Determination of the geomorphological instantaneous unit hydrograph using tracer experiments in a headwater basin

JENS MEHLHORN, FRANK ARMBRUSTER, STEFAN UHLENBROOK & CHRISTIAN LEIBUNDGUT
Institute of Hydrology, University of Freiburg, Werderring 4, D-79098 Freiburg, Germany

Abstract The geomorphological instantaneous unit hydrograph (GIUH) is a useful tool to describe basin response. In the present study, results of tracer experiments on a hillslope and the channel network were used to determine the GIUH. The area weighting function of the GIUH was determined using digital elevation data. The combination of hillslope pathway and channel length distributions were used as area weighting function. The resulting GIUH provides a reasonable production of an observed runoff event.

INTRODUCTION

The geomorphological instantaneous unit hydrograph (GIUH) was introduced by Rodriguez-Iturbe & Valdes (1979), who used the geomorphological structure of a fluvial basin to interpret the runoff hydrograph as a travel time distribution. This travel time distribution is controlled by the hillslope and the channel network response.

Often the GIUH is derived only by the use of network geometry like Strahler's ordering scheme (Rodriguez-Iturbe & Valdes, 1979) or channel length distributions (Gupta et al., 1986). In large river basins, the network response imposes the hillslope response, while in smaller catchments the importance of the hillslope response increases. In microscale catchments like headwater basins, the hillslope response governs the travel time distribution.

The objective of the present study was to determine the GIUH of a headwater catchment using information from dye tracer experiments. The following steps were conducted: (a) conducting dye tracer experiments on a hillslope and in river channels, (b) determining flow distance distributions for the hillslopes and river channels using digital elevation data and GIS techniques, (c) regionalizing the parameters obtained by the tracers to the flow distances in hillslopes and channels, (d) calibrating the GIUH with the regionalized tracer parameters, and (e) validating the GIUH with measured rainfall-runoff data.

STUDY SITE

The mountainous basin St Wilhelm Tal in southwest Germany covers an area of 15.2 km² and ranges in elevation from 636 to 1496 m (Fig. 1). More than 80% of the basin consists of slopes of more than 15°, covered with blocks and periglacial debris and 77% of the basin area is forested. An environmental tracer investigation showed
that the main runoff generation takes place in the debris cover of the slopes and a
minor portion of runoff is produced on saturated areas, which are directly connected
to the river network. Field mapping identified saturated areas in 6.6% of the basin
which do not increase during rainfall events. The mean annual rainfall is 1850 mm
which leads to a mean runoff of 1300 mm. Due to the basin mean altitude of 1030 m,
the runoff regime is nival.

TRACER EXPERIMENTS

In order to determine the response functions of the hillslopes and river channels dye
tracer experiments were conducted. The tracer experiments were evaluated using the
following convection dispersion model (CDM):

\[
C(t) = \frac{M}{Q} \frac{1}{t \sqrt{4 \pi P_D (t/t_m)}} \exp \left[ -\frac{(1 - t/t_m)^2}{4 P_D t/t_m} \right]
\]

where \( C \) is the tracer concentration at time \( t \), \( M \) is the injected mass of tracer, \( Q \) is
the flow rate, \( t_m \) is the mean tracer transit time and \( P_D \) is the dispersion parameter.
The dispersion parameter \( P_D \) is defined as:

\[
P_D = \frac{D}{v_x} = \frac{\alpha_l}{x}
\]
where $D$ is the dispersion, $v$ is the mean flow velocity, $x$ is the flow distance and $\alpha_l$ is the longitudinal dispersivity.

The hillslope tracer experiment was conducted in June 1996 after a period with high rainfall and therefore during high soil moisture conditions. The tracer—2.5 kg naphthionat dissolved in 50 l water—was instantaneously injected to a point on the surface of a slope in the study basin. The inclination of the hillslope is 30° and its periglacial debris and block cover was considered to be representative for the study basin. Sampling was done in a spring at the slope bottom. The distance between the injection point and sampling was 88 m. To avoid tracer retardation in the unsaturated zone, the injection point was irrigated for 36 h with 1 l s$^{-1}$ water. The hydraulic conditions were stable since the spring discharge did not increase. The irrigation was stopped 12 h before injection, so that field capacity could become established in the unsaturated zone.

