Predicting salt-water upconing due to wellfield pumping

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Abstract A steady-state, sharp-interface analytical solution for predicting salt-water upconing due to pumping from a single well in an aquifer overlain by a leaky confining bed has been extended to include the interference effects of other pumped wells. An example calculation illustrates how the resulting solution can be used to determine the critical pumping rate due to pumping from multiple wells in a wellfield. The critical pumping rate calculated using the analytical solution is conservatively low compared to literature results obtained using a density-dependent, solute transport numerical solution. The analytical solution can be used for an initial analysis of a wellfield and for calculating a lower bound to help verify results from other solutions.

NOTATION

\[ \begin{align*}
  b & \quad \text{thickness of aquifer;} \\
  b' & \quad \text{thickness of confining bed;} \\
  1/B & \quad (K'/b'T)^{1/2}; \\
  d & \quad \text{distance from top of aquifer to top of well screen;} \\
  f & \quad \text{partial penetration correction factor;} \\
  K_0 & \quad \text{modified Bessel function of the second kind, zero order;} \\
  K_r & \quad \text{horizontal hydraulic conductivity of aquifer;} \\
  K_z & \quad \text{vertical hydraulic conductivity of aquifer;} \\
  K' & \quad \text{vertical hydraulic conductivity of confining bed;} \\
  K'/b' & \quad \text{leakance of the confining bed;} \\
  \ell & \quad \text{distance from top of aquifer to bottom of well screen;} \\
  M & \quad \text{number of multiple wells;} \\
  m,n & \quad \text{summation indices;} \\
  Q & \quad \text{pumping rate of well;} \\
  Q_c & \quad \text{critical pumping rate of well;} \\
  r & \quad \text{radial distance;} \\
  r_w & \quad \text{radius of pumped well;} \\
  s_i & \quad \text{drawdown at } r \text{ along the salt-water fresh-water interface;} \\
  T & \quad \text{transmissivity;} \\
  z & \quad \text{vertical coordinate measured downward from the top of the aquifer;} \\
  \alpha_m & \quad \text{pumping rate coefficient;} \\
  \gamma_f & \quad \text{specific weight of fresh water;} \\
  \gamma_s & \quad \text{specific weight of salt water;} \\
  \Delta & \quad \text{rise of the salt-water fresh-water interface;}
\end{align*} \]
\[ \Delta_c \] critical rise of the salt-water fresh-water interface;
\[ \delta = \left[ \gamma_f / (\gamma_s - \gamma_f) \right] \]

INTRODUCTION

Fresh water in a groundwater aquifer overlies more dense salty, or brackish, water in many parts of the world. In response to pumping from a well in the fresh-water zone, the salt-water fresh-water interface moves vertically upward toward the pumped well (Fig. 1). Under some conditions, a stable cone in the interface will develop, and the well will continue to discharge fresh water. Under other conditions, however, the cone will exceed some critical height and become unstable, causing the interface to rise abruptly to the bottom of the pumped well, and resulting in the discharge becoming salty (Reilly & Goodman, 1985).

METHODS OF ANALYSIS

Both sharp-interface and density-dependent solute transport approaches have been used to analyse salt-water upconing and to determine the critical pumping rate, or the pumping rate at which the cone becomes unstable (Reilly & Goodman, 1987). The sharp-interface approach is considered an appropriate approximation if the thickness of the transition zone is relatively small compared to the thickness of the aquifer (Bear, 1979). A number of analytical and numerical solutions have been developed based on the sharp-interface approach, including an analytical solution for salt-water upconing due to pumping from a single well in an aquifer overlain by a leaky confining bed (Motz, 1992).
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In this solution, based on Muskat’s (1946) approach, it is assumed that the rise in the salt-water fresh-water interface is small, the interface acts as a streamline impervious to flow across it, and no flow occurs in the salt water beneath the interface.

SOLUTION FOR MULTIPLE WELLS IN A LEAKY AQUIFER

As described below, the single well leaky aquifer solution can be extended to include the interference effects of other pumped wells. As illustrated by an example, the resulting solution can be used to determine the critical pumping rate due to pumping from multiple wells in a wellfield.

