Modelling density-induced convection processes in regions containing saline disposal basins

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Abstract Intercepted saline groundwaters and drainage effluent from irrigation are commonly disposed in natural and artificial saline disposal basins throughout the Murray Basin of southeastern Australia. In this paper, the mixed convection processes which occur below a saline disposal basin are investigated numerically using the 2D model SUTRA. An experimental Hele-Shaw cell analog of a "dry" salt lake is simulated to determine whether SUTRA satisfactorily reproduces density-induced convection observed in the laboratory. The model was then applied to Lake Tutchewop, a saline disposal complex in north-central Victoria, using time-dependent boundary conditions for lake salinity and water levels. The calibrated model was then used to predict lake seepage rates, basin stability and sustainability. Concentration profiles and seepage rates from the basin show that Lake Tutchewop is inherently stable under its present operating regime with downward movement of salt mainly controlled by diffusion and dispersion. A sensitivity analysis of governing variables showed that salt fluxes were most sensitive to lake salinity levels.

INTRODUCTION

In the Murray Basin of southeastern Australia (see Fig. 1), large volumes of saline water are produced annually as a consequence of rising water tables and drainage effluent from irrigation. A major engineering approach adopted to reduce salt accessions to streams and rivers is the diversion of irrigation returns into saline disposal basins. The continued use of saline disposal basins requires a detailed understanding of their groundwater and solute transport dynamics since the environmental impacts of these basins are largely unknown.

Simmons & Narayan (1995a) showed that gravitational instabilities and density-induced convection could significantly exacerbate salt transfer to aquifers when the magnitude of a non-dimensional parameter combining a Rayleigh and modified Peclet number exceeded a certain critical value. The non-dimensional numbers were defined in terms of basin scale hydrogeological parameters. In this paper, we apply the stability criteria developed and the numerical modelling technique used in that work to the Lake Tutchewop saline disposal basin, Victoria.

Lake Tutchewop is located midway between the townships of Kerang and Swan Hill in the Riverine Plain of the Murray Basin. Since 1968, saline water has been pumped from Barr Creek into Lake Tutchewop and two surrounding basins, Lake William and Lake Kelly, via a number of artificial channels (Simmons & Narayan, 1995b). It was

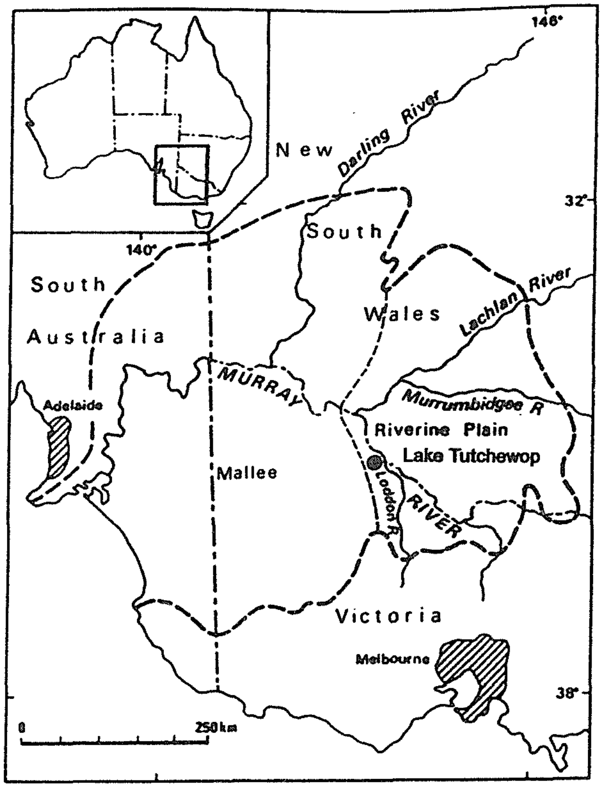

Fig. 1 Map of the Murray Basin and the Lake Tutchewop study areas.

necessary to pump water into these lakes to reduce the salinity input of Barr Creek into the River Murray. The groundwater and salinity dynamics below Lake Tutchewop are poorly understood and the long term stability, sustainability and leakage rates from this basin are unknown.

