Relation of hydraulic conductivity and dispersivity to scale of measurement in a carbonate aquifer

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Abstract Tests on several scales were analyzed and conducted in a dolomite aquifer of southeastern Wisconsin to examine the variation of hydraulic conductivity and longitudinal dispersivity with scale of measurement. Scale behavior of hydraulic conductivity was shown to be dependent on the hydrofacies. In vuggy facies dominated by porous flow hydraulic conductivity increased relative to scale linearly with an upper bound on a log-log plot. A similar increase of longitudinal dispersivity with scale for these facies is probable but not certain, due to insufficient data. In fractured-flow dominated mudstone facies hydraulic conductivity also increases with scale. However, its log-log slope was steeper, and neither an upper nor a lower bound is apparent.

INTRODUCTION
There are various reports of hydraulic conductivity and dispersivity increases with scale of measurement in the literature (Neuman, 1990; Wheatcraft & Tyler, 1988; Kinzelbach, 1990; Clauser, 1992). Most of the papers summarize different types of rocks and aquifers or treat scale behavior statistically (Neuman, 1994; Gelhar et al., 1985). Only a few authors have compiled data from the same aquifer or geologic formation and analyzed scale behavior in a specific rock type (Rovey & Cherkauer, 1994; Herzog & Morse, 1984). This paper will go a step further and introduce a facies approach to the scale behavior of hydraulic conductivity and longitudinal dispersivity. The study area is a carbonate aquifer of SE Wisconsin; an aquifer where porous, double-porosity and fractured hydrofacies are present.

Three formations of this aquifer will be introduced: the Thiensville Formation, the Racine Formation and the Mayville Formation. The Thiensville Formation consists of four different hydrofacies: a soil facies, a solutioned facies with big vugs, a mudstone facies and a mixed facies, which is mainly made up by recrystallized dolomite, algal laminates and packstones with thin layers of the previously mentioned facies. The four different facies can be distinguished by their hydraulic conductivities on the pressure-packer test scale (Fig. 1). The Thiensville Formation as a unit shows the porous Theis curve responses in pumping tests, and has a unimodal hydraulic conductivity distribution (as indicated by specific capacity data).

The Racine Formation consists of a mudstone facies and a vuggy, porous facies called the Romeo beds. The mudstone facies can be subdivided into a fracture-flow
Fig. 1 Differentiation of the hydrofacies of the Thiensville Formation on the pressure-packer test scale. Some increase of hydraulic conductivity with scale can be recognized. Scale was calculated by assuming cylindrical flow to the packed-off interval.

controlled facies where most of the flow is transmitted, and a porous flow dominated facies. The hydraulic conductivity distribution of the Racine Formation as unit is bimodal (as indicated by pressure-packer test results; Rovey, 1990) or even trimodal (as indicated by specific capacity data).

The last analyzed facies is the vuggy facies of the Mayville Formation. Fluid flow through the vuggy Mayville facies is primarily transmitted by the porous medium, secondarily by fractures.

**MEASURE OF SCALE**

While the common scale measure for dispersivity behavior is a distance, such as the distance to the advective front or a radius of influence, the scale measure of hydraulic conductivity behavior is better expressed as a volume of effected geological unit. For example, consider comparing slug tests and permeameter tests. The radius of influence of a slug test is only several cm into the geological unit, while its vertical scale is in the meter range. The flow distance through a lab sample in permeameter tests is also several cm, while the transverse radius is only centimeters. Using a radius of influence approach would project both tests basically on the same scale even though they are not. The effected volume of geological unit approach separates these two tests by orders of magnitude. The effected volume of the geological unit can generally be obtained by dividing the volume of water used in the test by the unit’s porosity. The porosity of a porous facies was obtained by the Principle of Archimedes, the porosity of a fractured facies by equation (1), which was modified from Snow (1968).

\[
\eta_f = 2 N_h b_h + N_v b_v
\]  

(1)
where $n_f$ is the fracture porosity, $N_h$ and $N_v$ are the number of fractures per distance in horizontal and vertical direction, and $b_h$ and $b_v$ are the mean apertures of the fractures in horizontal and vertical direction, respectively.