Drawdown along interface

The drawdown along the salt-water fresh-water interface due to pumping from a single well is given by (Motz, 1992) (Fig. 1):

\[ s_i = \frac{Q}{2\pi T} \left[ K_o \left( \frac{r}{B} \right) + \frac{f}{2} \right] \]

where:

\[ \frac{1}{B} = \left( \frac{K'}{b'K_r b} \right)^{1/2} \]

and:

\[ f = \frac{4b}{\pi(\ell - d)} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left[ \sin \left( \frac{n\pi\ell}{b} \right) - \sin \left( \frac{n\pi d}{b} \right) \right] \cdot K_o \left[ \left( \frac{K_i}{K_r} \right)^{1/2} \left( \frac{n\pi r}{b} \right) \right] \]

If multiple wells are present, the drawdown along the interface is the sum of the drawdowns due to \( M \) number of individual wells, or:

\[ s_i = \sum_{m=1}^{m=M} \frac{Q_m}{2\pi T} \left[ K_o \left( \frac{r_m}{B} \right) + \frac{f_m}{2} \right] \]

Upconing of interface

Based on the Ghyben-Herzberg relation, the rise in the salt-water fresh-water interface due to the drawdown along the interface is (Bear, 1979):

\[ \Delta = \delta s_i \]
where:

\[ \delta = \frac{\gamma_f}{\gamma_s - \gamma_f} \tag{6} \]

Combining equations (4) and (5) expresses the interface rise in terms of the pumping rates of the wells, or:

\[ \Delta = \delta \sum_{m=1}^{m=M} \frac{Q_m}{2\pi T} \left[ K_o \left( \frac{r_m}{B} \right) + \frac{f_m}{2} \right] \tag{7} \]

The largest drawdowns and resulting interface rises generally will occur beneath each of the pumped wells, or approximately at \( r = r_w \) for each well.

**Critical rise of the interface**

The critical rise of the interface can be assumed conservatively to occur when the interface has risen to a height equal to 0.3 times the distance between the initial location of the interface and the bottom of the pumped well (Motz, 1992), or:

\[ \Delta_c = 0.3(b - \ell) \tag{8} \]

Substituting equation (8) into equation (7) with \( r = r_w \) and solving for the critical pumping rate at a pumped well under consideration yields:

\[ Q_c = \frac{2\pi (0.3) T (b - \ell)}{\delta \left\{ \left[ K_o \left( \frac{r_w}{B} \right) + \frac{f_w}{2} \right] + \sum_{m=2}^{m=M} \alpha_m \left[ K_o \left( \frac{r_m}{B} \right) + \frac{f_m}{2} \right] \right\}} \tag{9} \]

where:

\[ \alpha_m = \frac{Q_m}{Q_c} \tag{10} \]

The first bracketed term in the denominator of equation (9) represents the drawdown effects of pumping from the pumped well under consideration, and the terms in the summation represent the interference effects from the other pumped wells in a wellfield. The coefficients \( \alpha_m \) in equation (10) express the ratio of the pumping rate of each of the other pumped wells to the pumping rate of the pumped well under consideration.

From equation (9), the critical pumping rate also can be written in a nondimensional form as:

\[ \frac{Q_c \delta}{ Tb} = \frac{2\pi (0.3) (b - \ell)}{b \left\{ \left[ K_o \left( \frac{r_w}{B} \right) + \frac{f_w}{2} \right] + \sum_{m=2}^{m=M} \alpha_m \left[ K_o \left( \frac{r_m}{B} \right) + \frac{f_m}{2} \right] \right\}} \tag{11} \]
EXAMPLE

Data describing the Geneva fresh-water lens in Seminole County, Florida, USA (Panday et al., 1990 and 1993) were selected to illustrate how equation (9) can be used to estimate the critical pumping rate for a wellfield. Seminole County is in the east-central part of peninsular Florida, and the area is underlain by limestone formations of Eocene and Oligocene ages that extend to a depth of about 450 m below sea level. These formations are overlain by unconsolidated sand, silt, and clay deposits of Miocene and post-Miocene ages. These geologic units form a hydrologic system that consists of a surficial aquifer, an intermediate confining bed, and the upper Floridan aquifer, which is part of a major regional aquifer system (Miller, 1986).