We use the US Geological Survey model SUTRA (Saturated-Unsaturated TRAnsport) developed by Voss (1984) to simulate variable density flow and solute transport processes in the Lake Tutchewop region. Firstly, a Hele-Shaw experiment of the convection processes associated with a "dry" salt lake was modelled to assess whether SUTRA satisfactorily reproduces density-induced convection phenomena. Then SUTRA was used in an attempt to answer questions about the stability and sustainability of the Lake Tutchewop disposal basin system. We calibrated our model with a suite of piezometric, salinity, hydrochemical and hydrogeological data collected in extensive field and laboratory studies.

GOVERNING EQUATIONS AND THEORY

SUTRA simultaneously solves two partial differential equations, the fluid mass balance equation and the solute mass balance equation. Further details may be found in Voss (1984). Simmons & Narayan (1995a) showed that the stability of a dense saline disposal basin brine was related to the magnitude of at least two non-dimensional numbers, a Rayleigh number and a modified Peclet number. Both of these parameters are defined in terms of basin scale hydrogeological parameters. The stability criteria is stated as:

$$Ra* = \frac{Ra}{1 - Pe*} \leq 1250 \qquad (1)$$

Here, the Rayleigh number Ra is defined by:

$$R_a = \frac{U_c H}{D_T} = \frac{gk\beta(C_{\max} - C_{\min})H}{\epsilon\nu_0(D_0 + \alpha_T V_{amb})} = \frac{\text{Buoyancy forces}}{\text{Resistance forces}} \qquad (2)$$

where U_c is the convective velocity, H is depth of the porous layer, D_T is the transverse dispersion coefficient, D_0 is the molecular diffusivity, g is acceleration due to gravity, k is the intrinsic permeability, $\beta = \rho_0^{-1} - (\partial\rho/\partial C)$ is the linear expansion coefficient, $C_{\max}$ and $C_{\min}$ are the maximum and minimum values of concentration respectively (expressed as solute weight relative to weight of solution), ϵ is the aquifer porosity, $\nu_0 = \mu_0/\rho_0$ is the kinematic viscosity of the fluid, α_T is the transverse dispersivity and V_{amb} is the ambient velocity due to external head gradients.

The modified Peclet number $Pe*$ is used to account for dispersion due to ambient groundwater flow and is defined as:

$$Pe^* = \frac{V_{amb}\alpha_L}{D_0 + V_{amb}\alpha_L} \qquad (3)$$

In the case where the ambient velocity V_{amb} is assumed negligible, free convection dominates and $Pe*$ is negligibly small.

HELE-SHAW CELL

For visual laboratory studies on flow through porous media, the Hele-Shaw cell, a physical analog for saturated flow through porous media, has been used. See Wooding *et al.* (1995) for a review of Hele-Shaw cell work. A simple Hele-Shaw cell model was devised by Wooding *et al.* (1995). Only cases corresponding to evaporation from a "dry" salt lake are modelled in this paper. The Hele-Shaw cell had height $H = 7.5$ cm and length $L = 15$ cm. Along the top boundary of the cell, evaporation occurred for $0 \leq x \leq 5$ cm (representing half of an evaporating salt lake with line of symmetry at $x = 0$). A source representing regional groundwater recharge was also located at the surface of the cell at $10 \leq x \leq 15$ cm. All other boundaries in the Hele-Shaw cell are taken to be no-flow boundaries.

Numerical modelling

A summary of parameters used in the Hele-Shaw cell numerical modelling appears in Table 1 for $Ra = 4870$. Voss (1984) has provided some general guidelines for proper discretization. In most cases, a rule of thumb which guarantees spatial stability is defined by $\Delta_L \leq 4\alpha_L$, where Δ_L is the local distance between sides of an element measured in

Table 1 Hele-Shaw cell parameters.

