A method to predict final subsidence basins by means of a finite difference computer code

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Abstract In this paper a method to predict subsidence due to total extraction of flat coal seams is presented. The method is based on the adequate characterization and the correct definition of the constitutive model of behaviour of rock masses, and is implemented in a finite difference numerical code. Due to the existence of an enormous quantity of data related to material properties as well as to subsidence measurements in the UK, the method is validated with acceptable results for standard British carboniferous coal measures rock masses.

INTRODUCTION

During the last decades many researches have been devoted to understand and adequately predict the phenomenon of subsidence due to total extraction of coal seams. In the recent years, research has been focused not only on the surface effects, but on the actual behaviour of cover rock. The main aim of our work has been to model correctly the behaviour of rock masses in such a way that displacements at the surface and plastification of the overburden could be predicted and compared with measured data.

In order to reach this objective, it has been decided to model standard English coalfields rock masses. The main reasons are on the one hand the many data about the features and properties of materials present in English coalfields (Coal Measures) and on the other hand the existing sufficiently accurate empirical prediction methods to check the results.

The prediction method to compare our results to, has been an empirical graphical method produced by the National Coal Board from subsidence measurements taken in more than one hundred collieries in different British coalfields for fifteen years. This prediction system was first published in the so called Subsidence Engineer's Handbook (SEH) in 1965 and updated 10 years later (National Coal Board, 1965, 1975). This method presents a series of formulae and graphs which can be used to predict future subsidence effects. Thus, it applies only to mining and ground conditions similar to those where the measurements were taken, i.e. to an overburden consisting predominantly of mudstone and siltstone, for which cases the model can predict the subsidence with an accuracy better than 10% of the extraction height.

When a wide enough longwall coal panel is mined, the cover rocks suffer different degrees of deformation and damage. Peng (1992) differentiates four zones in the overburden:
- **Caved Zone**: The immediate roof caves irregularly, filling the void. The strata lose their continuity and bedding planes disappear.
- **Fractured Zone**: Located over the caved zone, its main feature is the loss of continuity and breaking or yielding of materials but some bedding planes may remain.
- **Continuous Bending Zone**: In which strata bend downwards without breaking. Only occasional tension cracks can be observed.
- **Soil Zone**: The surface layer, whose behaviour is very site-dependent.

The first two zones can be considered plastified, that's to say their forming materials yield, so they can be simulated by means of an elasto-plastic constitutive behaviour model. According to Peng (1992) the combined height of these two zones is in general 20 to 30 times the extraction height, being bigger for hard strata and vice versa. The two upper zones do not suffer yield, they just deform elastically, so they can be considered elastic.

Following Farmer (1983) and Hoek & Brown (1980), and after some numerical validation studies, it was decided to use an elasto-plastic model with the following features:
- Elastic transversely isotropic behaviour before yield, as was used by Yao et al. (1993), in a similar finite element analysis.
- Anisotropic rock strength due to the presence of weakness planes, as it was observed in slates by Jaeger & Cook (1976). This can be modelled by the "ubiquitous joint model" provided in the used code, in which the Hoek-Brown failure criterion is used for the material, while the strength of the planes of weakness is expressed by means of the Mohr-Coulomb parameters.
- Isotropic elastic post-failure behaviour, because it is considered that the rock mass loses its anisotropy after yielding.

**COALFIELD ROCK MASS CHARACTERIZATION**

A typical English coalfield, as those where the SEH measurements were taken, is basically formed by mudstone and siltstone. Although sandstone and underclay may exist in small amounts, they are not going to be taken into account in this analysis.

To characterize a rock mass in order to define its mechanical behaviour, Hoek (1994) defined the Geological Strength Index (GSI). The calculation of the GSI of the British Coal Measures containing 50% mudstone and 50% siltstone is presented in Table 1.