**TESTS USED FOR DIFFERENT SCALES**

The test on the lowest scale is a falling head permeameter test for low conductivity carbonate units ($\leq 5 \times 10^{-8}$ m s$^{-1}$) and a constant head permeameter test for high conductivity carbonate units ($\geq 5 \times 10^{-8}$ m s$^{-1}$). The tested samples had a length between 2.5 and 4 cm and are 3.3 cm in diameter. The lab columns are so designed to measure both hydraulic conductivity and longitudinal dispersivity. The hydraulic conductivity is measured by the flow rate under a specified gradient. Then a KCl tracer is applied to measure the breakthrough curve via electrical resistance to obtain the longitudinal dispersivity. The scale of the lab tests for hydraulic conductivity is the volume of the rock sample. The dispersivity scale is the flow distance through the sample.

Slug tests were conducted in several piezometers, which were screened in a specific geological unit. The head rise or drop was measured and the hydraulic conductivity was obtained (Freeze & Cherry, 1979). The scale of the slug tests is the removed or introduced volume of water divided by the porosity of the hydrofacies.

Pressure-Packer tests were conducted in sets (MMSD, 1984). Pressures of 0.07, 0.14, 0.21, 0.34 and 0.69 MPa (10, 20, 30, 50 and 100 psi) were applied and a geometric mean of the hydraulic conductivity was calculated for each set. The scale is calculated by measuring the volume introduced into the unit and dividing by the porosity of the facies.

Assigning a scale to single and multiple well pumping tests was somewhat problematic, since in most cases the hydraulic conductivity was found to increase with discharge volume during the pumping test (Fig. 2). Therefore a method, which uses successive five-point sets of observation data, was applied to obtain average hydraulic conductivities for the pumping tests (detail of the method is provided in Schulze-Makuch & Cherkauer, 1994). In general, the scale was calculated by multiplying the pumping rate by half of the time length of the test and then dividing the product by the porosity of the facies.

A huge data base is available from well construction reports, since such reports must be filed with the state for all water supply wells in Wisconsin. During construction of each well a specific capacity test is done. The specific capacities have been analyzed by an computerized technique using an algorithm to estimate hydraulic conductivity (Bradbury & Rothschild, 1985). The scale of measurement is calculated by dividing the volume of water removed by the porosity of the facies.

One multiple well tracer test conducted in the Mayville Formation was used to determine a longitudinal dispersivity (MMSD, 1983). The scale measure of dispersivity was the distance between the observation well and the pumping well where breakthrough occurred.

Due to high pumpage in the dolomite aquifer and the construction of a sewage tunnel, Lake Michigan water is induced into the dolomite aquifer of SE Wisconsin. Since Lake Michigan water has a different chemical composition than ambient ground
Fig. 2 Scale dependency during pumping test. Hydraulic conductivity of the geological unit increases with discharge volume. Hydraulic conductivities were calculated for successive five-point sets of observation data utilizing the Theis matching curve procedure.

water, the breakthrough of Lake Michigan water can be observed by analyzing the chemical composition of water in nearby piezometers. The scale for dispersivity is the breakthrough distance, which is the distance from the lake shore to the piezometer, where breakthrough occurred. The scale for hydraulic conductivity is obtained by multiplying the breakthrough distance with the transverse distance and the facies thickness. The transverse distance depends on the geology but is not larger than the breakthrough distance.

One computer model (Mueller, 1992) was used to obtain hydraulic conductivity values for the porous-flow facies on the largest scale. The scale for hydraulic conductivity is determined by multiplying the flow distance of the model by its transverse distance and facies thickness.

**HYDRAULIC CONDUCTIVITY BEHAVIOR**

Enough data were collected for the mixed facies of the Thiensville Formation to establish a scale behavior of hydraulic conductivity (Fig. 3). Even though the mixed facies is quite heterogeneous, hydraulic conductivity seems to increase linearly on a log-log plot. At a higher scale the increase of hydraulic conductivity with scale levels off (upper cutoff), and the average hydraulic conductivity is constant with scale. This leveling off is a homogenization effect. The effect of the inherent heterogeneities decreases with scale to a certain point, where the geological unit becomes so homogeneous, that no further increase of hydraulic conductivity with scale occurs.