Freshwater density (ρ_0)	1000 kg m^{-3}
Background concentration (C_0)	84 000 mg l^{-1}
Salt lake concentration (C_{sat})	110 000 mg l^{-1}
Intrinsic permeability ($b^2/12$)	3.68×10^{-9} m^2
Porosity (ϵ)	1
Diffusion coefficient (D_o) K_2SO_4	0.9×10^{-9} m^2 s^{-1}
Initial evaporation rate (= recharge)	1.03×10^{-6} m s^{-1}
Fluid dynamic viscosity (μ)	1.1×10^{-3} kg m^{-1} s^{-1}
Coefficient of fluid density change($\partial\rho/\partial C$)	780 kg m^{-3}
Water compressibility	4.5×10^{-10} Pa^{-1}
Longitudinal dispersivity (α_L)	0.9×10^{-9} m
Transverse dispersivity (α_T)	0.9×10^{-9} m
Cell angle (θ)	5°
Acceleration due to gravity (g sinθ)	0.855 m s^{-2}

the direction parallel to local flow and α_L is the longitudinal dispersivity. A non-uniform mesh was generated and contained 4876 nodes and 4725 elements and was discretized such that if $(X,Y) = (0,0)$ is taken to be the bottom left hand corner of the model, $\Delta x = 0.8$ mm for $0 < x < 60$ mm, $\Delta x = 3$ mm for $60 < x < 150$ mm, $\Delta y = 2.5$ mm for $0 < y < 50$ mm and $\Delta y = 1$ mm for $50 < y < 75$ mm. In this modelling exercise, all transient simulations were run with 400 time steps to cover a simulation time of 6 h. The time step size was held constant at 60 s intervals.

Model calibration and the use of numerical perturbations

In the field, instabilities are caused by preferential flow paths set up by pore scale and regional scale heterogeneities. In the Hele-Shaw cell, instabilities may be caused by non-uniform evaporative deposition and small irregularities in cell plate spacing. In order to calibrate this model satisfactorily, it was considered appropriate to perturb the system by applying random noise of zero mean to the top row of the evaporating lake surface for the duration of the simulation. The magnitude of the perturbation was taken to be 1% of the total salinity difference at saturation.

HELE-SHAW CELL RESULTS

A detailed comparison of the numerical results which follow have been made with experimental results. However at the time this paper was written, we were unable to include the experimental Hele-Shaw cell results. Figures 2(a)-2(h) provide the developing convection pattern obtained numerically using SUTRA. Time was made