**Strength properties**

Hassani & Scoble (1981) present the following data of uniaxial compressive strength (UCS) and triaxial failure behaviour of mudstone and siltstone, according to Bieniawski's yield criterion:

\[
mudstone: \text{UCS} = 39 \text{ MPa} \quad \frac{\tau_m}{\sigma_c} - 0.05 = 0.888 \cdot \left( \frac{\sigma_m}{\sigma_c} \right)^{0.767} \quad (1)\]
Table 1 Estimation of the GSI of a Coal Measures rock mass.

<table>
<thead>
<tr>
<th></th>
<th>Mudstone</th>
<th></th>
<th>Siltstone</th>
<th></th>
<th>Average rock mass</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Values</td>
<td>Rating</td>
<td>Value</td>
<td>Rating</td>
<td>Rating</td>
<td>Rating</td>
</tr>
<tr>
<td>UCS</td>
<td>30 MPa *</td>
<td>4</td>
<td>53.9 MPa *</td>
<td>7</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>R.Q.D.***</td>
<td>58% **</td>
<td>12</td>
<td>90% **</td>
<td>19</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>Joint spacing</td>
<td>250 mm</td>
<td>13</td>
<td>300 mm</td>
<td>14</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>Conditions joints</td>
<td>rough-hard</td>
<td>20</td>
<td>rough-hard</td>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Groundwater</td>
<td>-</td>
<td>10</td>
<td>-</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>GSI</td>
<td>59</td>
<td></td>
<td>70</td>
<td></td>
<td>64.5</td>
<td></td>
</tr>
</tbody>
</table>

* Data estimated from Hassani & Scoble (1981).
** Data obtained from Spears & Taylor (1972).
*** R.Q.D. = Rock Quality Designation. The rest of the data are estimates.

Table 2 Strength properties of the rock masses.

<table>
<thead>
<tr>
<th>Material</th>
<th>Mudstone</th>
<th>Siltstone</th>
<th>Rock mass (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSI</td>
<td>59</td>
<td>70</td>
<td>64.5</td>
</tr>
<tr>
<td>Hoek-Brown parameters of rock</td>
<td>UCS = 30 MPa</td>
<td>UCS = 53.86 MPa</td>
<td>UCS = 41.92 MPa</td>
</tr>
<tr>
<td></td>
<td>m = 12.14</td>
<td>m = 16.39</td>
<td>m = 14.26</td>
</tr>
<tr>
<td>Hoek-Brown parameters of &quot;rock mass&quot;</td>
<td>m(i) = 2.8077</td>
<td>m(i) = 5.6139</td>
<td>m(i) = 4.014</td>
</tr>
<tr>
<td></td>
<td>m(b) = 0.6492</td>
<td>m(b) = 1.9228</td>
<td>m(b) = 1.13</td>
</tr>
<tr>
<td></td>
<td>s(i) = 0.00107 U.T.S.</td>
<td>s(i) = 0.0356</td>
<td>s(i) = 0.0194</td>
</tr>
<tr>
<td></td>
<td>s(b) = 0.0014</td>
<td>s(b) = 0.00674</td>
<td>s(b) = 0.00269</td>
</tr>
<tr>
<td></td>
<td>0.112 MPa</td>
<td>0.341 MPa</td>
<td>0.2019 MPa</td>
</tr>
</tbody>
</table>

Table 3 Strength properties of the discontinuities of the rock mass.

<table>
<thead>
<tr>
<th></th>
<th>Mudstone</th>
<th>Siltstone</th>
<th>Average rock mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion (kPa)</td>
<td>Range</td>
<td>0-200</td>
<td>100-300</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Friction (°)</td>
<td>Range</td>
<td>21-33</td>
<td>28-33</td>
</tr>
<tr>
<td></td>
<td>Mean</td>
<td>27</td>
<td>30.5</td>
</tr>
</tbody>
</table>

\[ \text{siltstone: } \text{UCS} = 63 \text{ MPa } \frac{\tau_m}{\sigma_c} - 0.06 = 0.944 \cdot \left( \frac{\sigma_m}{\sigma_c} \right)^{0.813} \] (2)

