Entropy approach — an alternative for effective prediction of land subsidence from mining

ROSSEN HALATCHEV, GEORGE ANDREEV
Higher Institute of Mining and Geology, Sofia-1156, Bulgaria

DELIANA GABEVA
Geological Institute, Bulgarian Academy of Sciences, Sofia-1113, Bulgaria

Abstract A new approach for thermodynamic modelling of the rock mass is developed, which is intended for predicting the land subsidence due to underground mining activity. It is based on the postulates of Non-Equilibrium Thermodynamics and the rock mass is treated as a dissipative system being in a mechanical non-equilibrium state as a result of receiving and dissipating mechanical energy. The natural system is identified as a viscous-elastic medium, for which is inherent the phenomenon of creep. The influence of the temperature field on the stress-strain state of the rock mass is also taken into account. The thermodynamic criterion for assessing the land subsidence state is the entropy variation.

INTRODUCTION

To be able to predict the land subsidence phenomena is vital for man to be able live in harmony with the environment. In this context the investigations made till the present mark a rich history of the engagement of scientific thought and considerable success at the solution of the problem.

Without launching out into a detailed review of the existed approaches we will summarize only their more considerable shortcomings as:
- predomination of the static approach ignoring time as a factor;
- emphasis mainly on modelling the subsidence form through displacements and/or strains and ignoring the variety of chemical-physical processes occurring in the rock mass;
- using criteria for assessing the subsidence state which are characteristic of the different mechanical processes then ones of the state of a given system;
- neglecting the importance of the fact that the exchange of energy (including thermal energy) between the structural elements of the rock mass determines its real state.

In the present paper a new approach is suggested which aims to solve the above shortcomings. The new approach uses Non-Equilibrium Thermodynamics (Glansdorff & Prigogine, 1973; Nicolis & Prigogine, 1979) which open wide possibilities for adequate modelling of land subsidence. The specific feature of the approach is the inclusion of the dynamic characteristics of the rock mass state in the thermodynamic model (TDM) for assessing the viscosity of the medium which can be determined in situ with geophysical monitoring.
MODEL FORMULATION

The development of TDM needs to be defined some terms in the context of the Non-Equilibrium Thermodynamics. In this connection a question arises about the concrete thermodynamic system that should be modelled. From logical considerations such a system can be identified with a zone of the rock mass with a border $\Gamma_2$ as shown in Fig. 1, in which land subsidence is expected from underground mining. We adopt this system as a common system. It also can be classified as a closed continuous thermodynamic system that exchanges thermal energy at the external border $\Gamma_1$ through solar radiation and at the internal border $\Gamma_2$ through thermal conduction due to the geothermal gradient. The common system is divided into two subsystems A and B with the border $\Gamma_s$ determined by the spatial location of the probable subsidence border. The subsystem A represents the zone of subsidence and it behaves dynamically. Its location is changed in time and space due to the variation of the physical properties of the rock mass and mostly to the influence of the gravitational (and sometimes tectonic) field. This subsystem also represents a closed system exchanging a thermal energy at the borders $\Gamma_1$ and $\Gamma_s$. The subsystem B is identified with the rest part of the common system. This division allows the prediction land subsidence in time and within the framework of the common system. In this way every irreversible process can be localized at the border $\Gamma_s$ with the exception of processes dealing with the chemical reactions (Westerhoff & Van Dam, 1992).

![Fig. 1 Scheme of the thermodynamic model of land subsidence.](image)

The state of the common system can be described in accordance with the postulates of linear Non-Equilibrium Thermodynamics only if the requirement for a local equilibrium at every small element of the rock mass is satisfied (Glansdorff & Prigogine, 1973). Here local equilibrium means that the local entropy has to be the same function of the local macroscopical variables as for the system in an equilibrium state. The admission, however, for a local equilibrium does not contradict the fact that the common system as a whole is in a non-equilibrium state, because irreversible processes can take place in it. Such a kind of admission can be acceptable for the present investigation because a similar approach was already used successfully by Sokolovski (Turchaninov et al., 1989), although in a pure mechanical aspect, applying the conception for a local limit equilibrium at every point of the critical zone affected by engineering work.
Entropy of the system

For the subsidence zone identified with a closed thermodynamic system the following expression of the entropy variation is valid (Nicolis & Prigogine, 1979):

\[
dS = dS^{(i)} + dS^{(e)}
\]

where \(dS^{(i)}\) is the entropy production in subsystem \(A\), \(dS^{(e)}\) is the entropy flow to the subsystem \(A\) at the borders \(\Gamma_1\) and \(\Gamma_s\) (Fig. 1).

The division of the entropy into production (source) and flow aims to be assessed the state of the thermodynamic equilibrium in the case that the entropy production does not exist. Equation (1) can be presented also for an element of the common system as follows:

\[
\sigma[S] = \partial_t (\rho s) + \text{div} \Phi[S] \geq 0
\]

where \(\sigma[S]\) and \(\Phi[S]\) have the meaning of \(dS^{(i)}\) and \(dS^{(e)}\) respectively; \(\rho\) is density; \(s\) is specific entropy.