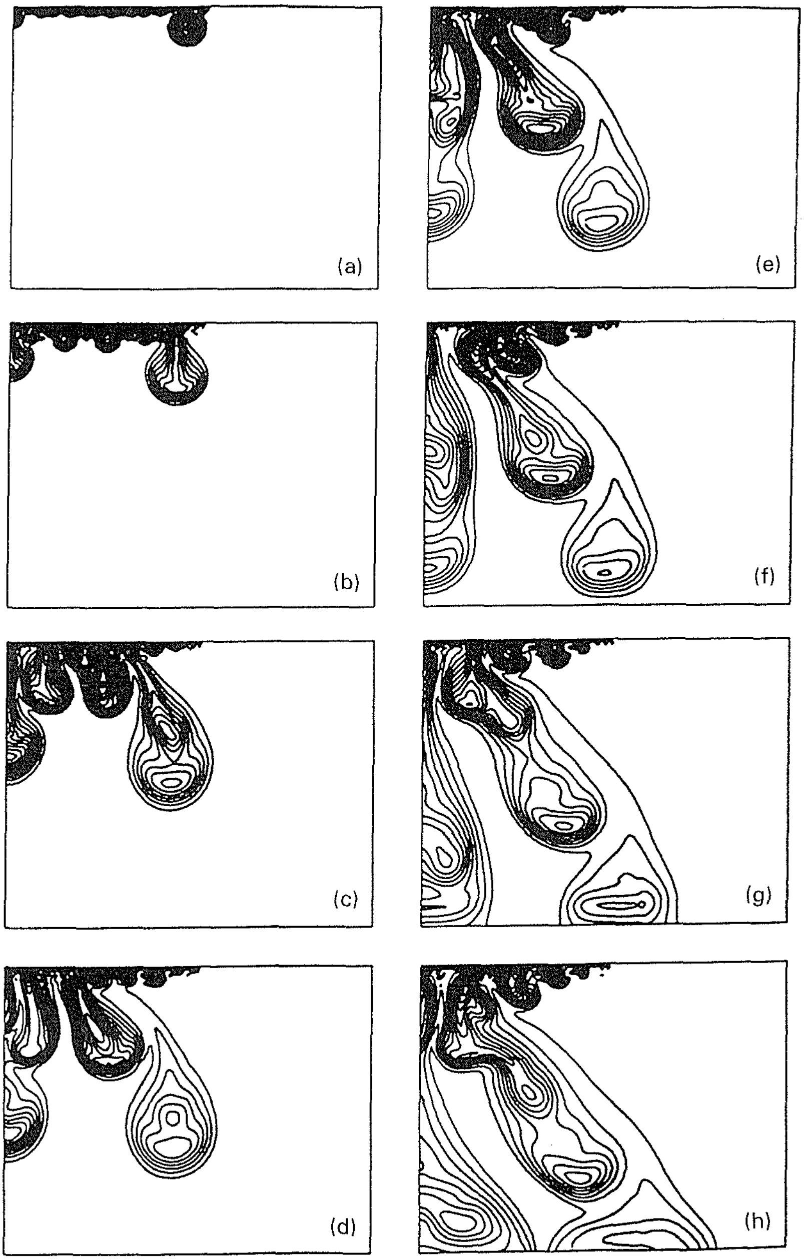

Fig. 2 Developmental stages of unstable plumes in Hele-Shaw cell using the SUTRA solute transport model. Elapsed times (a,b,...,h) given in dimensionless times are T = 2.01, 4.02, 8.03, 10.05, 12.06, 14.06, 16.07, 18.08, measured from start of numerical simulation.

dimensionless using $T = u_c t/H$ with the value for H as defined previously. In the laboratory experiment, one unit of dimensionless time T was equivalent to about 21.4 min. Overall, it was seen that the corresponding plume developments appeared to be well matched both spatially and temporally. A test with Hele-Shaw cell data allowed the reliability of the model to be assessed. We could then proceed to field scale modelling with a model capable of simulating the required convective processes.

LAKE TUTCHEWOP: A CASE STUDY

Site description and Barr Creek salinity management plan

Lakes Tutchewop, William and Kelly form part of the Kerang Lakes system of natural depressions in north-central Victoria (Fig. 1). Prior to 1968, Barr Creek, a natural drainage system in the area, delivered on average 180 000 tonnes of salt to the River Murray each year. It is estimated that Barr Creek carries over 11 000 m^3 of disposal water each year at an average salinity of 4000 mg l^{-1}. To reduce the amount of salt entering the Murray River, water is being pumped from Barr Creek via a number of artificial channels into Lakes Tutchewop, Kelly, and William, which act as evaporative disposal basins.

Numerical modelling

The hydrostratigraphy of the Lake Tutchewop area is presented in the northwesterly transect in Fig. 3. Several bores which lie along this transect were used in model calibration (Bore numbers 6016, 26826, 49558, 49569) and in vertical boundary conditions. A no-flow boundary condition is specified along the bottom of the mesh at depth 90 m where the base of the Parilla Sand is considered to be impervious. Time-dependent heads $h_1(t)$ and $h_2(t)$ were used along the left and right hand vertical boundaries respectively and were extracted from piezometer readings taken in the field.