With these data and the previously obtained GSI, the values "m" and "s" of intact and broken rock mass for mudstone, siltstone and average rock, as shown in Table 2, can be deduced from the equations of Hoek (1994). The Mohr-Coulomb parameters, namely cohesion and friction angle, at the natural interfaces obtained by Hassani & Scoble (1981) are presented in Table 3.
Deformability properties

The deformability parameters have a great influence on the final shape of the subsidence basin, that’s why they have been extensively studied. Hassani & Scoble (1981) in his laboratory studies obtained a Young’s modulus of elasticity of 5000 MPa and 6000 MPa for mudstone and siltstone rocks respectively. The data of deformability of the rock mass estimated after several sources are presented in Table 4.

Table 4 Deformability properties of the rock mass.

<table>
<thead>
<tr>
<th></th>
<th>Intact rock mass (transversely isotropic elastic model)</th>
<th>Broken rock mass (elastic isotropic model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>980 MPa*</td>
<td>$E = 300$ MPa†</td>
</tr>
<tr>
<td>$E_y/E_x$</td>
<td>$1.72^{**}$</td>
<td>$v = 0.275^{***}$</td>
</tr>
<tr>
<td>$G_{xy}$</td>
<td>33.02 MPa**</td>
<td></td>
</tr>
<tr>
<td>$v_x$</td>
<td>$0.2^{**}$</td>
<td></td>
</tr>
<tr>
<td>$v_y$</td>
<td>$0.2^{**}$</td>
<td></td>
</tr>
</tbody>
</table>

* Data obtained by means of a benchmarking procedure with computer code analysis.
** Data estimated from Yao et al. (1993).
*** Datum extrapolated from Hoek (1994).

One relation has been found in literature (Afrouz, 1992) giving more or less the same ratio between the Young’s modulus of rock and rock mass obtained in this study. This equation is:

$$E_{\text{rock mass}} = E_{\text{rock}} \cdot \exp(0.564 \cdot RMR - 5.64)$$  \hspace{1cm} (3)

The Young’s modulus of the broken rock mass may be stress dependent, as it usually happens with backfill materials.

NUMERICAL SIMULATION

The main goal of the simulation is to assess the mechanical behaviour of the Coal Measures rock mass, by comparing the results obtained from the computer model to the ones provided by the SEH, taking also into account the plastified zone height.

The code FLAC version 3.22 (Itasca, 1993) has been used to carry out the simulation. This is a two dimensional explicit finite difference code which simulates the mechanical behaviour of rocks, as a continuum, which may undergo plastic flow.

Assumptions, initial conditions, mesh and model restrain

Since FLAC is a bidimensional code, the simulation has to be performed in plane-strain. As far as the coal seam is flat, a vertical symmetry axis exists in its centre, and so only half of the model has to be simulated, saving computer time.

The natural stress field in the area of the mine has proved to be a fairly important and very site-specific issue in subsidence. According to different types of stress measurements carried out in several British coalfields (British Coal Corporation, 1994) it can be concluded that the average horizontal and vertical stresses are very much alike
A finite difference method to predict final subsidence basins

(usually $\sigma_v$ is a little bigger than $\sigma_h$) in most of the deposits studied. Therefore it has been decided to use as initial condition:

$$\sigma_h = \sigma_v = \rho \cdot g \cdot h$$

To make a first approach to the problem a specific mining situation has been selected ($W =$ width $= 600 \text{ m}$, $h =$ depth $= 300 \text{ m}$ and $M =$ extraction height $= 2 \text{ m}$). After several tentative runs it has been decided to use an area of discretization of 850 m wide and 600 m deep. The mesh size is big enough to ensure that the influence of the boundaries is not important in the model behaviour. The zone size is variable, being smaller in zones where important gradients of stress or displacement will occur (around the excavation) and wider close to the boundaries. The mesh is restrained in the horizontal direction at the symmetry line (right border) and at the left boundary, and in the vertical direction at the bottom boundary. The model and its restraints are shown in Fig. 1.

Fig. 1 Area of discretization and boundary conditions.

