FORMULATION

The hydrodynamic model of the Azov Sea is based on the following modified system of equations of shallow water theory with retention of the non-linear terms (Krukier et al., 1991) and advection-diffusion equation (Volsinger et al., 1977):

\[
\frac{\partial U}{\partial t} + \frac{1}{2} \left( \frac{\partial U}{\partial x} + \frac{\partial uU}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial vU}{\partial y} \right) + gH^{1/2} \frac{\partial \xi}{\partial x} = A_1 \Delta U + \Omega V - \alpha U + \tau_{nx}
\]

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \left( \frac{\partial V}{\partial x} + \frac{\partial uV}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial vV}{\partial y} \right) + gH^{1/2} \frac{\partial \xi}{\partial y} = A_1 \Delta V - \Omega U - \alpha V + \tau_{ny}
\]

\[
g \frac{\partial \xi}{\partial t} + \frac{\partial gH^{1/2} U}{\partial x} + \frac{\partial gH^{1/2} V}{\partial y} = 0
\]

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \Delta C + HF
\]

derived from the divergent form of shallow water equations (Volsinger et al., 1977) by substituting $H = H^{1/2}H^{1/2}$, where $u$, $v$ — velocity components averaged by the depth; $U = H^{1/2}u$, $V = H^{1/2}v$ — flows; $g$ — acceleration of gravity; $H = h + \xi$ is the total depth; $\xi$ — perturbation of the surface level; $h$ — sea depth; $\Omega$ — the Coriolis
Numerical investigation of contaminant transport in shallow water bodies

parameter; $A_x$ — coefficient of horizontal turbulent exchange of momentum; $\Delta$ the Laplace operator; $\tau_{w1}, \tau_{w2}$ are $x$ and $y$ of components of wind stress; $C$ — depth integrated concentration; $D$ — turbulent diffusion coefficient; $F(x,y,z)$ — the function, describing external sources and sinks of pollution, changing concentration of substances as well as chemical reactions and biological processes, if necessary.

The origin of coordinates is chosen on an undisturbed sea surface and the initial conditions are:

$$U|_{t=0} = \varphi_1(x,y); \quad V|_{t=0} = \varphi_2(x,y); \quad \xi|_{t=0} = \varphi_3(x,y)$$

$$C|_{t=0} = \varphi_0(x,y)$$

and the boundary conditions for the whole water body:

$$U|_s = \psi_1(s,t); \quad V|_s = \psi_2(s,t)$$

$$\frac{\partial C}{\partial n}|_s = 0$$

are added to the system of equations (1)-(4), where $\varphi_i(x,y)$ ($i = 0, \ldots, 3$) — the given functions, $S$ — shore-line contour; $s \in S$ — parameter; $\psi_1, \psi_2$ — the given functions differing from zero only in places of sinks and sources; $n$ — the normal to the water body boundary.

The computation of the velocity field and water surface elevation in a shallow sea is determined by equations (1)-(3). It should be noted that the form of equations (1)-(3) differs from the standard form. The specific form of the initial system equations (1)-(3) is due to the necessity to use numerical finite-difference algorithms without limitation of the number of steps in space and time.

After solving the hydrodynamic problem and obtaining the velocity field in the water body, pollutant distribution is simulated.

The model can be applied either to the entire water body or to a part of it, such as a bay or river estuary.

For the open water body, the following conditions must be satisfied for the open part of the boundary:

$$(D \frac{\partial C}{\partial n} + qC)|_s = 0$$

$$q = \begin{cases} v_n, & v_n \geq 0 \\ 0, & v_n < 0 \end{cases}$$

where $v_n$ — normal velocity.

The initial-boundary problem of equations (4), (6), (8), (9) can describe many processes which take place during the pollution of water bodies. The characteristics of sources might be reflected by means of the functions $F$ and $\lambda$. If the pollutant is conservative, then $\lambda = 0$. If the pollutant is not conservative, and either decomposition or formation occurs, then the coefficient has to reflect these processes. In the model, the source of pollution can be fixed or moving, it may be single or multiple, it may be point or non-point source. The pollutant discharge may be represented in the form of single impulse or steady input, following a certain sequence but random.
METHOD OF ANALYSIS

The finite difference method is used for solving the initial-boundary problem equations (1)-(4) according to which the area of the shallow water basin, $G$, is approximated by a rectangular grid, $W$, with the $h_x$ step along the $Ox$ axis and the $h_y$ step along the $Oy$ axis. The time interval is split into a number of integer steps, $\tau$. The conditions on the grid border are determined from boundary conditions on $\partial G$. On the solid border (shoreline) conditions of nonflow are used.