Three hours after injection the maximum tracer concentration was measured in the spring and after 24 h the breakthrough had nearly passed the sampling point (Fig. 2). The measured tracer breakthrough was fitted with the CDM. The best fit was obtained for a $v$ of 8.5 m h$^{-1}$ and a $P_D$ of 0.44 yielding a $\alpha_l$ of 38.7 m. The high $P_D$ indicates a strong heterogeneity of the hillslope causing a wide spectra of flow velocities. This high heterogeneity is based on the wide variety of grain sizes ranging from blocks over debris to loamy sands as weathering residuals. These different substrates effect the formation of macropores having different diameters adjacent to micropore matrices and an associated wide range of flow velocities.

![Fig. 2 Measured and fitted tracer breakthrough of the hillslope experiment.](image)

In the river channel experiment, 2.0 g naphthionat dissolved in 10 l water was instantaneously injected to the river during mean flow. Samples were taken at four different sampling sites. The flow distances in the stream ranged from 300 to 6800 m (Table 1) from the injection point. With this information it was possible to determine the river response at different flow distances. The flow velocities in the stream range from 1000 to 1500 m h$^{-1}$ and are two orders of magnitude higher than those for the hillslope. In contrast, $D$ is very low. The $\alpha_l$ increases slowly with $x$ (Table 1).
Table 1 Parameters of the river tracer experiment at the four sampling sites.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow distance</td>
<td>300</td>
<td>2200</td>
<td>4400</td>
<td>6800</td>
</tr>
<tr>
<td>v (m h⁻¹)</td>
<td>1510</td>
<td>1080</td>
<td>1370</td>
<td>1400</td>
</tr>
<tr>
<td>Dispersivity a</td>
<td>2.7</td>
<td>4.1</td>
<td>5.3</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Although \( x \) increases more than 20 times from 300 to 6800 m, \( a \) increases only 3 times.

**DERIVATION OF THE GIUH**

The discharge at the basin outlet \( Q(t) \) is given by (Robinson & Sivapalan, 1995):

\[
Q(t) = \int_0^L w(x)p_{\text{eff}}(\tau)h(t - \tau)\,dx\,d\tau
\]

and

\[
h(t) = \frac{1}{t\sqrt{4\pi P_D(t/t_m)}} \exp\left[-\frac{(1 - t/t_m)^2}{4P_D t/t_m}\right]
\]

where \( w(x) \) is the area weighting function and \( p_{\text{eff}} \) is the effective precipitation. Neglecting the first term of equation (1), the equations (1) and (4) are identical. Therefore, the parameters determined in the tracer experiments are useful for modelling the discharge at the basin outlet.

Fig. 3 Hillslope flow pathway distances in the study basin.
A main point of interest was the determination of \( w(x) \), because in combination with \( t_m \) and \( P_D \), \( w(x) \) represents the basin characteristics. The used approach combines the channel and the hillslope flow pathway length distributions. It was assumed that the hillslope-pathway length distribution, in combination with the tracer results of the hillslope experiment, represents the hillslope response. The hillslope response was not additionally routed through the river network due to the short transit times in the channel network compared to those in the hillslopes. The channel-length distribution and the tracer parameters of the river experiments were used to model the response function of the saturated areas adjacent to the channel segments.

To determine \( w(x) \) the DEDNM software system (Cluis et al., 1996) and a 50 x 50 m digital elevation model was used. DEDNM produces three outputs:
- flow distance of each hillslope raster cell to the associated river cell,
- flow distance of each river cell to the basin outlet, and
- total flow distance for each raster cell to the basin outlet summarizing hillslope distance and river network distance.

The flow pathway in a hillslopes was assumed to be strongly influenced by slope and aspect of its surface and therefore, equal to the surface flow plane (Fig. 3). Hillslope cells, which were simultaneously saturated areas, were not used in the flow-length distribution. To determine \( w(x) \) of the saturated areas, the saturated area layer was overlain by the total flow distances layer, producing a flow-length distribution of the saturated areas to the basin outlet.