The Racine Formation was divided into three hydrofacies: the fractured mudstone facies, the non-fractured mudstone facies and the vuggy Romeo facies. The two
Relation of hydraulic conductivity and dispersivity to scale of measurement

Fig. 3 Scale dependency of the mixed facies of the Thiensville Formation. Hydraulic conductivity increases relative to scale but displays an upper bound. Number of observations are given in parenthesis. Lab and slug tests are given as geometric mean with standard deviation.

Fig. 4 Scale dependency for facies of the Racine Formation. The Romeo facies shows a linear increase of hydraulic conductivity with upper bound, while the mudstone facies shows an unbounded linear increase with a steeper slope. Numbers of observations upon which geometric mean is based: from left to right; Romeo facies $N=(7,6,18,2,51,2,1,1)$; fractured mudstone facies $N=(10,7,1,64)$; non-fractured mudstone facies $N=(3,21,114)$. WCR = well construction reports (specific capacity data). The hydraulic conductivities of the non-fractured mudstone facies on the lab scale were below the detection limit of the permeameter ($5 \times 10^{-11} \text{ m s}^{-1}$).
mudstone facies appear as one line on the log-log plot without upper or lower bound (Fig. 4). No separation of the two mudstone facies seems to occur on the plot. Further data collecting is in progress to obtain high and low scale data for the mudstone facies to reveal a probable homogenization effect at some scale. The scale behavior of the vuggy Romeo facies is revealed "nicely". The slope of the line on the log-log plot is less than that of the mudstone facies (Fig. 4). At the scale of the well construction tests the homogenization effect occurs and hydraulic conductivity remains constant with scale thereafter.

The vuggy facies of the Mayville Formation shows a very similar increase of hydraulic conductivity with scale (Fig. 5) to the mixed facies of the Thiensville Formation. Although the absolute hydraulic conductivities of the Mayville facies are somewhat lower, the linear increase of hydraulic conductivity with an upper bound on a log-log plot is also present.

![Scale Dependency of Mayville Facies](image)

**Fig. 5** Scale dependency of Mayville facies indicating a hydraulic conductivity increase relative to scale with an upper bound. Packer and lab test results are given as geometric mean with standard deviation. Other data points are single observations.

**DISPERIVITY BEHAVIOR**

Scale behavior of dispersivity is more difficult to examine, since most aquifer tests are not designed to measure dispersivity. The in-field measured dispersivities were obtained from one multiple well tracer test in the Mayville facies and from one breakthrough of Lake Michigan water in the Romeo facies. Lab test dispersivities are given in Table 1. Due to the shortage of test results, data from other carbonate aquifers were acquired (Baker, 1977; Lallemand-Barrès & Peaudecerf, 1978; Gelhar et al., 1985). Figure 6 is a plot of longitudinal dispersivity relative to scale, which shows that
Table 1 Dispersivity results from permeate tests, vuggy carbonate hydrofacies, scale: 2.5-4 cm.

<table>
<thead>
<tr>
<th>Facies</th>
<th>Observations N</th>
<th>Mean Dispersivity (cm)</th>
<th>Standard Deviation (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed, Thiensville</td>
<td>7</td>
<td>0.96</td>
<td>0.47</td>
</tr>
<tr>
<td>Romeo-style</td>
<td>1</td>
<td>1.73</td>
<td>----</td>
</tr>
<tr>
<td>Mayville</td>
<td>3</td>
<td>0.98</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Scale Dependency of Longitudinal Dispersivity

Fig. 6 Scale dependency of longitudinal dispersivity. Lab tests produce lower dispersivities than field tests. The data from other carbonate aquifers were obtained from Gelhar et al. (1985), Lallemand-Barrès & Peaudecerf (1978), and from Baker (1977).

Lab dispersivities are much smaller than in-field measured dispersivities. Data are still needed on the scale of a meter. We speculate on a similar relationship to that of the hydraulic conductivity-scale plots (line on log-log plot with an upper bound). If this is the case, a simple relationship between hydraulic conductivity and longitudinal dispersivity should follow. More dispersivity data have to be collected from the same or similar hydrofacies in order to lead to conclusions.

REFERENCES