Lake Tutchewop was represented as a time-dependent concentration source $C(t)$, 3800 m long, located on the surface of the model with position described mathematically

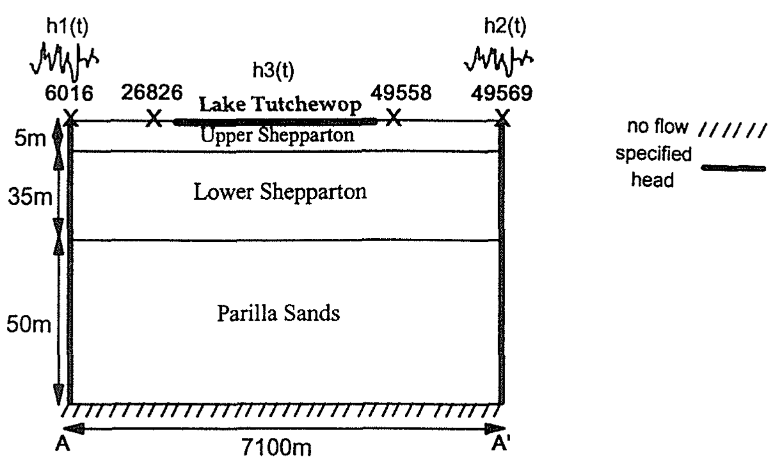

Fig. 3 2D cross-sectional view of Lake Tutchewop model, with boundary conditions.

by $1500 < x < 5300$ m where x is the coordinate variable for the horizontal direction. A fluctuating time-dependent lake head $h_3(t)$ extracted from field data was also employed on the lake surface.

The cross-sectional vertical slice, 7100 m long and 90 m deep was discretized to form 2736 rectangular elements and 2842 (48 × 49) nodes. The horizontal spacing varied from coarser elements (Δx = 200 m) near model boundaries to finer elements (Δx = 100 m) under Lake Tutchewop. The vertical spacing was determined by trial-and-error in an attempt to account for non-uniform vertical conditions in the Shepparton formation and the Parilla Sands group below Lake Tutchewop. Stable solutions were obtained with vertical spacings of Δy = 1 m and Δy = 2 m in the Upper and Lower Shepparton formations respectively, and with Δy = 2 m in the Parilla Sands group. In this modelling exercise, the time step size was held constant at 1 month intervals throughout all simulations to cover a 25 year period.

Model calibration

A linear approximation to the salinity data for Lake Tutchewop measured during 1984-1994 (Fig. 4) was used. The model was calibrated over the 5 year period January 1989-December 1993 using a lake salinity equation given by:

$$C(T) = [73500 + (T - 1)570] \text{ mg l}^{-1}$$

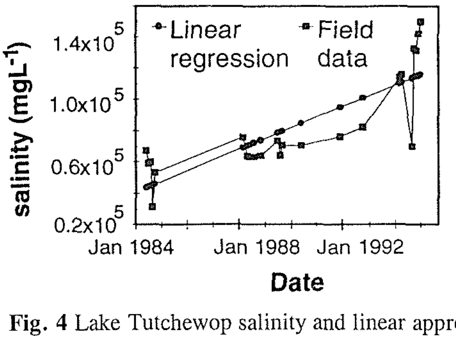

Fig. 4 Lake Tutchewop salinity and linear approximation used in model.

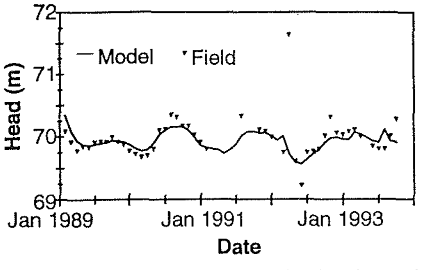

Fig. 5 Model calibration results for the period January 1989-December 1993 at Bore 49558.