The Geneva fresh-water lens is an isolated lens of fresh water surrounded laterally and vertically on the bottom by brackish water in the upper Floridan aquifer. The lens is sustained by fresh-water recharge from the surficial aquifer through the confining bed into the upper Floridan aquifer. It occupies a horizontal area of approximately $1.1 \times 10^8 \text{ m}^2$, and its vertical extent is on the order of 135 m or more below the top of the upper Floridan aquifer (Fig. 2).

The critical pumping rate was calculated for a simulation that corresponds to pumping from four wells located at the centre of the lens using the data from Panday et al. (1990 and 1993). The chloride concentration at the bottom of the aquifer is $c_0 = 10 \ 000 \ \text{mg} \ \text{l}^{-1}$, and it was assumed that the salt-water fresh-water interface could be represented by the $c/c_0 = 0.5$ concentration contour. The distance from the top of the aquifer to the interface at the centre of pumping is $b = 129 \ \text{m}$ (Fig. 2). Values for $T = 1.76 \times 10^{-2} \ \text{m}^2 \ \text{s}^{-1}$, $K_r = 1.36 \times 10^{-4} \ \text{m} \ \text{s}^{-1}$, and $K_z = 2.00 \times 10^{-7} \ \text{m} \ \text{s}^{-1}$ were obtained by averaging the appropriate horizontal and vertical hydraulic conductivities in Table 1 in Panday et al. (1993), and then $K_z/K_r = 0.001 47$ was obtained. The spatially averaged leakance of the confining bed in the vicinity of the lens is $K'/b' = 1.16 \times 10^{-9} \ \text{s}^{-1}$. For $c_0 = 10 \ 000 \ \text{mg} \ \text{l}^{-1}$, the density of salt water is $1.013 \ \text{g cm}^{-3}$ and $\delta = 76.9$ from equation (6).

![Fig. 2 Relative concentration of chloride through A-A' for steady-state simulation of the Geneva fresh-water lens (Panday et al., 1990 and 1993).]
The simulated wells were open from \( d = 24.4 \text{ m} \) to \( t = 64 \text{ m} \), and \( d/b = 0.189 \) and \( t/b = 0.496 \). Values of \( Q_c \) are relatively insensitive to changes in \( r_w \) that would be encountered in typical field situations, and a value of \( r_w = 0.3 \text{ m} \) was chosen to represent the radii of each of the pumped wells. The wells were located in plan view in an approximately square pattern 610 m on each side, and values of \( r_2 = r_3 = 610 \text{ m} \) and \( r_4 = 863 \text{ m} \) were used to determine interference effects at the pumped well under consideration. Also, it was assumed that the pumping rates at each of the wells would be the same, and the pumping rate coefficients were \( \alpha_2 = \alpha_3 = \alpha_4 = 1.0 \).

Using these parameters and equations (9) and 10, the critical pumping rate was determined to be \( Q_c = 0.006 42 \text{ m}^3 \text{s}^{-1} \) for one well. Due to the symmetry of the square array, the drawdowns and interface rises would be the same at each well, and thus the total \( Q_c = 0.0257 \text{ m}^3 \text{s}^{-1} \) for the four wells in the simulated wellfield.

**DISCUSSION**

In obtaining the analytical solution for the critical pumping rate (equation (9)), it is assumed that \( \Delta/(b - \ell) = 0.3 \) is the critical rise for the salt-water fresh-water interface. This assumption is necessary in order for the interface rise to be relatively small in the analytical solution. Results calculated by Panday et al. (1990 and 1993) using a density-dependent, finite-element flow and transport code called DSTREAM indicate that stable interface conditions can exist at rises greater than \( \Delta/(b - \ell) = 0.3 \) with correspondingly greater \( Q_c \) values. Thus, the value of \( Q_c \) calculated using equation (9) appears to represent a conservatively low value for \( Q_c \). Within the limits of its applicability, equation (9) could be used to estimate the critical pumping rate as part of an initial, or a preliminary, analysis; also, it could be used to calculate a lower bound to help verify results from other solutions.

**REFERENCES**


Miller, J. A. (1986) Hydrogeologic framework of the Floridan aquifer system in Florida and in parts of Georgia, Alabama, and South Carolina. USGS Prof. Pap. 1403-B.