![Flow distance distributions](image)

**Fig. 4** Flow distance distributions for (a) hillslope and (b) saturated areas.
Both distributions differ markedly (Fig. 4). The hillslope distribution is nearly unimodal and the distances range from 0 to 1090 m with an arithmetic mean of 309 m. Due to a positive skewness, the median (285 m) is smaller than the arithmetic mean. In contrast, the saturated area distribution is multimodal and the distances range from 150 to 7000 m, with an arithmetic mean of 4340 m.

To account for increasing dispersivity by increasing flow distances, \( \alpha_i \) had to be transformed before it was used in the modelling. The change of \( \alpha_i \) for the channel network could be determined with the results of the tracer experiment. The relation between dispersivity and flow distance was best expressed by \( \alpha_i = 0.22 x^{0.41} \). Using equation (2) \( \alpha_i \) was transformed to \( P_D \) with respect to \( x \).

\[ \text{Fig. 5 Hillslope response for different flow velocities and dispersivities.} \]
The change of dispersivity for the hillslopes could not be determined due to the lack of results for different flow distances. In the case of the hillslope response, the dispersivity was held constant with respect to $x$. The total dispersion establishes after 50 m (one raster cell) and its effects decrease with increasing $x$.

RESULTS AND DISCUSSION

The effect of different $\alpha_i$ and $v$ on the GIUH considering only the hillslope $w(x)$ is shown in Fig. 5. The response functions are calculated for 1 mm effective precipitation. The dispersivity is less sensitive with respect to the shape of the hillslope response than to the flow velocity. For fixed flow velocities and varying dispersivities (Figs 5(a) and (b)), the resulting hydrograph changes are minor, while for fixed dispersivities, the variation of flow velocity yields markedly differing response functions (Figs 5(c) and (d)). The maximum decreases markedly with increasing velocity and tailing becomes more distinct. All response functions are unimodal.

The response functions of the saturated areas calculated for 1 mm effective precipitation and different flow velocities are shown in Fig. 6. The multimodal distribution of the saturated area $w(x)$ also produces a multimodal response function. Due to the small dispersion in the river network, the effects of the multimodal distribution are not removed. Although the flow distances for the saturated areas were one magnitude longer than those for the hillslopes, the response of the saturated areas only occurs for hours, while that for the hillslope response occurs for days, because $v$ is two orders of magnitude higher in the river network.

To validate the derived GIUH, the response of the hillslope and saturated areas was calculated for one precipitation runoff event. The hillslope response was calculated with a $v$ of 8.5 m h$^{-1}$ and a $\alpha_i$ of 39 m. For the saturated areas, a $v$ of 1200 m h$^{-1}$ and the regionalized $\alpha_i$ were used. All of these values were derived from the tracer experiments. The modelled direct runoff fits the measured direct runoff.

![Flow velocity:](image)

**Fig. 6** Response of saturated areas for different flow velocities.
exceptionally well (Fig. 7). The difference between predicted and observed values is high during the beginning of the event, but is low during the recession. The differences between predicted and observed hydrographs during the rising limb and peak may result from less volume of effective precipitation for the quick response of the saturated areas and slow response of the saturated areas. To account for the differences on the rising limb, a third process of runoff generation, which contributes to quick delivery of near-stream groundwater, may occur in the study catchment. Piston flow would produce a high runoff volume at the beginning of the event, caused by displaced groundwater. However, this process cannot be determined with dye tracer experiments. The slow response of saturated areas could be corrected by the use of $\nu$ determined during flood events, which would produce a quicker response due to higher flow velocities during high flow conditions.

**CONCLUSION**

The modelling results suggest that runoff generation on the hillslopes is the main process controlling the transit time distribution of water in a headwater basin. The dye tracer experiments provided valuable information for modelling catchment flow processes. Additional investigations using environmental tracers are needed to evaluate all contributing processes.

**Acknowledgement** The authors gratefully thank Daniel Cluis (INRS-EAU, University of Quebec, Canada) for providing and explaining the software system DEDNM.
REFERENCES