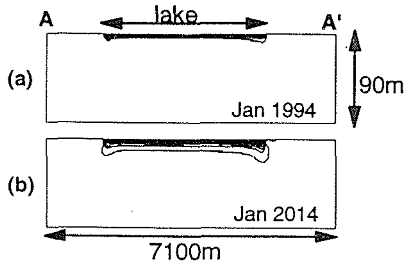

Fig. 6 Computed concentration profiles for below the lake at (a) January 1994 and (b) January 2014.

where T is a month number index or time step counter. The results of one piezometric head calibration are given in Fig. 5. In addition, the salinity profile (see Fig. 6(a)) was compared with salinity data for a borehole at the site (Ferguson, 1993). This borehole did not lie directly on the transect and therefore serves as only an approximate salinity calibration tool (see Fig. 7). The field salinity data and those derived from the model are in good agreement, showing the leakage effects of the current disposal regime to be confined to within 3-4 m of the lake bottom. The parameters determined from this model calibration are given in Table 2.

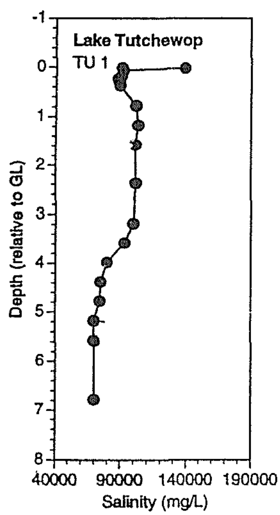

Fig. 7 Measured salinity profile below Lake Tutchewop (Ferguson, 1993).

LAKE TUTCHEWOP RESULTS AND DISCUSSION

Concentration and hydraulic head profiles

Cross-sectional concentration profiles corresponding to January 1994 and January 2014 are given in Fig. 6. In January 1994, model results show that leakage from the current disposal regime is confined to within 3-4 m of the lake bottom which is consistent with

Table 2 Lake Tutchewop model parameters.

Water density (ρ)	1000 kg m^{-3}
Groundwater salinity	70 000 mg l^{-1}
Lake salinity (C_L)	$C(t)$ mg l^{-1}
Fluid dynamic viscosity (μ)	10^{-3} kg m^{-1} s^{-1}
Coefficient of fluid density change($\partial\rho/\partial C$)	700 kg m^{-3}
Water compressibility	4.5×10^{-10} Pa^{-1}
Soil compressibility	1×10^{-8} Pa^{-1}
Hydraulic conductivity of Upper Shepparton	$K_{HUS} = 10^{-1}$ m day^{-1}; $K_{VUS} = 10^{-2}$ m day^{-1}
Hydraulic conductivity of Lower Shepparton	$K_{HLS} = 6 \times 10^{-1}$ m day^{-1}; $K_{VLS} = 10^{-3}$ m day^{-1}
Hydraulic conductivity of Parilla Sands	$K_{HPS} = 1$ m day^{-1}; $K_{VPS} = 10^{-2}$ m day^{-1}
Porosity of Upper Shepparton (ϵ_{US})	0.35
Porosity of Lower Shepparton (ϵ_{LS})	0.35
Porosity of Parilla Sands (ϵ_{PS})	0.25
Longitudinal dispersivity (α_L)	50 m
Transverse dispersivity (α_T)	5 m
Molecular diffusivity (D_0)	2.8×10^{-9} m^2 s^{-1}
Acceleration due to gravity (g)	9.81 m s^{-2}
Aquifer depth (H)	90 m
Aquifer length (L)	7100 m
Lake dimension along the slice	3800 m
Upper Shepparton thickness	5 m
Lower Shepparton thickness	35 m
Parilla Sands thickness	50 m

measured field observations (Ferguson, 1993). After 25 years, results for January 2014 suggest that leakage is confined to within 10 m of the lake bottom. This lake system is inherently stable with very little vertical movement of salt from Lake Tutchewop to the underlying aquifer system. This is in response to the small vertical hydraulic conductivities of the Shepparton formation ($\sim 10^{-2}$-10^{-3} m day^{-1}). Lateral leakage to neighbouring regions is also minimal owing to the very small horizontal hydraulic gradients in the area below Lake Tutchewop (head gradients as small as 200 mm in lateral distances of 1 km). This is also supported by hydrochemical and isotopic data around the lake (Ferguson, 1993; HydroTechnology, 1994).