Fig. 2 Plastification zone after the first run of the case with $W =$ 600 m, $h =$ 300 m and $M =$ 2 m, by means of the ubiquitous joint model. The height of the plastified zone over the longwall coincides with Peng's observations.
Implementation of the predefined constitutive model

The predefined constitutive model cannot be directly implemented in FLAC. To adequately simulate the behaviour of the rock mass the following strategy has been implemented. First, the whole material is modelled as an ubiquitous joint model with isotropic elastic softened post-failure behaviour (this involves isotropic elastic pre-failure behaviour, too). This first run shows up to what height over the seam the plastification reaches (this should accord with Peng’s results). Then, a second and definitive model is run, in which the material up to the height previously obtained is modelled as before and the rest of the rock mass is considered to behave according to be elastic. In Fig. 2 the area that has suffered plastification after the first run is presented.

In this way the caved and fractured zones are simulated by the ubiquitous joint model, because plastification is the most important aspect in the mechanical response of these zones; while the continuous deformation and soil zones are simulated by the transversely isotropic model, since the anisotropic deformation is the basic behavioral phenomenon in this upper part of the model.

RESULTS

The above mentioned supercritical case has been run in order to test up to what extent the subsidence profile calculated resembles the one predicted by SEH. In Fig. 3 both profiles are shown and it can be seen that the maximum subsidences (S) are very similar. All the subsidences obtained are within 10% M of the values predicted by the SEH, but over the ribside the difference is a little bigger. This is probably due to the fact that FLAC models a continuum, where as in reality the presence of strata originates a "bridging effect" making subsidence smaller over the ribside. The code is not capable to simulate this effect.

If the limit angle is defined like the angle formed by a vertical line on the ribside and a line passing through the ribside point and the closest surface point with no subsidence

![Distance from Basin Center](image)

**Fig. 3** Subsidence profiles for the case with $W = 600$ m, $h = 300$ m, and $M = 2$ m, obtained by the proposed method and SEH.
at all; then the limit angle obtained by the proposed method would be more than 45°, while according to SEH this limit angle has to be 35°. Nevertheless, if a second line is defined as passing through the ribside and the ground point where subsidence is not null but very small (0.025 $S = 45$ mm), then the limit angle predicted by the proposed method will be 36°. Taking into account that it is actually impossible to locate precisely the first point with no movement, the above mentioned divergence is meaningless.

The surface horizontal displacements predicted by both methods are shown in Fig. 4. In this case the proposed method overestimates the values of this parameter.

In order to extrapolate the results to different $W/h$ ratios, more cases have been run, varying the extraction height and depth as well. The results of $S/M$ against $W/h$ obtained have been plotted together with the standard SEH curve in Fig. 5. According to this figure the results of $S/M$, foreseen by FLAC for each $W/h$ ratio, are always within 10% of the SEH predictions. So, the method can be considered acceptable.

![Fig. 4 Horizontal displacement profiles for the case with $W = 600$ m, $h = 300$ m, and $M = 2$ m, obtained by the proposed method and SEH.](image)

![Fig. 5 Subsidence/extraction height at various width/depth ratios. Standard SEH curve and different values obtained by the proposed method for different mining geometries.](image)
CONCLUSIONS AND FUTURE WORK

The characterization data of the English Coal Measures and the subsidence prediction method SEH have been successfully used to define a new subsidence prediction numerical method. Although very time-consuming and heavily dependent on the quality of the characterization of the overburden, this method could help not only to deepen the understanding of the phenomenon of mining subsidence due to longwall extraction, but to assess the influence of the overburden mass behavioral properties and to study more complicated mining geometries like inclined seam and multiple-seam mining.

Among the topics on the subsidence phenomenon, found to be definitely important, the natural stress field and the deformability properties of rock masses ought to be highlighted. Specially, this last aspect is of primordial interest and more research should be focused on the estimation of Young’s modulus and Poisson’s ratio of rock masses.

Starting from the results obtained so far, future work will be oriented to the extrapolation of the method to inclined seams and to the realization of some site-specific case studies.

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REFERENCES