Having used the equation of the entropy balance (Glansdorff & Prigogine, 1973) and ignoring its terms which reflect the exchange of substance and passing of chemical reactions, we obtain the equation of the entropy production for an element of the common system:

\[
\sigma[S] = \frac{1}{T^2} \sum_{j=1}^{3} w_j \frac{\partial T}{\partial x_j} - \frac{1}{T} \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij} \frac{\partial v_j}{\partial x_i}
\]

where \(T\) is temperature; \(w_j\) is heat flow; \(\partial T/\partial x_j\) is the \(j\)th component of the temperature gradient; \(\sigma_{ij}^{d}\) is tensor of the dissipative (inelastic) stresses; \(\partial v_j/\partial x_i\) is the \(j\)th gradient of velocity in \(i\)th direction.

In the context of Non-Equilibrium Thermodynamics equation (3) for the 3-D case can be expressed as follows:

\[
\sigma[S] = \sum_{\alpha=1}^{3} J_\alpha X_\alpha + \sum_{\alpha=4}^{12} J_\alpha X_\alpha = \sum_{\alpha=1}^{12} J_\alpha X_\alpha
\]

where \(J_{\alpha(1,3)}\), \(X_{\alpha(1,3)}\) are the flows and forces of the thermal conduction:

\[
J_\alpha = w_j \quad \text{and} \quad X_\alpha = -\frac{1}{T^2} \frac{\partial T}{\partial x_j}
\]

\(J_{\alpha(4,12)}\), \(X_{\alpha(4,12)}\) are the flows and forces of the viscous deformation:

\[
J_\alpha = \sigma_{ij}^{d} \quad \text{and} \quad X_\alpha = -\frac{1}{T} \frac{\partial v_j}{\partial x_i}
\]

The flows and forces in equation (4) indicate the passing of two basic processes in the subsidence zone — thermal conduction and viscous deformation. The first process is a vector process while the second is a tensor process. The difference in the character of these processes does not allow them to be combined as it is stated in Glansdorff &
Prigogine (1973) with the aim to be applied the Onsager’s relationship and hence, every flow and force can have its own contribution to the entropy production.

The acceptance of the tensor of the dissipative stresses as a group of flows, respectively the strain rates as forces, is debatable. For example, an attempt was already made for classifying the strains and stresses as flows and forces respectively (Chelidze, 1987). In our case, however, we took into account the conclusion of Glansdorff & Prigogine (1973) that the dissipative stresses represent flows because they correspond to carrying an impulse into the system.

We note also that the entropy production describes the dissipative irreversible processes in the subsidence zone, for which the requirement $\sigma[S] > 0$ is always satisfied. In this case the entropy production represents a thermodynamic quantity due to forces $X_\alpha$ and a kinetic quantity due to flows $J_\beta$. When a state of thermodynamic equilibrium is reached the entropy production becomes $\sigma[S] = 0$.

The full entropy of the subsidence zone with a volume $V$ and for time $t$ can be evaluated using the expression:

$$ S = -\frac{1}{T^2} \int_0^t \left[ \sum_{i=1}^3 \sum_{j=1}^3 w_{ij} \frac{\partial T}{\partial x_j} dV dt - \frac{1}{T} \int_0^t \left[ \sum_{i=1}^3 \sum_{j=1}^3 \sigma_{ij} d \frac{\partial v_i}{\partial x_j} dV dt + \int_{\Gamma_1} w_{\Gamma_1} d\Gamma_1 dt + \int_{\Gamma_2} w_{\Gamma_2} d\Gamma_2 dt \right] \right] (9) $$

where $w_{\Gamma_1}$ and $w_{\Gamma_2}$ are the heat flows to the subsystem A at the borders $\Gamma_1$ and $\Gamma_2$, respectively.

The evaluation of the entropy production for every element of the discretization (if a numerical approach is used for stress-strain modelling of the system) and for the common system allows conclusions to be made about the state of the land subsidence. For example, one of the possible states is the thermodynamic equilibrium ($\sigma[S] = 0$), which definition has a disparate meaning regarding the limit equilibrium of the system in a purely mechanical sense. Also, a non-equilibrium stationary state can take place ($\Phi[S] = -\sigma[S] < 0$), for which is inherent the equality of the quantity of the flow of the energy and the dissipating energy. Hence, the thermodynamic approach suggests wider possibilities for differentiating the state of land subsidence than the pure mechanical approach, predominant in mining practice, which explores only the criterion of the limit equilibrium.