A computed hydraulic head contour of the aquifer system for January 1994 is given in Fig. 8 and shows the groundwater hydraulics below Lake Tutchewop. These results show that flow is radial into Lake Tutchewop around its periphery and that the regional groundwater flow is in a northwesterly direction, as expected from piezometric observations.

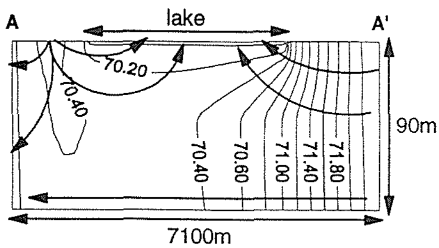

Fig. 8 Modelled hydraulic head (in m) for January 1994.

Lake Tutchewop stability

Neglecting ambient groundwater flow in the region below Lake Tutchewop, equation (2) is evaluated to be $Ra \sim 1$ (at the start of the simulation) for the Upper Shepparton formation. This is well below the critical Rayleigh number of $Ra_{crit} = 1250$ (Simmons & Narayan, 1995a) for the case of free convection alone. Lake Tutchewop is therefore inherently stable owing to both the low vertical permeability and low concentration gradients below its lake bed.

Sensitivity analysis

To assess the sensitivity of seepage to parameter variations, the calibrated model values (see Table 2) for vertical hydraulic conductivity, lake salinity levels and dispersion and diffusion coefficients were varied by ±20%. Relative sensitivity (S) to the model parameter (P) was determined from the relative change in salt seepage (U) ie.

$$S = \frac{U_h - U_l}{U_c} \tag{4}$$

where the subscripts h, l and c refer to the high (+20%), low (−20%) and calibrated values of the parameter being tested (see Table 3). Of the parameters tested, results show that salt flux rates are most sensitive to lake salinity C and least sensitive to molecular diffusion D_o. The effects of increased salinity on hydraulic conductivity has not been accounted for in this study.

Table 3 Effect of model parameters on relative sensitivity.

Parameter (P)	Sensitivity parameter (S)
Lake salinity level (C)	0.39342
Dispersivity (α_L, α_T)	0.11
Vertical hydraulic conductivity (k_v)	0.05
Molecular diffusivity (D_o)	0.01

CONCLUSION

The SUTRA model was firstly used to simulate variable density flow and solute transport at the laboratory scale using a Hele-Shaw cell and by using random noise perturbations so simulate heterogeneity. It was then applied at the field scale to investigate solute transport at Lake Tutchewop, a saline disposal basin in north-central Victoria, Australia. In this case, time-dependent boundary conditions were a critical part of model calibration. Results show that the present effect of the current disposal regime is felt at depths no greater than about 2-3 m, and that even 25 years from present time, the effects of leakage will not be felt at depths greater than 10 m below the lake bed.

Rayleigh number calculations showed that Lake Tutchewop exhibits stable behaviour in the Upper Shepparton formation. This means that leakage of salt is predominantly caused by mechanical dispersion in the aquifer and not by free convective instabilities. It is unlikely that this lake will become unstable owing to the very low permeability of the basin lining.

With several developments to the existing SUTRA model (e.g. use of time-dependent boundaries and random noise functions) calibration at both the laboratory scales and field scales could be achieved. The models and concepts developed in this work may find immediate application in the future design and management of many other saline disposal complexes and natural salt lakes which are a common feature of many arid and semiarid landscapes throughout the world.

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