Unknown values in the count cell of the grid are determined as follows (Fig. 1): velocity components, water surface elevation are calculated in the centre of the cells; fluxes, if they are given instead of the velocities, in the middle of the corresponding sides. The count cell is obtained by the number on the left lower corner.

The boundary of the domain is approximated by the rectangular dashed line. On the building grid area the terms of equation (5) are approximated by the scheme, in which the first derivations are calculated using "against stream" differences (Roache, 1976).

Specially created software based on the mathematical model mentioned above is intended to solve unsteady hydrodynamic problems in the arbitrary regions. This model has the flexibility to carry out different types of numerical experiments: simulation of the processes in the whole area of the sea or in bays or in river estuaries.

The model helps to study possible changes in hydrodynamics of water bodies under different stages of dam construction under operation.

Construction of dams in a water body is equivalent to the change of one type of boundary condition in the mathematical model to another, for example, setting velocities to zero within the water body affected.

In the simulation experiment the average and extreme wind situations (strong wind in one direction) were considered.

RESULTS OF SIMULATIONS

The software was verified by using the data on suspended-sediment distribution in the Don River estuary (Fig. 1A) in calm weather. There are two reasons for selecting suspended sediment to model. Most of the pollutants, such as heavy metals, enter the Don Estuary and the Azov Sea with suspended sediment. This explains the importance of modelling the suspended-sediment distribution and loading. Besides, there are detailed field data allowing the testing of the mathematical model. It is necessary to note, however, that the created mathematical model allows the calculation for different pollutants if reliable initial data are available.

At first, the hydrodynamic problem corresponding to the given hydrometeorological situation was solved and the velocity field in the water body was obtained. These data were then used for the calculation of pollutant distribution in the study area. The results of the simulations (Fig. 2) were compared with the map of concentrations measured during the largest magnitude spring flood in 1963 (since the Don River impoundment) (Fig. 3) (Bronfman & Khlebnikov, 1985). Spatial patterns of calculated and observed suspended sediment concentrations generally are similar.
The second part of the numerical experiment consisted of the calculations of the pollutant distribution in the Taganrog Bay (Fig. 1B) for typical south-west wind conditions. The velocity field was obtained using the program of hydrological simulations. Two versions of calculations were made: (1) using the natural area of the Taganrog Bay, and (2) using the scenario of narrowing the neck of the Bay.

The project narrowing the neck of the Taganrog Bay supposes the construction of a dam, narrowing the neck from 26 to 10 km. The computation of water velocities has shown that for the scenario of the dam construction in the Taganrog Bay, a circular flow will be formed having the same direction as the circular flow in the sea itself.
Water circulation in the Taganrog Bay two days after the beginning of a constant south-west wind of 10 m s\(^{-1}\) (establishing the steady state hydrodynamic regime) during the construction phase of the dam is shown on the Fig. 4.

The following experiments were made to check the influence of the designed dam on distribution of the pollutants entering the Bay with the Don River water. Two cases were considered: single impulse, and steady input from the Don River. The time-step for the calculations was three hours.

The pictures of the pollutants distribution in a natural area and for a scenario of the dam construction in the Taganrog Bay after 10 days for single impulse and steady input from the Don River, is shown on Figs 5 and 6.

Simulation results showed that in case of impulse discharge, contamination will be spread over the area of the Bay with a simultaneous decrease of peak concentrations from the river. Five days after contaminant discharge the area with maximum

---

Fig. 4 Water circulation in the Taganrog Bay two days after the beginning of a steady 10 m s\(^{-1}\) southwest wind. A — in the actual bay; B — for the dam scenario.
contaminant concentrations will be removed from the mouth of the Don to the middle of the Bay with decreased absolute values and this picture of pollutant distribution was stable for a long period. Differences for three versions of the calculations are insignificant until the pollutant reaches the neck of the Taganrog Bay.

For the case of steady input, maximum pollutant concentrations are predicted near the mouth of the Don River and then decrease gradually towards the open Bay.
Changes of pollutant distribution in the Taganrog Bay in case of dam construction were simulated. Concentration in the vicinity of the dam is higher than in the neck of the open Bay. It should be pointed out that the results were obtained for a particular wind condition. It can be supposed, however, that realization of any scenario of the dam construction can lead to the build-up concentration of pollutants in the restricted part of the Taganrog Bay.

REFERENCES