**Stress-strain state**

The mathematical description of the stress-strain state of the land subsidence zone identified with a continuous medium, which is subordinated to the laws of the linear theory of the viscous-elasticity, is reached with the solution of the following system of differential equations:

(a) Equations of the medium movement:

$$ \sigma_{ij,j} + \rho F_i = 0 \quad 1 \leq j, i \leq 3 $$

where $\sigma_{ij} = \sigma_{ij}^e + \sigma_{ij}^d$ is the tensor of the stresses presented by the tensors of the elastic ($\sigma_{ij}^e$) and dissipative ($\sigma_{ij}^d$) stresses; $F_i$ is mass force.

The right part of equation (10) does not include inertial forces because the non-equilibrium process of deformation being discussed from the positions of the
Thermodynamics is realized indeed very slowly and for every discretization in time the movement of the rock mass due to the action of the viscous forces satisfies the requirement for static equilibrium, i.e. the right-hand part should be equal zero. The elastic stresses satisfy Hooke's law while the dissipative stresses are described with an integral equation which can be presented in the following way:

$$\sigma_{ij}^d = f(c_p, c_s, Q_p, Q_s, \nu, t)$$  \hspace{0.5cm} (11)

where $c_p$, $c_s$ are velocities of the longitudinal and transverse seismic waves; $Q_p$, $Q_s$ are quality factors regarding $c_p$, $c_s$; $\nu$ is a strain rate; and $t$ is time.

The model (11) is suggested by Gubkin (1984) and solves successfully the problem for assessing the dissipation of the energy in a solid. It uses the conception that the dissipative stresses are determined by the loading history. The dissipative stresses are proportional to the rate of strains and they relax continuously in time. The action of these stresses leads to a dissipation of a mechanical energy that is reduced into a heat.

(b) Cauchy's equations describing the relationship between the tensor of strains and the vector of the displacements.

(c) Saint Venant's equations for the medium continuity before and after the deformation.

The solution of the above system of equations is made with the Finite Element Method using fixed initial and boundary conditions (Zienkiewicz, 1971; Hematian & Porter, 1993).

The identification of the rock mass within the zone of land subsidence as a viscous-elastic medium is motivated by the fact that the process of deformation in time is a consequence of the creep phenomenon, which is inherent for that kind of medium. The approach suggested for modelling stress-strain has a specific feature due to the inclusion of Gubkin's model. This solution replaces the classical approach involving the use of the well-known Kelvin-Voigt and Maxwell units (Pytel & Chugh, 1992). Also, Gubkin's model allows the organization of a geophysical monitoring of the land subsidence and realization of an automated control of its state. Namely this circumstance that is in unison with the new tendencies in the Rock Mechanics applications for mining purposes (Bawden, 1993; Ferrero et al., 1993; Mendecki, 1993; Halatchev et al., 1993) determines the actuality of the model and approach as a whole.

**Temperature field**

The modelling of the temperature field of the zone of land subsidence in this investigation has a methodological character because of the fact that every concrete mining object has own specific features exceeding the bounds of the standard. From this point of view we treat the case of a stationary process of the thermal conduction which is described with Laplace's equation (Dmitriev & Gontcharov, 1983). The solution of this equation is made with the Boundary Integral Equations Method using fixed boundary conditions at the borders $\Gamma_1$ and $\Gamma_2$ (Gabeva, 1992).

From a thermodynamic point of view Laplace's equation describes a fixed non-equilibrium thermal process in the rock mass that means an invariability of the
temperature in time for every point of the medium but with different assessments at the different points. The support of the thermal process is due to the geothermal gradient.

The evaluation of the heat flow ($w$) in the TDM is made with the use of Fourier's law.

**EXAMPLE SOLUTION**

A computer program TFEM written in QuickBASIC-4.5 version of Microsoft Corp is created for the practical realization of the thermodynamic approach. The program is developed on a module principle and includes also the codes FEM and BIEMH written in FORTRAN 77 and intended for calculation procedures with the Finite Element Method and Boundary Integral Equations Method for the 2-D case (Halatchev, 1993).

A hypothetical land subsidence is investigated with a height of 100 m. The length of the goaf in the rock mass is 65 m. The homogeneous strata are characterized with: $\rho = 20$ kN m$^{-3}$, $c_p = 1950$ m s$^{-1}$, $c_s = 800$ m s$^{-1}$, $Q_s = 30$. The time of the subsidence
Prediction is 6 years. The results about the shear stress distribution obtained by the FEM program are illustrated in Fig. 2. The distribution of the local specific entropy is given in Fig. 3 and its analysis shows an increase of the degree of the dissipation of a mechanical energy in close proximity to the goaf. In the upper central zone of the subsidence the entropy is minimum which means the rock mass subsides with a preservation of the medium continuity, i.e. with minimum deformations. Figures 4 and 5 show the distributions of the forces and flows at points of the probable border $\Gamma_s$ fixed in the Fig. 2. The assessment of the subsidence entropy for the border $\Gamma_s$ is 29.91 MJ K$^{-1}$.

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REFERENCES


